Name: Schlansky

Common Core Geometry Regents Review Packet!

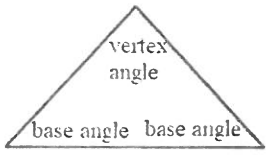
Mr. Schlansky

2025
~~2024~~

Isosceles Triangles with Vocabulary

Isosceles triangles have congruent sides opposite congruent angles. The congruent angles are called base angles and the non-congruent angle is called the vertex angle.

The angles of a triangle add to equal 180°.



1. In ΔXYZ, $\angle X$ and $\angle Z$ are the base angles. If $m\angle Z = 41°$, find the measure of $\angle Y$.

41 + 41 = 82

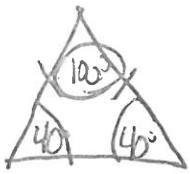
180 − 82 = 98

2. In ΔPQR, $\angle Q$ is the vertex angle. If $\angle Q = 94°$, find the measure of $\angle P$.

180 − 94 = 86

$\frac{86}{2} = 43$

3. An isosceles triangle has a base angle of 40°. What is the measure of the vertex angle?



40 + 40 = 80

180 − 80 = 100

4. An isosceles triangle has a vertex angle of 46°. What is the measure of one of the base angles?

180 − 46 = 134

$\frac{134}{2} = 67$

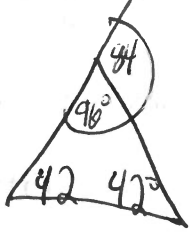
~~184~~ ~~136~~

2

5. The measure of one of the base angles of an isosceles triangle is 42°. The measure of an exterior angle at the vertex of the triangle is

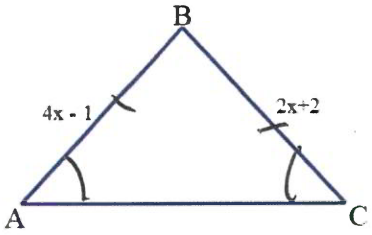
1) 42°
2) 84°
3) 96°
4) 138°

180 − 96 = 84



42 + 42 = 84 ; 180 − 84 = 96

6. In ΔABC, $\angle A$ and $\angle C$ are the base angles. Find $\overline{BC}$



$4x-1=2x+2$

$-2x \quad -2x$

$2x-1=2$

$+1 \quad +1$

$2x=3$

$\frac{2x}{2}=\frac{3}{2}$

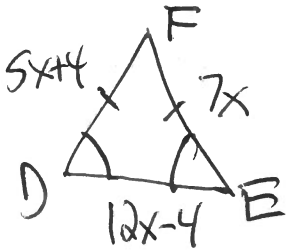
$x=1.5$

$BC=2x+2$

$BC=2(1.5)+2$

$BC=5$

7. In ΔDEF, $\angle F$ is the vertex angle. If $\overline{DF}=5x+4$, $\overline{DE}=12x-4$, and $\overline{EF}=7x$, find $\overline{DE}$.



$5x+4=7x$

$-5x \quad -5x$

$4=2x$

$\frac{4}{2}=\frac{2x}{2}$

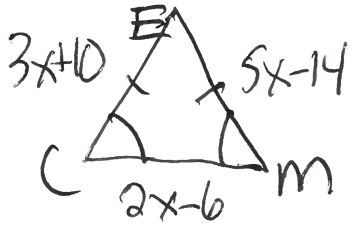
$2=x$

$DE=12x-4$

$DE=12(2)-4$

$DE=20$

8. In triangle CEM, $CE = 3x + 10$, $ME = 5x - 14$, and $CM = 2x - 6$. Determine and state the value of x that would make CEM an isosceles triangle with the vertex angle at E.



$3x+10=5x-14$

$-3x \quad -3x$

$10=2x-14$

$+14 \quad +14$

$24=2x$

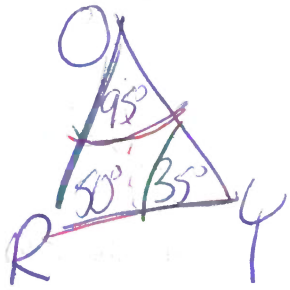
$\frac{24}{2}=\frac{2x}{2}$

$12=x$

~~180~~ ~~134~~

Largest/Smallest Sides/Angles in a Triangle
The largest side is opposite the largest angle
The smallest side is opposite the smallest angle

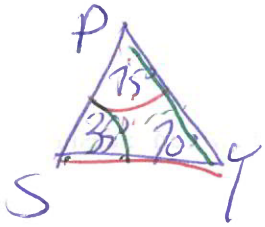
1. In ΔROY, $m\angle R = 50°$ and $m\angle O = 95°$. What is the largest angle of ΔROY? What is the smallest angle of ΔROY?



95 180
+50 −145
145 35

largest $\overline{RY}$
Smallest $\overline{OR}$

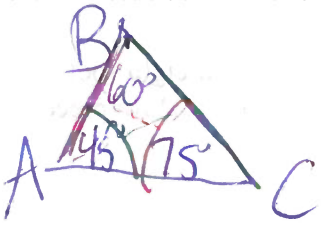
2. In triangle SPY, $m\angle S = 35°$ and $m\angle Y = 70°$. What is the largest side of triangle SPY?



70 180
+35 −105
105 75

largest $\overline{SY}$
Smallest $\overline{PY}$

3. In ΔABC, $m\angle A = 45$ and $m\angle B = 60$. What is the largest side of ΔABC ? What is the smallest side of ΔABC ?

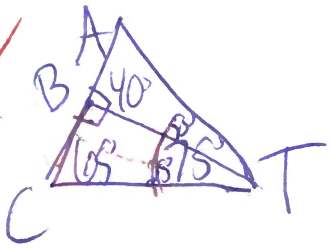


60 180
+45 −105
105 75

Smallest $\overline{BC}$
largest $\overline{AB}$

4. In ΔCAT, $m\angle C = 65$, $m\angle A = 40$, and B is a point on side $\overline{CA}$, such that $\overline{TB} \perp \overline{CA}$. Which line segment is shortest?

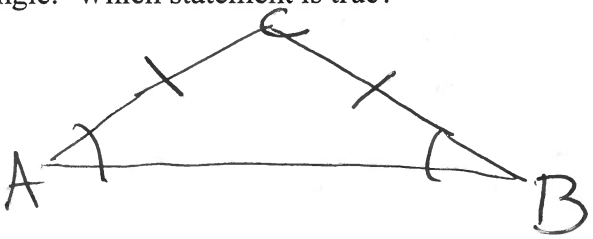
1) $\overline{CT}$ ✗
2) $\overline{BC}$ ✓
3) $\overline{TB}$ ✗
4) $\overline{AT}$ ✗



65 180
+40 −105
105 75

5. In $\triangle ABC$, $\angle A \cong \angle B$ and $\angle C$ is an obtuse angle. Which statement is true?

1) $\overline{AC} \cong \overline{AB}$ and $\overline{BC}$ is the longest side.
2) $\overline{AC} \cong \overline{BC}$ and $\overline{AB}$ is the longest side.
3) $\overline{AC} \cong \overline{AB}$ and $\overline{BC}$ is the shortest side.
4) $\overline{AC} \cong \overline{BC}$ and $\overline{AB}$ is the shortest side.



6. In $\triangle ABC$, $m\angle A = 60$, $m\angle B = 80$, and $m\angle C = 40$. Which inequality is true?

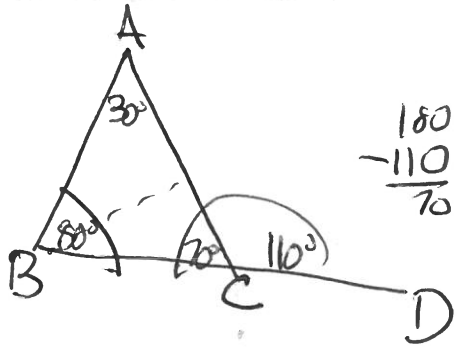
1) $AB > BC$ S > M
2) $AC > BC$ L > M
3) $AC < BA$ L < S
4) $BC < BA$ M < S

AB = small
BC = ~~medium~~ middle
AC = largest

7. In $\triangle ABC$, side $\overline{BC}$ is extended through C to D. If $m\angle A = 30°$ and $m\angle ACD = 110°$, what is the longest side of $\triangle ABC$?

1) $\overline{AC}$
2) $\overline{BC}$
3) $\overline{AB}$
4) $\overline{CD}$

70 + 30 = 100
180 − 100 = 80°

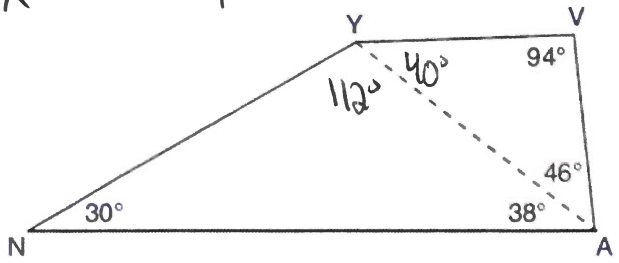


180 − 110 = 70

8. In the diagram of quadrilateral $NAVY$ below, $m\angle YNA = 30°$, $m\angle YAN = 38°$, $m\angle AVY = 94°$, and $m\angle VAY = 46°$.
Which segment has the shortest length?

1) $\overline{AY}$ ~~SAS~~ $\triangle AYN$ ~~$\cong \triangle AYV$~~
2) $\overline{NY}$ $m\triangle AYN$
3) $\overline{VA}$ $S\triangle AYV$
4) ~~$\overline{VY}$~~ $m\triangle AYV$

30 + 38 = 68
180 − 68 = 112



94 + 46 = 140
180 − 140 = 40

Name Schlansky
Mr. Schlansky

Date _____
Geometry

Triangle Inequality Theorem

1. Which of the following cannot make up the three sides of a triangle?

1) {3,5,4} 3+4>5 ✓
2) {2,2,3} 2+2>3 ✓
3) {9,7,5} 5+7>9 ✓
4) {6,1,4} 1+4>6 ✗

2. Which of the following can make up the three sides of a triangle?

1) {2,4,2} 2+2>4 ✗
2) {1,7,4} 1+4>7 ✗
3) {8,1,6} 1+6>8 ✗
4) {5,5,7} 5+5>7 ✓

3. Which numbers could represent the lengths of the sides of a triangle?

1) 5,9,14 5+9>14 ✗
2) 7,7,15 7+7>15 ✗
3) 1,2,4 1+2>4 ✗
4) 3,6,8 3+6>8 ✓

4. Which of the following cannot make up the three sides of a triangle?

1) {5,1,6} 1+5>6 ✗
2) {9,14,8} 8+9>14 ✓
3) {3,5,6} 3+5>6 ✓
4) {7,10,4} 4+7>10 ✓

5. Which set of numbers represents the lengths of the sides of a triangle?

1) {5, 18, 13} 5+13>18 ✗
2) {6, 17, 22} 6+17>22 ✓
3) {16, 24, 7} 7+16>24 ✗
4) {26, 8, 15} 8+15>26 ✗

6. Which of the following cannot make up the three sides of a triangle?

1) {3,9,7} 3+7>9 ✓
2) {2,7,5} 2+5>7 ✗
3) {8,12,15} 8+12>15 ✓
4) {9,3,7} 3+7>9 ✓

7. In ΔBLA, $\overline{BL} = 12$ and $\overline{AL} = 8$. What is a possible values of $\overline{BA}$?

1) 17 $8+12>17$ ✓
2) 4
3) 2
4) 21

$12-8=4$ $12+8=20$

8. In ΔCAM, $\overline{CM} = 10$ and $\overline{CA} = 4$. What is a possible value of $\overline{MA}$?

1) 4
2) 6
3) 15
4) 13 $4+10>13$

$10-4=6$ $10+4=14$

9. In ΔABC, $AB = 5$ feet and $BC = 3$ feet. Which *cannot* represent the value for the length of $\overline{AC}$, in feet?

1) 3
2) 5
3) 7
4) 9 $3+5>9$ ✗

$5-3=2$ $5+3=8$

10. Two sides of a triangle are 7 and 11. The third side of the triangle can measure:

1) 4
2) 18
3) 8 $7+8>11$
4) 21

$11-7=4$ $11+7=18$

11. Jacquie is building a triangular fence for her tomato garden. She has an eight foot piece of fence and a four foot piece of fence. Which can be the length of the third piece of fence?

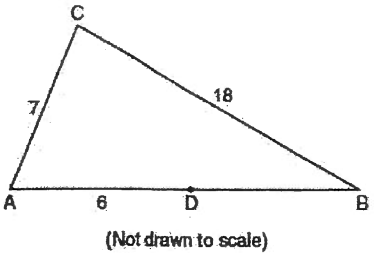
1) 2 feet
2) 5 feet $4+5>8$
3) 4 feet
4) 12 feet

$8-4=4$ $8+4=12$

12. In the diagram below of ΔABC, D is a point on $\overline{AB}$, $AC = 7$, $AD = 6$, and $BC = 18$. The length of $\overline{DB}$ could be

1) $5+6=11$
2) $12+6=18$
3) $19+6=25$
4) $25+6=31$

$11+18>25$



(Not drawn to scale)

$18-7=11$ $18+7=25$

Name Schlansky
Mr. Schlansky

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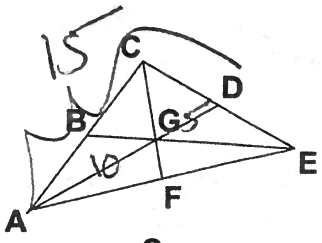
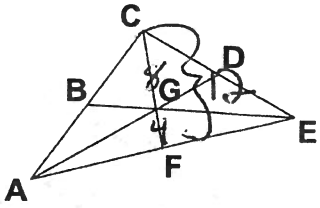
Geometry

Intersecting Medians (Centroid Problems)

1. In the given triangle, all three medians are drawn in. If $\overline{AG} = 10$, find
   a) $\overline{GD}$ 5
   b) $\overline{AD}$ 15

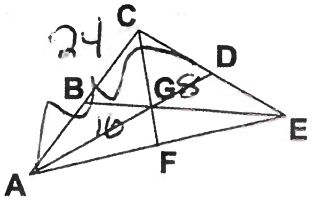


2. In the given triangle, all three medians are drawn in. If $\overline{FG} = 4$, find
   a) $\overline{CG}$ 8
   b) $\overline{CF}$ 12

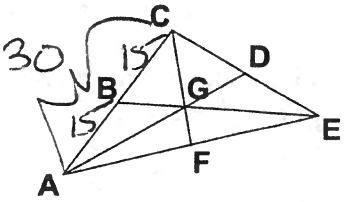


3. In the given triangle, all three medians are drawn in. If $\overline{AD} = 24$, find
   a) $\overline{AG}$ 16
   b) $\overline{DG}$ 8

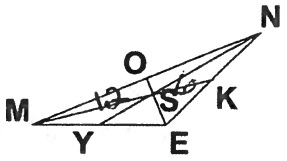
$2x+1x=24$
$\frac{3x}{3}=\frac{24}{3}$
$x=8$



4. In the given triangle, all three medians are drawn in. If $\overline{AC} = 30$, find
   a) $\overline{AB}$ 15
   b) $\overline{BC}$ 15

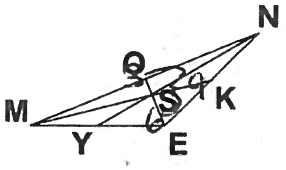


5. In the given triangle, all three medians are drawn in. If $\overline{MS} = 12$, find
   a) $\overline{SK}$ 6
   b) $\overline{MK}$ 18



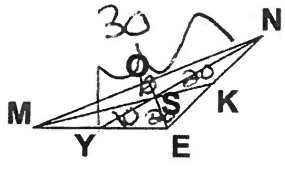
6. In the given triangle, all three medians are drawn in. If $\overline{OE} = 9$, find
   a) $\overline{OS}$ 3
   b) $\overline{SE}$ 6

$2x+1x=9$
$\frac{3x}{3}=\frac{9}{3}$
$x=3$

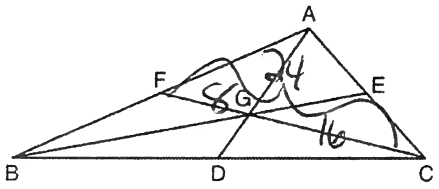


7. In the given triangle, all three medians are drawn in. If $\overline{YN} = 30$, find
   a) $\overline{YS}$ 10
   b) $\overline{SN}$ 20

$2x+1x=30$
$\frac{3x}{3}=\frac{30}{3}$
$x=10$



8. In the diagram below of $\triangle ABC$, medians $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ intersect at G. If $CF = 24$, what is the length of $\overline{FG}$?



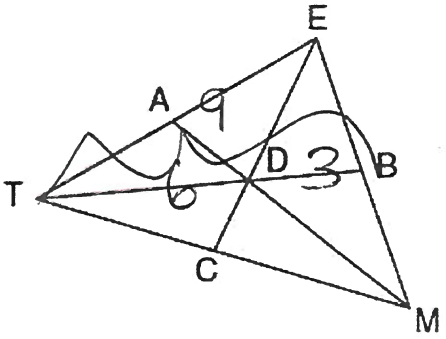
$2x+1x=24$

$\frac{3x}{3}=\frac{24}{3}$

$x=8$

$\overline{FG}=8$

9. In the diagram below of $\triangle TEM$, medians $\overline{TB}$, $\overline{EC}$, and $\overline{MA}$ intersect at D, and $TB = 9$. Find the length of $\overline{TD}$.



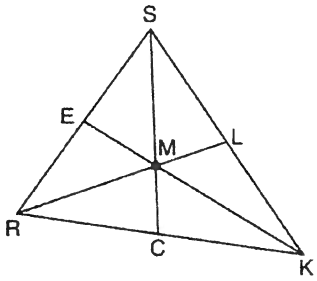
$2x+1x=9$

$\frac{3x}{3}=\frac{9}{3}$

$x=3$

$\overline{TD}=6$

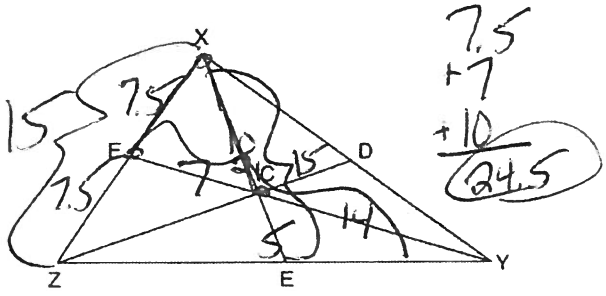
10. In triangle SRK below, medians $\overline{SC}$, $\overline{KE}$, and $\overline{RL}$ intersect at M.



Which statement must always be true?

1) $3(MC) = SC$ ✓
2) $MC = \frac{1}{3}(SM)$ ✗
3) $RM = 2MC$ ✗
4) $SM = KM$ ✗

11. In $\triangle XYZ$, shown below, medians $\overline{XE}$, $\overline{YF}$, and $\overline{ZD}$ intersect at C. If $CE = 5$, $YF = 21$, and $XZ = 15$, determine and state the perimeter of triangle CFX.



7.5
+7
+10
24.5

$2x+1x=21$

$\frac{3x}{3}=\frac{21}{3}$

$x=7$

Performing Transformations

Reflections

Flip (Count to what you are reflecting over)

*Switch the coordinates for reflection over $y = x$

y = # is horizontal line, x = # is vertical line. You must graph these lines before you can reflect over them.

Rotations

$R_{90} = (-y, x)$

$R_{180} = (-x, -y)$

$R_{270} = (y, -x)$

Translation

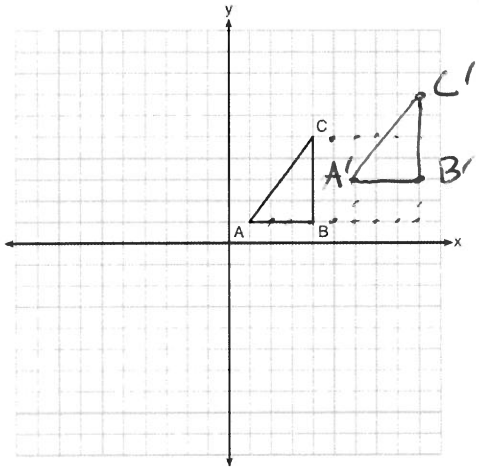
Slide. Count out the translation on the grid

Dilations

If centered at the origin: multiply the coordinates by the scale factor

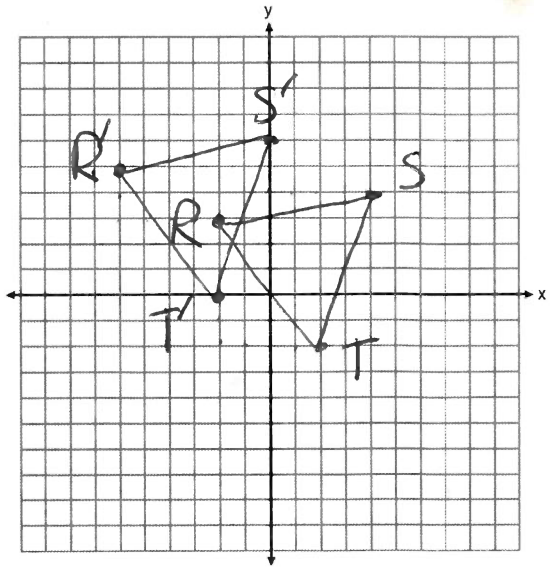
If centered at a point: Count from the center to each point the number of times of the scale factor.

1. In the diagram below, $\triangle ABC$ has coordinates $A(1, 1)$, $B(4, 1)$, and $C(4, 5)$. Graph and the image of $\triangle ABC$ after the translation five units to the right and two units up.

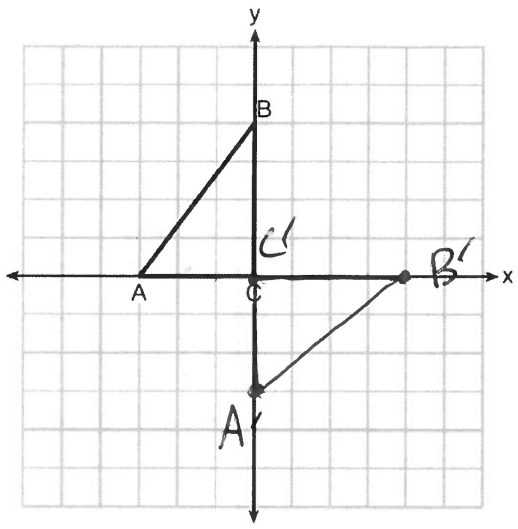


2. The coordinates of the vertices of $\triangle RST$ are $R(-2, 3)$, $S(4, 4)$, and $T(2, -2)$. Graph $\triangle RST$.

Graph and label $\triangle R'S'T'$, the image of $\triangle RST$ after a translation 4 units to the left and 2 units up.



3. Triangle ABC is graphed on the set of axes below. Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a reflection over the line $y = x$.



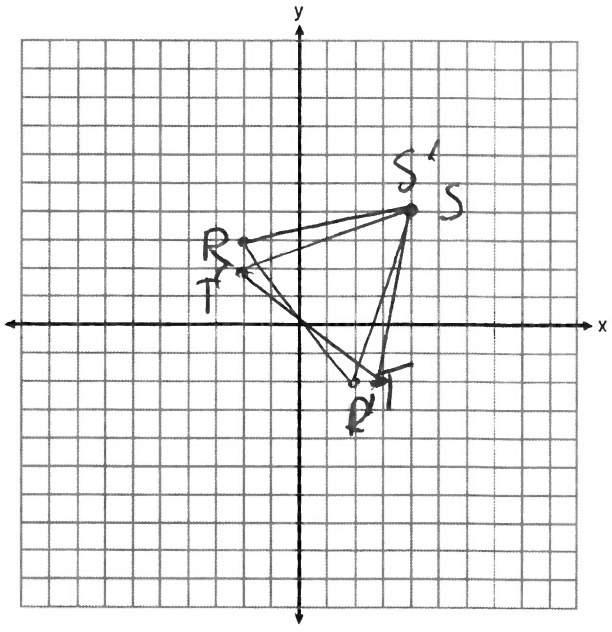
— Switch x and y

A (−3,0) → (0,−3) A'
B (0,4) → (4,0) B'
C (0,0) → (0,0) C'

4. The coordinates of the vertices of $\triangle RST$ are $R(-2,3)$, $S(4,4)$, and $T(2,-2)$. Graph $\triangle RST$. Graph and label $\triangle R'S'T'$, the image of $\triangle RST$ after a reflection in the line $y = x$.

Switch x and y

R(−2,3) → (3,−2) R'
S(4,4) → (4,4) S'
T(2,−2) → (−2,2) T'

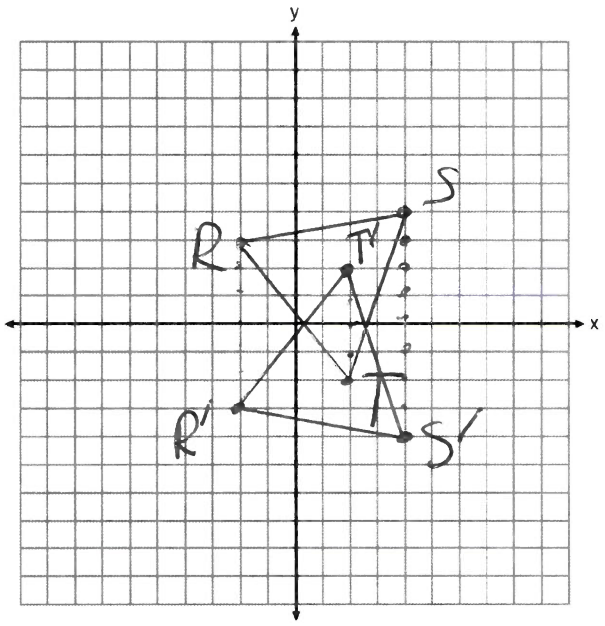


5. The coordinates of the vertices of $\triangle RST$ are $R(-2,3)$, $S(4,4)$, and $T(2,-2)$. Graph $\triangle RST$. Graph and label $\triangle R'S'T'$, the image of $\triangle RST$ after a reflection in x-axis.

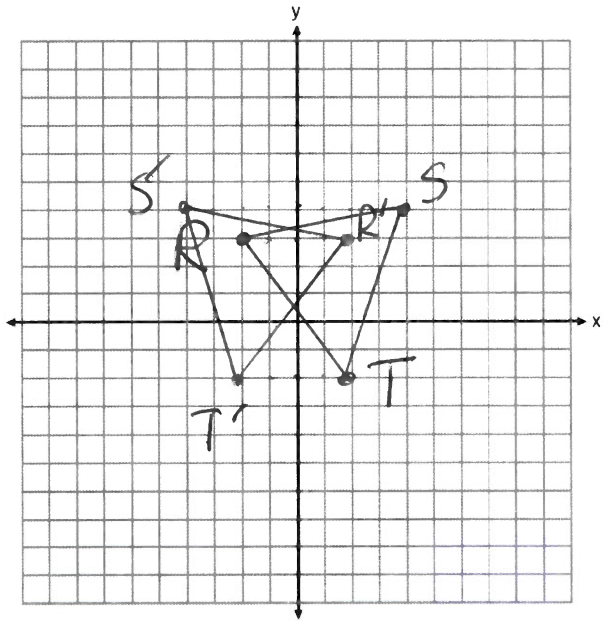
$R(-2,3) \rightarrow (-2,-3)$

$S(4,4) \rightarrow (4,-4)$

$T(2,-2) \rightarrow (2,2)$

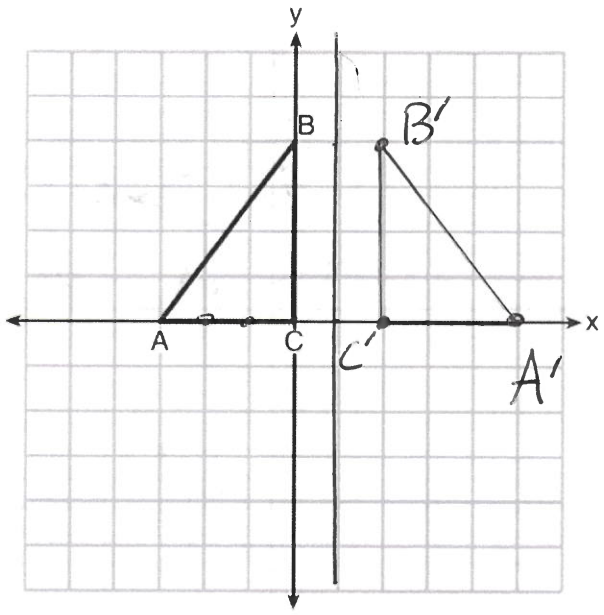


6. The coordinates of the vertices of $\triangle RST$ are $R(-2,3)$, $S(4,4)$, and $T(2,-2)$. Graph $\triangle RST$. Graph and label $\triangle R'S'T'$, the image of $\triangle RST$ after a reflection in y-axis.



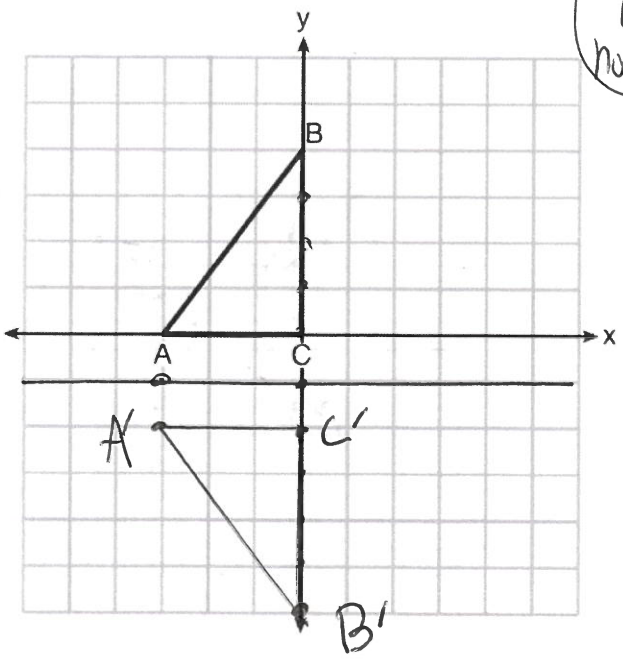
7. Triangle ABC is graphed on the set of axes below. Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a reflection over the line $x = 1$.

4H XV vertical



8. Triangle ABC is graphed on the set of axes below. Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$ after a reflection over the line $y = -1$.

4H XV horizontal



$R_{-270}^{CW} = R_{90}^{CCW}(x,y) = (-y,x)$

$R_{-180} = R_{180}(x,y) = (-x,-y)$

$R_{-90} = R_{270}(x,y) = (y,-x)$

Switch and negate:

$(x,y) \rightarrow (-y,x)$ 90

$(-x,-y)$ 180

$(+y,-x)$ 270

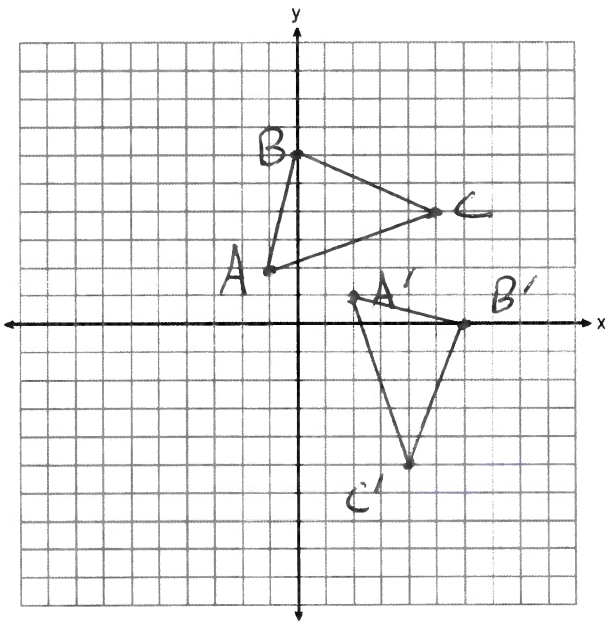
9. On the accompanying set of axes, graph $\triangle ABC$ with coordinates $A(-1,2)$, $B(0,6)$, and $C(5,4)$. Then graph $\triangle A'B'C'$, the image of $\triangle ABC$ after a counter-clockwise rotation of 270 centered at the origin.

$R_{270}(x,y) = (y,-x)$

$A(-1,2) \xrightarrow{y,-x} (2,1)A'$

$B(0,6) \rightarrow (6,0)B'$

$C(5,4) \rightarrow (4,-5)C'$



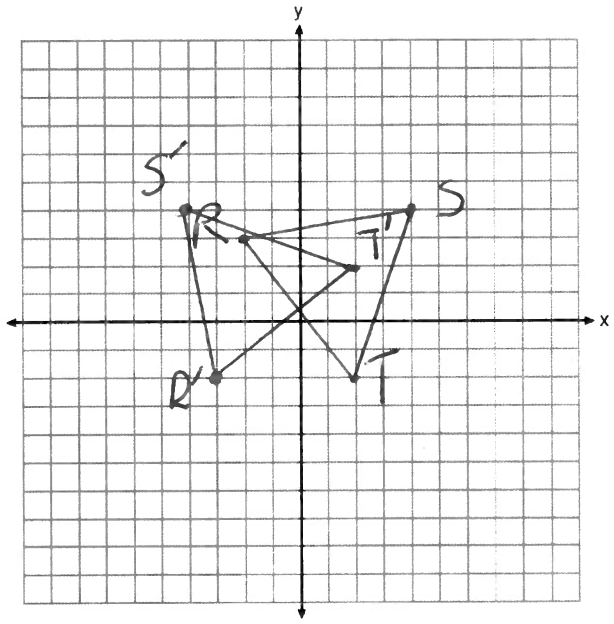
10. The coordinates of the vertices of $\triangle RST$ are $R(-2,3)$, $S(4,4)$, and $T(2,-2)$. Graph $\triangle RST$. Graph and label $\triangle R'S'T'$, the image of $\triangle RST$ after a counter-clockwise rotation of 90 centered at the origin.

$R_{90}(x,y) = (-y,x)$

$R(-2,3) \xrightarrow{-y,x} (-3,-2)R'$

$S(4,4) \rightarrow (-4,4)S'$

$T(2,-2) \rightarrow (2,2)T'$



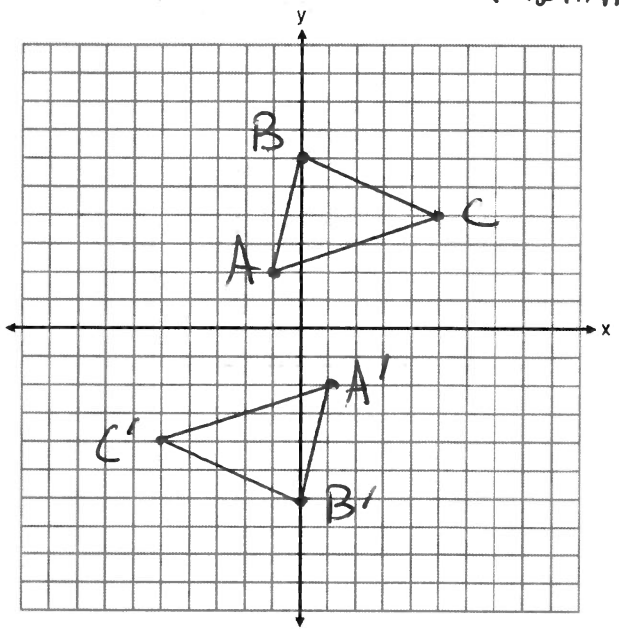
11. On the accompanying set of axes, graph $\triangle ABC$ with coordinates $A(-1,2)$, $B(0,6)$, and $C(5,4)$. Then graph $\triangle A'B'C'$, the image of $\triangle ABC$ after a clockwise rotation of 180 centered at the origin.

$R_{-180}(x,y) = (-x,-y)$

$A(-1,2) \xrightarrow{-x,-y} (1,-2)\ A'$

$B(0,6) \rightarrow (0,-6)\ B'$

$C(5,4) \rightarrow (-5,-4)\ C'$



12. The coordinates of the vertices of $\triangle RST$ are $R(-2,3)$, $S(4,4)$, and $T(2,-2)$. Graph $\triangle RST$. Graph and label $\triangle R'S'T'$, the image of $\triangle RST$ after a clockwise rotation of 90 centered at the origin.

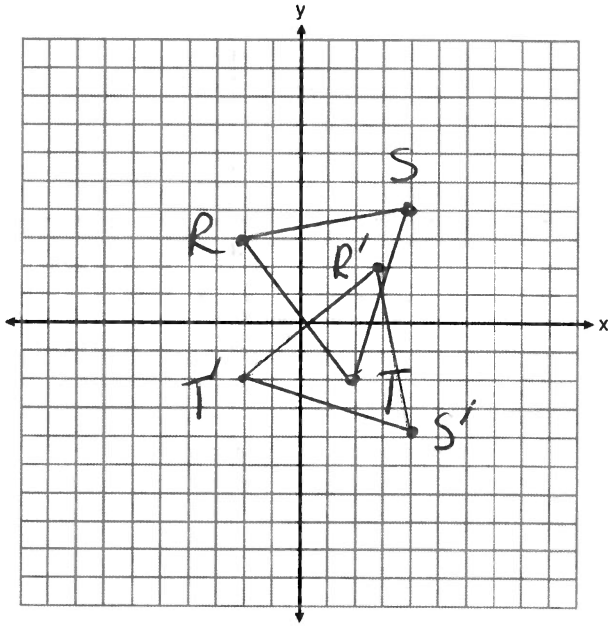
R_{-90}

$R_{-90}(x,y) = (y,-x)$

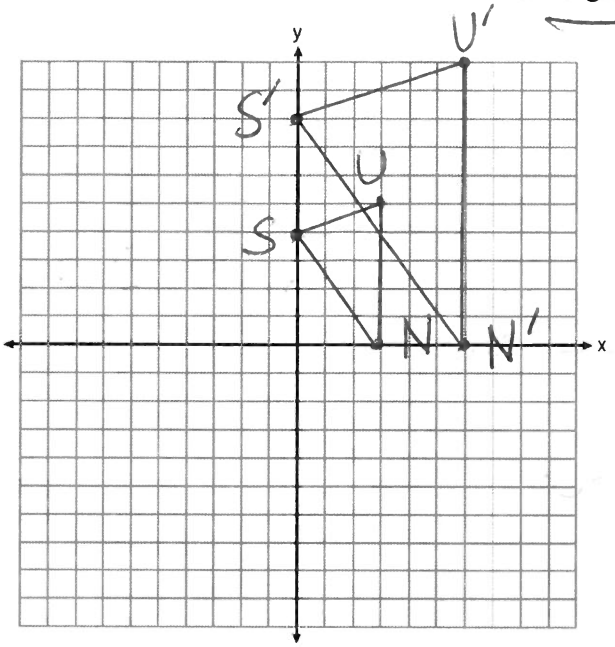
$R(-2,3) \xrightarrow{y,-x} (3,2)\ R'$

$S(4,4) \rightarrow (4,-4)\ S'$

$T(2,-2) \rightarrow (-2,-2)\ T'$



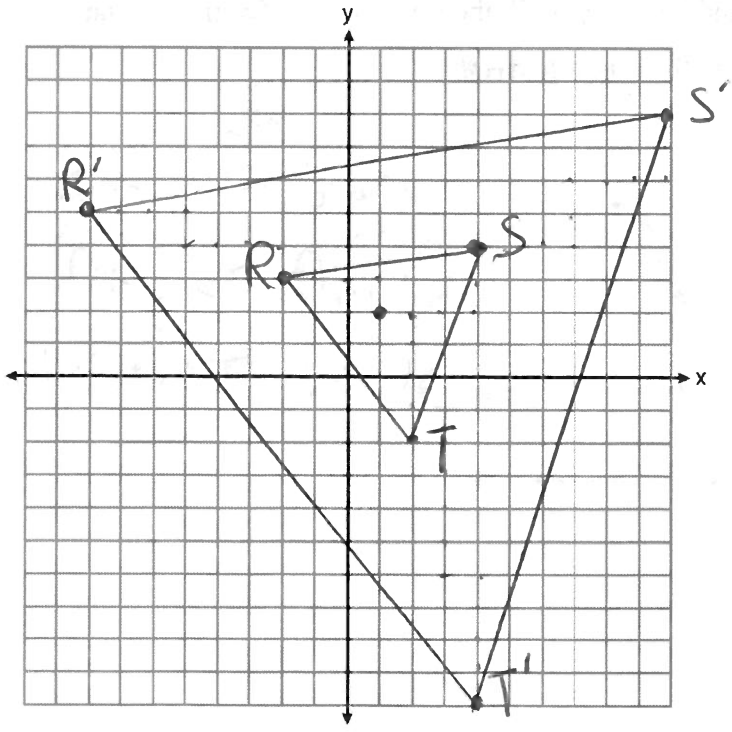
13. Triangle SUN has coordinates $S(0,4)$, $U(3,5)$, and $N(3,0)$. On the accompanying grid, draw and label $\triangle SUN$. Then, graph and state the coordinates of $\triangle S'U'N'$, the image of $\triangle SUN$ after a dilation of 2 centered at the origin. multiply



$S(0,4) \xrightarrow{\cdot 2} (0,8)\ S'$
$U(3,5) \rightarrow (6,10)\ U'$
$N(3,0) \rightarrow (6,0)\ N'$

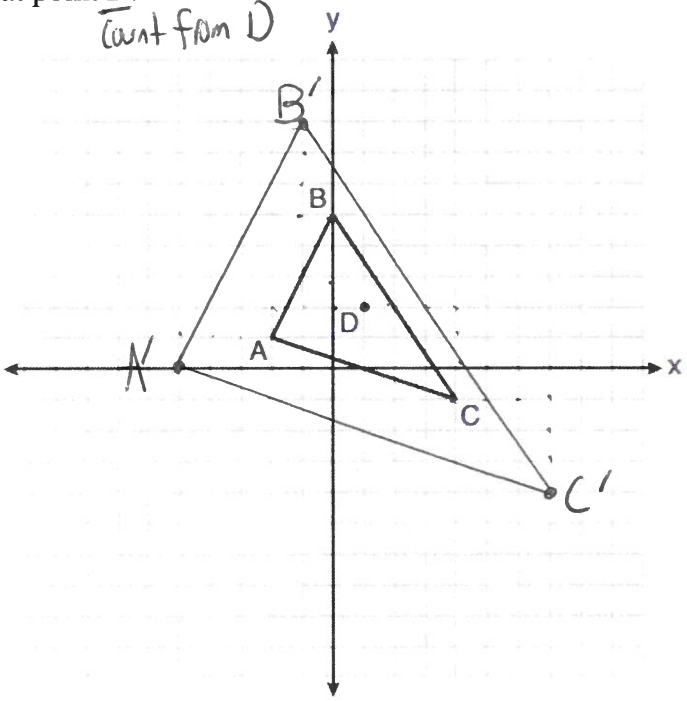
14. The coordinates of the vertices of $\triangle RST$ are $R(-2,3)$, $S(4,4)$, and $T(2,-2)$. Graph $\triangle RST$ and $\triangle R'S'T'$, the image of $\triangle RST$ after a dilation of 3 centered at (1,2).

count from the center

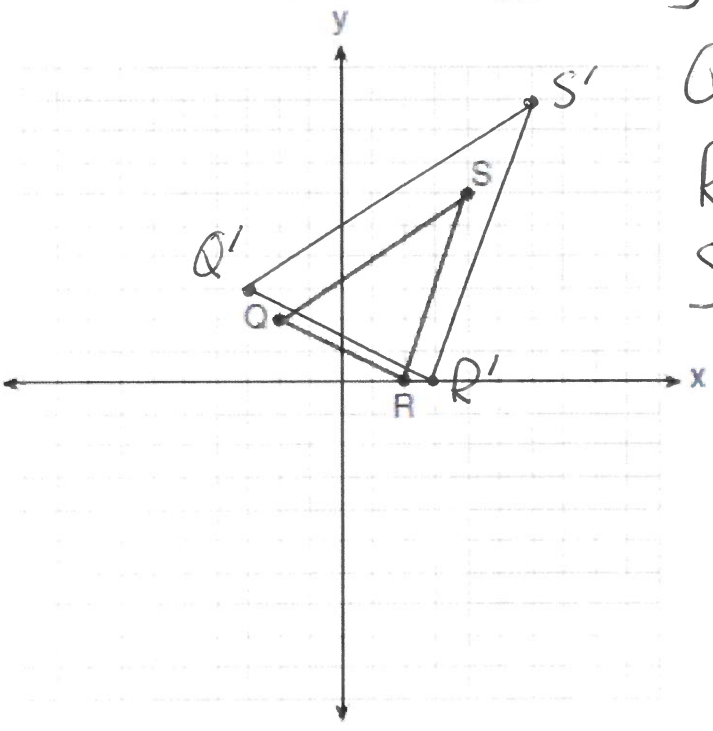


15. Triangle ABC and point $D(1, 2)$ are graphed on the set of axes below.
Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$, after a dilation of scale factor 2 centered at point D.

count from D



16. Triangle QRS is graphed on the set of axes below.
On the same set of axes, graph and label $\triangle Q'R'S'$, the image of $\triangle QRS$ after a dilation with a scale factor of $\frac{3}{2}$ centered at the origin. multiply



$\cdot \frac{3}{2}$

$Q(-2,2) \rightarrow (-3,3)$

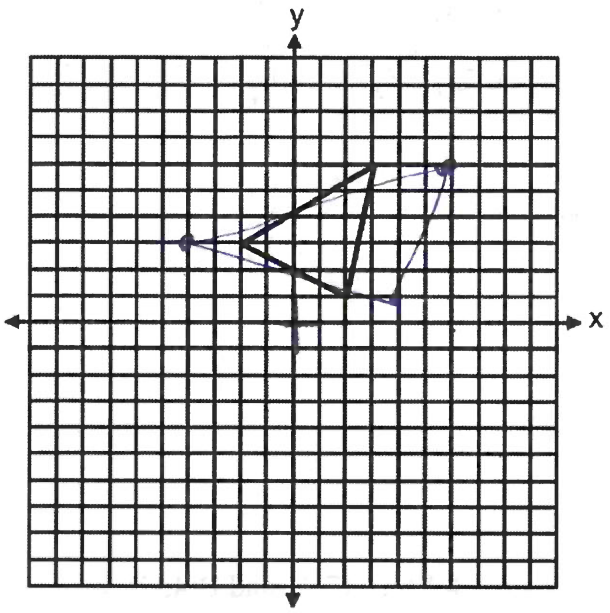
$R(2,0) \rightarrow (3,0)$

$S(4,6) \rightarrow (6,9)$

Vertical and Horizontal Dilations

Multiply the given variable by the given scale factor. Leave the other variable.

1. A triangle with vertices at (-2,3), (3,6), and (2,1), is graphed on the set of axes below. A horizontal stretch of scale factor 2 with respect to x = 0 is represented by $(x, y) \rightarrow (2x, y)$. Graph the image of this triangle, after the horizontal stretch on the same set of axes.



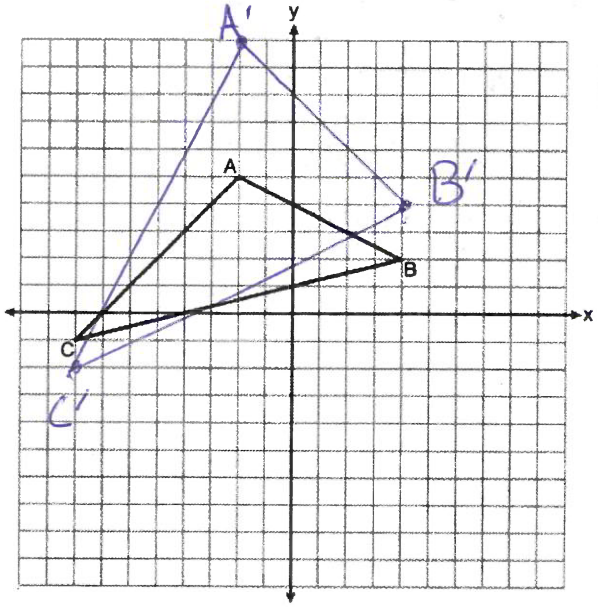
$(x,y) \Rightarrow (2x,y)$

$(-2,3) \rightarrow (-4,3)$

$(3,6) \rightarrow (6,6)$

$(2,1) \rightarrow (4,1)$

2. The triangle graphed below with vertices at A(-2,5), B(4,2), and C(-8,-1), is graphed on the set of axes below. A vertical stretch of scale factor 2 with respect to y = 0 is represented by $(x, y) \rightarrow (x, 2y)$. Graph the image of this triangle, after the vertical stretch on the same set of axes.



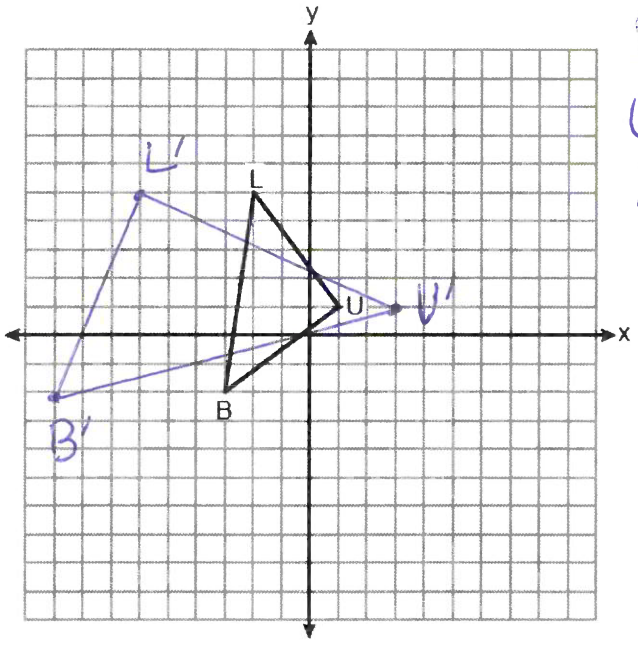
$(x,2y)$

$A(-2,5) \rightarrow (-2,10)\ A'$

$B(4,2) \rightarrow (4,4)\ B'$

$C(-8,-1) \rightarrow (-8,-2)\ C'$

3. The triangle graphed below with vertices at B(-3,-2), U(1,1), and L(-2,5), is graphed on the set of axes below. A horizontal stretch of scale factor 3 with respect to x = 0 is represented by $(x, y) \rightarrow (3x, y)$. Graph the image of this triangle, after the horizontal stretch on the same set of axes.

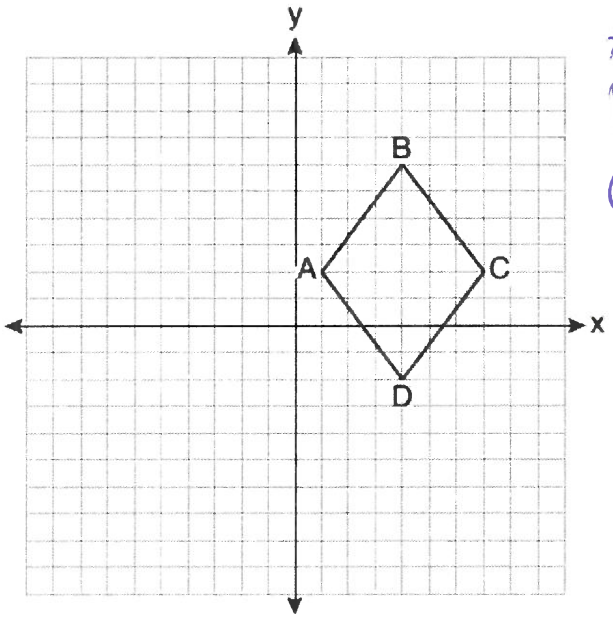


$B(-3,-2) \xrightarrow{(3x,y)} (-9,-2)\ B'$

$U(1,1) \rightarrow (3,1)\ U'$

$L(-2,5) \rightarrow (-6,5)\ L'$

4. The rhombus graphed below with vertices at A(1,2), B(4,6), C(7,2), and D(4,-2), is graphed on the set of axes below. A vertical shrink of scale factor $\frac{1}{2}$ with respect to y = 0 is represented by $(x, y) \rightarrow (x, \frac{1}{2}y)$. Graph the image of this rhombus, after the vertical shrink on the same set of axes.



$A(1,2) \xrightarrow{(x,\frac{1}{2}y)} (1,1)$

$B(4,6) \rightarrow (4,3)$

$C(7,2) \rightarrow (7,1)$

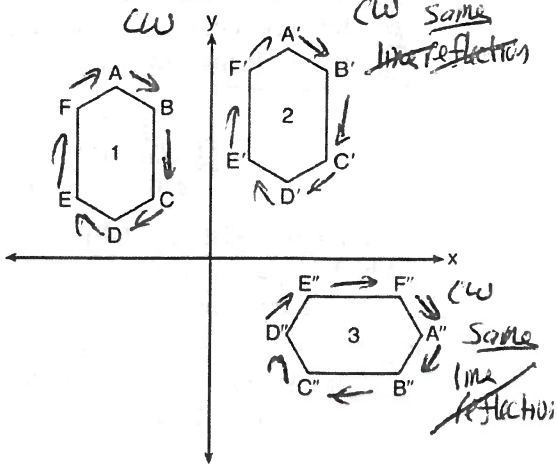
$D(4,-2) \rightarrow (4,-1)$

Name Schlansky
Mr. Schlansky

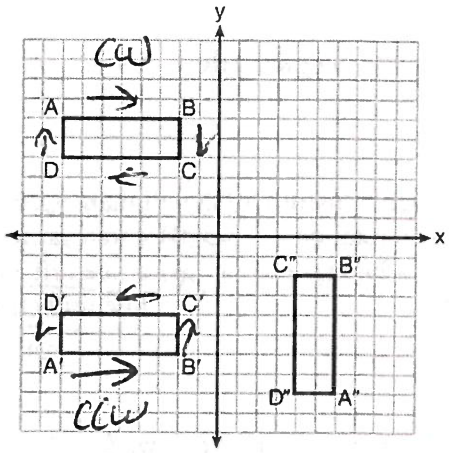
Date _____
Geometry

Identifying Sequences of Rigid Motions Multiple Choice

1. In the diagram below, congruent figures 1, 2, and 3 are drawn.
Which sequence of transformations maps figure 1 onto figure 2 and then figure 2 onto figure 3?
1) a line reflection followed by a translation
2) a point reflection followed by a translation
3) a translation followed by a reflection
4) a translation followed by a rotation

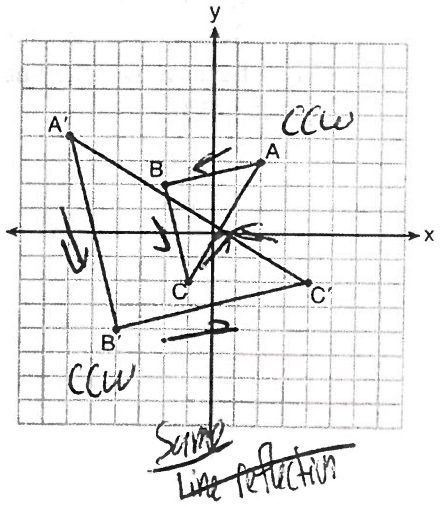


2. A sequence of transformations maps rectangle *ABCD* onto rectangle *A"B"C"D"*, as shown in the diagram below.
Which sequence of transformations maps *ABCD* onto *A'B'C'D'* and then maps *A'B'C'D'* onto *A"B"C"D"*?
1) a line reflection followed by a rotation
2) a line reflection followed by a translation
3) a translation followed by a rotation
4) a translation followed by a line reflection

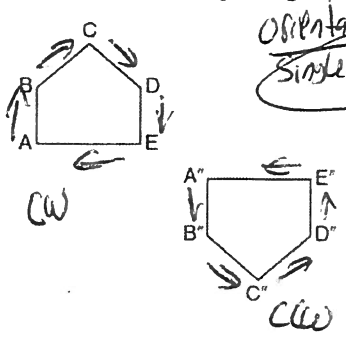


different
single line reflection

3. Which sequence of transformations will map $\triangle ABC$ onto $\triangle A'B'C'$?
1) line reflection and translation
2) point reflection and line reflection
3) translation and dilation
4) dilation and rotation



4. Identify which sequence of transformations could map pentagon $ABCDE$ onto pentagon $A''B''C''D''E''$, as shown below.



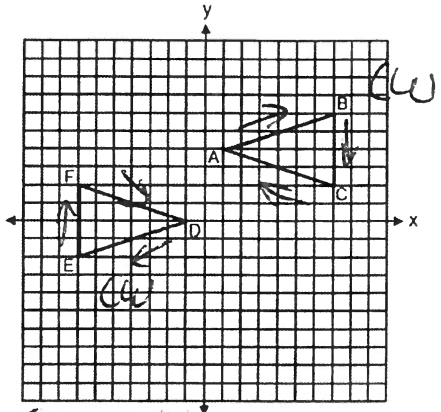
orientation different
Single line reflection

1) dilation followed by a rotation
2) translation followed by a rotation
3) line reflection followed by a translation
4) line reflection followed by a line reflection

double line reflection
keeps orientation

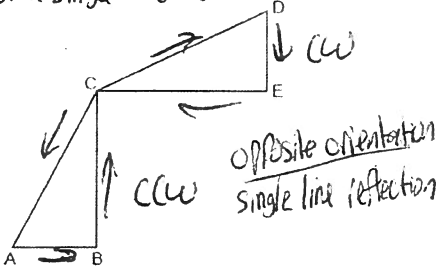
5. Triangles ABC and DEF are graphed on the set of axes below.
Which sequence of rigid motions maps ΔABC onto ΔDEF ?

1) A reflection over $y = -x + 2$
2) A point reflection through (0,2)
3) A translation 2 units left followed by a reflection over the x-axis
4) A translation 4 units down folowed by a reflection over the y-axis



Same orientation
not a single line reflection

6. In the diagram below, $\triangle ABC \cong \triangle DEC$.



opposite orientation
single line reflection

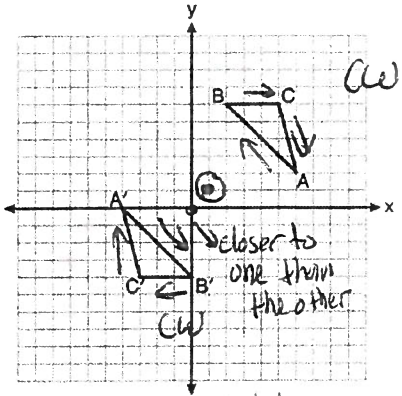
Which transformation will map $\triangle ABC$ onto $\triangle DEC$?

1) a rotation
2) a line reflection
3) a translation followed by a dilation
4) a line reflection followed by a second line reflection

not a single line reflection

7. On the set of axes below, $\triangle ABC \cong \triangle A'B'C'$.
Triangle ABC maps onto $\triangle A'B'C'$ after a

1) reflection over the line $y = -x$
2) reflection over the line $y = -x + 2$
3) point reflection through (1,1)
4) rotation of 180° centered at the origin

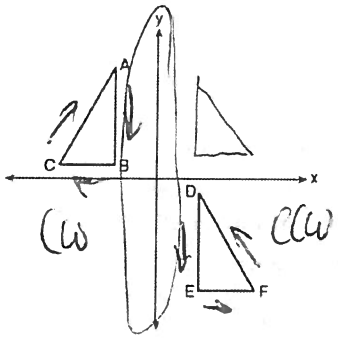


closer to one than the other

Same orientation
not a single reflection

3 2 1

8. In the diagram below, $\triangle ABC \cong \triangle DEF$.



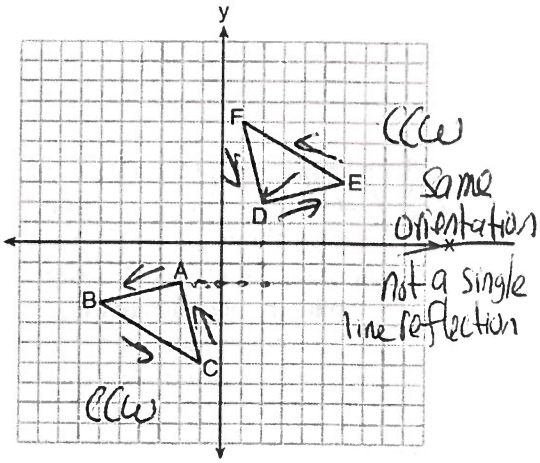
Opposite orientation
Single line reflection

Which sequence of transformations maps $\triangle ABC$ onto $\triangle DEF$?

1) a reflection over the x-axis followed by a translation
2) a reflection over the y-axis followed by a translation
3) a rotation of 180° about the origin followed by a translation
4) a counterclockwise rotation of 90° about the origin followed by a translation

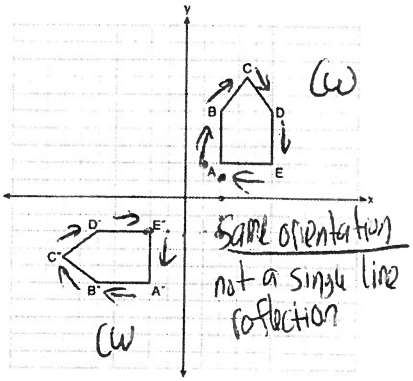
9. Triangle ABC and triangle DEF are graphed on the set of axes below.
Which sequence of transformations maps triangle ABC onto triangle DEF?

1) a reflection over the x-axis followed by a reflection over the y-axis
2) a point reflection through the origin followed by a reflection over the line $y = x$
3) a 90° clockwise rotation about the origin followed by a reflection over the y-axis
4) a translation 8 units to the right and 1 unit up followed by a 90° counterclockwise rotation about the origin



10. On the set of axes below, pentagon $ABCDE$ is congruent to $A''B''C''D''E''$.
Which describes a sequence of rigid motions that maps $ABCDE$ onto $A''B''C''D''E''$?

1) a rotation of 90° counterclockwise about the origin followed by a reflection over the x-axis
2) a rotation of 90° counterclockwise about the origin followed by a translation down 7 units
3) a reflection over the y-axis followed by a reflection over the x-axis doesn't work
4) a reflection over the x-axis followed by a rotation of 90° counterclockwise about the origin

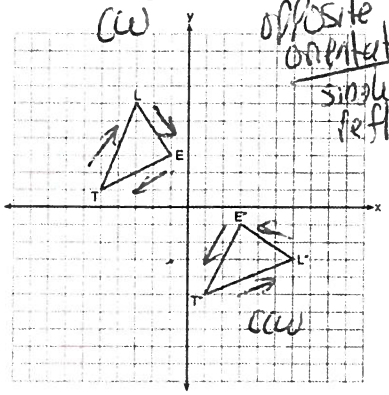


11. On the set of axes below, $\triangle LET$ and $\triangle L"E"T"$ are graphed in the coordinate plane where $\triangle LET \cong \triangle L"E"T"$.

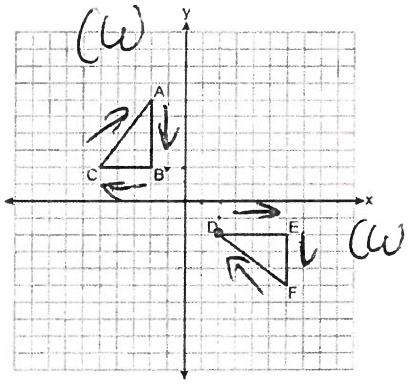
Which sequence of rigid motions maps $\triangle LET$ onto $\triangle L"E"T"$?

1) a reflection over the y-axis followed by a reflection over the x-axis
2) a rotation of 180° about the origin
3) a rotation of 90° counterclockwise about the origin followed by a reflection over the y-axis
4) a reflection over the x-axis followed by a rotation of 90° clockwise about the origin

doesn't work



12. On the set of axes below, congruent triangles ABC and DEF are drawn.



Same orientation
not a single line reflection

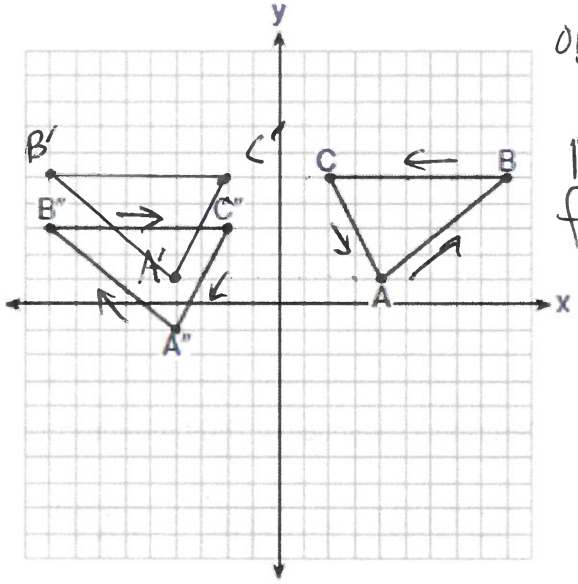
Which sequence of transformations maps $\triangle ABC$ onto $\triangle DEF$?

1) A counterclockwise rotation of 90 degrees about the origin, followed by a translation 8 units to the right.
2) A counterclockwise rotation of 90 degrees about the origin, followed by a reflection over the y-axis.
3) A point reflection through the origin, followed by a translation 4 units down.
4) A clockwise rotation of 90 degrees about the origin, followed by a reflection over the x-axis.

doesn't work

Identifying Transformations (Open Response)
CHECK FOR ORIENTATION!!!!
Same orientation (rotation first, then translation)
-Rotate any point the appropriate degree measure and direction.
-Translate the rest of the way by counting from that point to its image.
Opposite orientation (reflection first, then translation)
-Reflect over the appropriate axis (use y=x if it needs to be reflected diagonally)
-Translate the rest of the way by counting from any new point to its image.

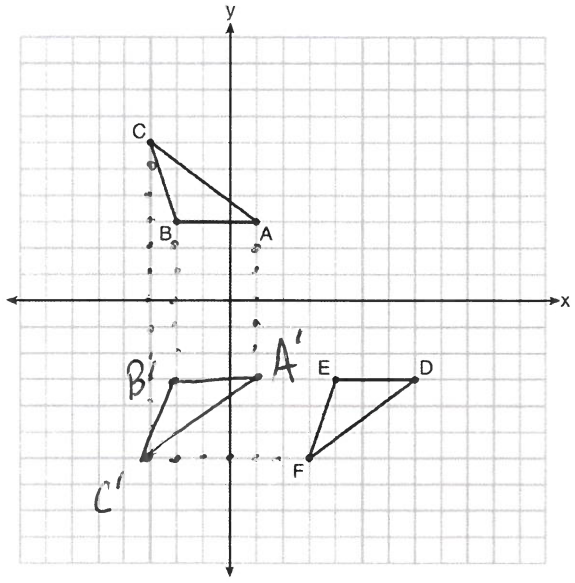
1. The graph below shows $\triangle ABC$ and its image, $\triangle A''B''C''$.
Describe a sequence of rigid motions which would map $\triangle ABC$ onto $\triangle A''B''C''$.



opposite orientation
reflection

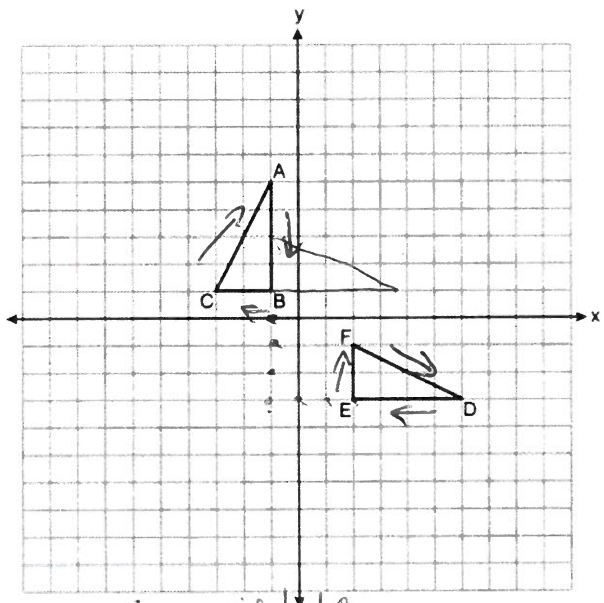
1) Reflect $\triangle ABC$ over the y-axis followed by a translation 2 units down

2. Describe a sequence of transformations that will map $\triangle ABC$ onto $\triangle DEF$ as shown below.



1) reflect $\triangle ABC$ over the x-axis followed by a translation 6 units to the right.

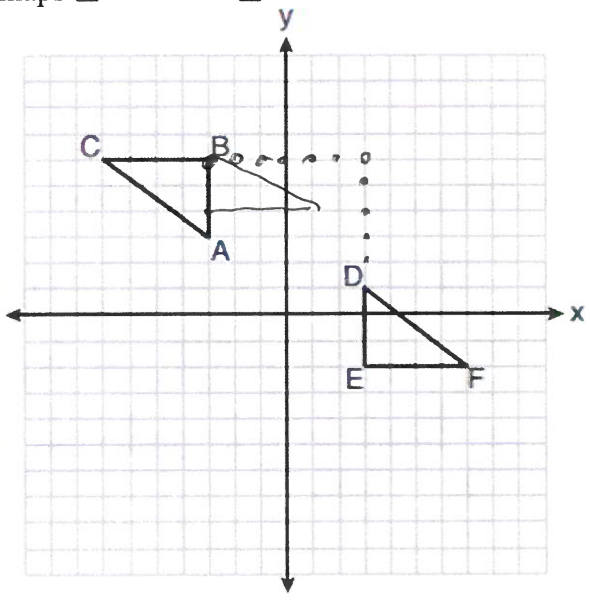
3. On the set of axes below, $\triangle ABC$ and $\triangle DEF$ are graphed. Describe a sequence of rigid motions that would map $\triangle ABC$ onto $\triangle DEF$.



rotate $\triangle ABC$ 90° clockwise centered at B followed by a translation 4 units down and 3 units right.

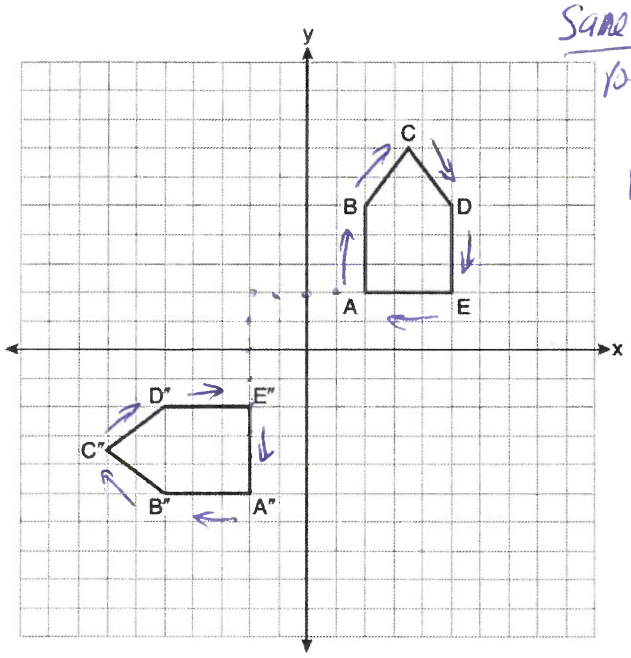
Same orientation rotation

4. On the set of axes below, $\triangle ABC \cong \triangle DEF$. Describe a sequence of rigid motions that maps $\triangle ABC$ onto $\triangle DEF$.



rotate $\triangle ABC$ 180° counter-clockwise centered at B followed by a translation 6 units right and 5 units down

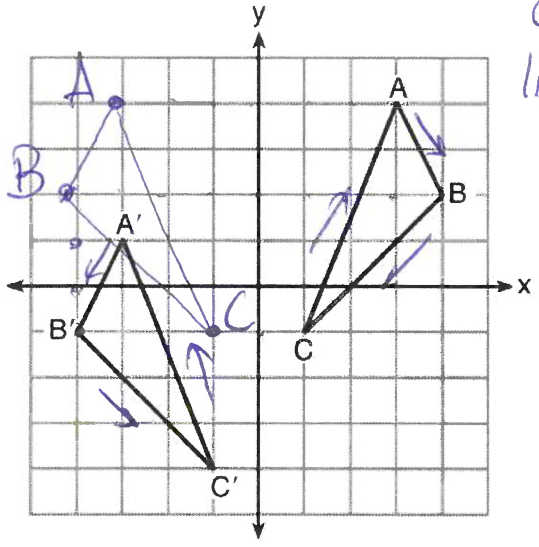
5. On the set of axes below, pentagon *ABCDE* is congruent to *A"B"C"D"E"*. Describe a sequence of rigid motions that maps pentagon *ABCDE* onto *A"B"C"D"E"*.



Same orientation
rotation

rotate ABCDE 90° counter-clockwise centered at A followed by a translation left 4 and down 7.

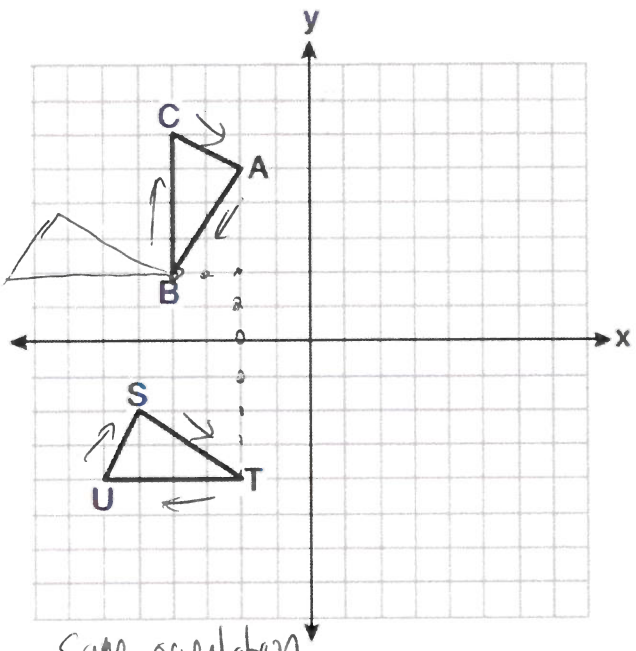
6. As graphed on the set of axes below, $\triangle A'B'C'$ is the image of $\triangle ABC$ after a sequence of transformations.



different orientation
line reflection

reflect ΔABC over the y-axis followed by a translation 3 units down.

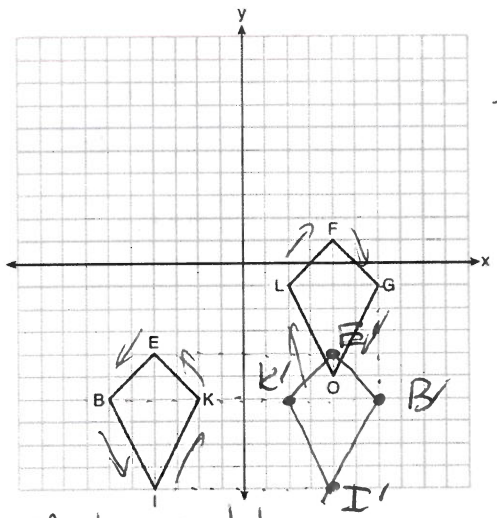
7. On the set of axes below, $\triangle ABC \cong \triangle STU$. Describe a sequence of rigid motions that maps $\triangle ABC$ onto $\triangle STU$.



rotate $\triangle ABC$ 90° counterclockwise centered at B followed by a translation 2 right and 6 down.

Same orientation
rotation

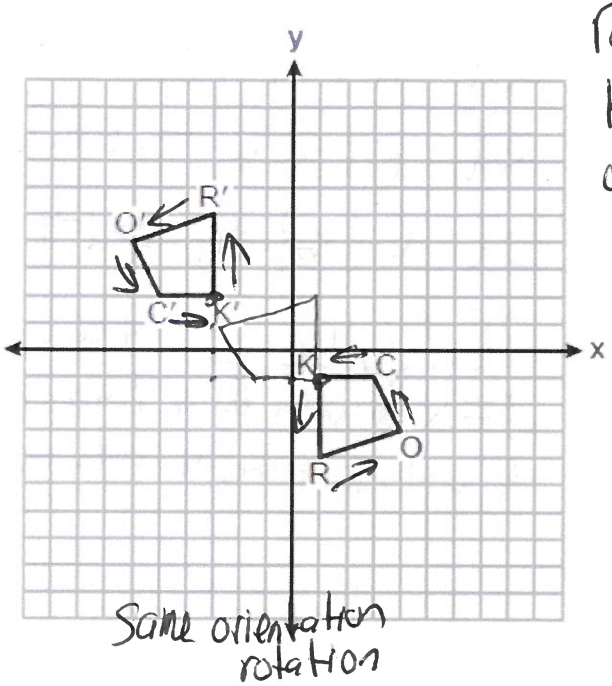
8. Quadrilaterals *BIKE* and *GOLF* are graphed on the set of axes below. Describe a sequence of transformations that maps quadrilateral *BIKE* onto quadrilateral *GOLF*.



Reflect BIKE over the y-axis followed by a translation 5 units up

Opposite orientation
reflection

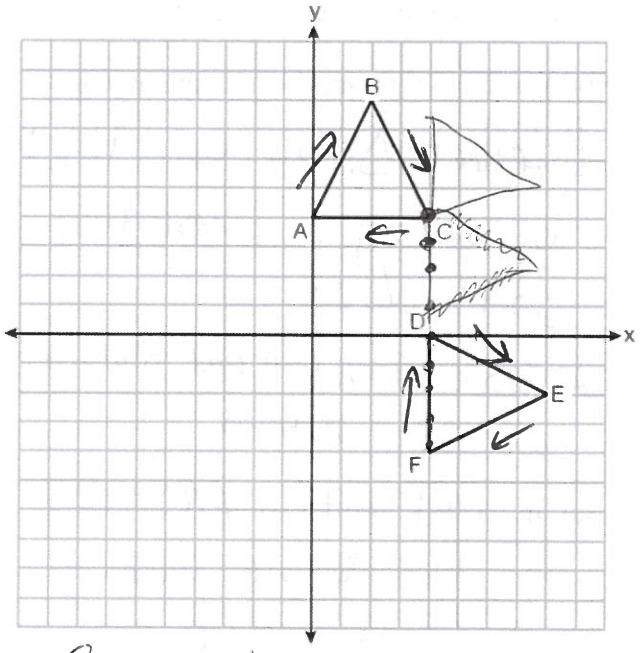
9. On the set of axes below, congruent quadrilaterals *ROCK* and *R'O'C'K'* are graphed. Describe a sequence of transformations that would map quadrilateral *ROCK* onto quadrilateral *R'O'C'K'*.



Rotate ROCK 180° centered at K followed by a translation 4 left and 3 up

Same orientation rotation

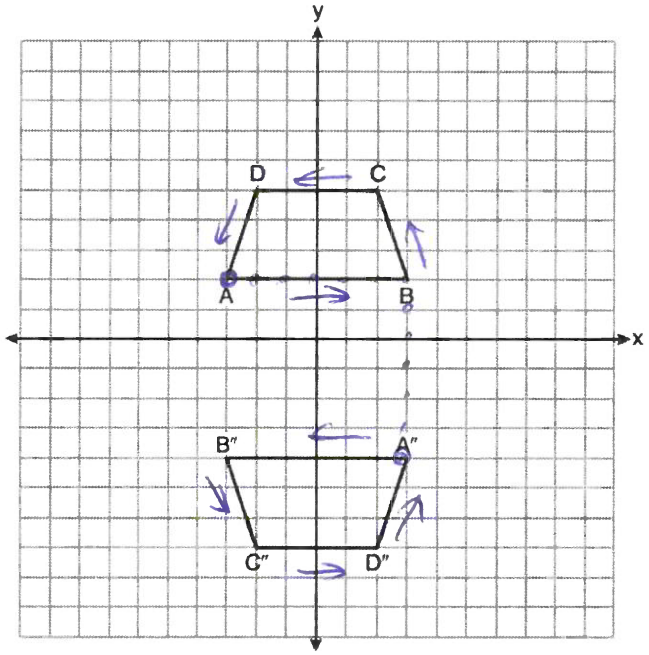
10. Triangles *ABC* and *DEF* are graphed on the set of axes below.



Rotate △ABC 90° clockwise centered at C followed by a translation 8 units down

Same orientation rotation

11. Trapezoids *ABCD* and *A"B"C"D"* are graphed on the set of axes below. Describe a sequence of transformations that maps trapezoid *ABCD* onto trapezoid *A"B"C"D"*.

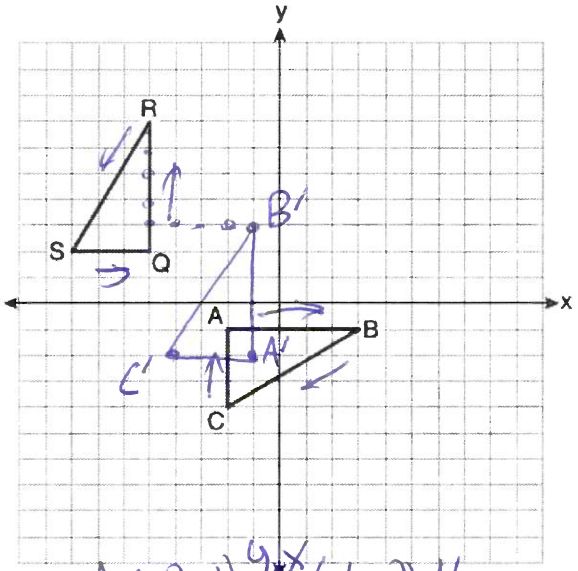


Same orientation

rotation

rotate ABCD 180° clockwise centered at A followed by a translation right 6 and down 6.

12. On the set of axes below, $\triangle ABC$ is graphed with coordinates $A(-2,-1)$, $B(3,-1)$, and $C(-2,-4)$. Triangle QRS, the image of $\triangle ABC$, is graphed with coordinates $Q(-5,2)$, $R(-5,7)$, and $S(-8,2)$. Describe a sequence of transformations that would map $\triangle ABC$ onto $\triangle QRS$.



opposite orientation

reflection

* line of reflection must be diagonal (y=x)

reflect ΔABC over y=x followed by a translation left 4 and up 4.

A (-2,-1) → (-1,-2) A'
B (3,-1) → (-1,3) B'
C (-2,-4) → (-4,-2) C'

Rigid Motion Properties
A rigid motion preserves size and angle measure producing a congruent figure
They all produce a congruent figure except dilation.

1. Which transformation would *not* always produce an image that would be congruent to the original figure?
1) translation 2) dilation 3) rotation 4) reflection

2. The vertices of $\triangle JKL$ have coordinates $J(5,1)$, $K(-2,-3)$, and $L(-4,1)$. Under which transformation is the image $\triangle J'K'L'$ *not* congruent to $\triangle JKL$?
1) a translation of two units to the right and two units down
2) a counterclockwise rotation of 180 degrees around the origin
3) a reflection over the x-axis
4) a dilation with a scale factor of 2 and centered at the origin

3. If $\triangle A'B'C'$ is the image of $\triangle ABC$, under which transformation will the triangles *not* be congruent?
1) reflection over the x-axis
2) translation to the left 5 and down 4
3) dilation centered at the origin with scale factor 2
4) rotation of 270° counterclockwise about the origin

4. Under which transformation would $\triangle A'B'C'$, the image of $\triangle ABC$, *not* be congruent to $\triangle ABC$?
1) reflection over the y-axis
2) rotation of 90° clockwise about the origin
3) translation of 3 units right and 2 units down
4) dilation with a scale factor of 2 centered at the origin

5. The image of $\triangle DEF$ is $\triangle D'E'F'$. Under which transformation will he triangles *not* be congruent?
1) a reflection through the origin
2) a reflection over the line $y = x$
3) a dilation with a scale factor of 1 centered at (2, 3)
4) a dilation with a scale factor of $\frac{3}{2}$ centered at the origin

6. The vertices of $\triangle PQR$ have coordinates $P(2, 3)$, $Q(3, 8)$, and $R(7, 3)$. Under which transformation of $\triangle PQR$ are distance and angle measure preserved?
1) $(x,y) \rightarrow (2x, 3y)$ 2) $(x,y) \rightarrow (x+2, 3y)$ 3) $(x,y) \rightarrow (2x, y+3)$ 4) $(x,y) \rightarrow (x+2, y+3)$

dilations dilation dilation

7. Which transformation would result in the perimeter of a triangle being different from the perimeter of its image?
1) $(x,y) \rightarrow (y,x)$
2) $(x,y) \rightarrow (x,-y)$
3) $(x,y) \rightarrow (4x, 4y)$
4) $(x,y) \rightarrow (x+2, y-5)$

dilations

Rigid Motion Proofs
To prove triangles are congruent using rigid motions/transformations
1) A _____ and _____ are rigid motions.
2) **A rigid motion preserves size and angle measure producing a congruent figure.**

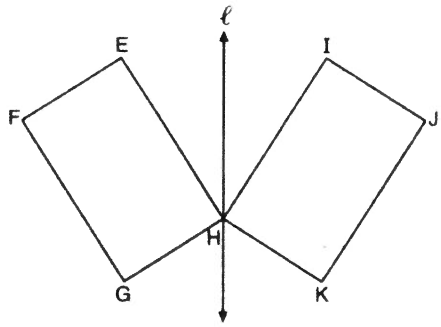
1. Triangle $A'B'C'$ is the image of triangle ABC after a translation of 2 units to the right and 3 units up. Is triangle ABC congruent to triangle $A'B'C'$? Explain why.

Yes, a translation is a rigid motion. A rigid motion preserves size and angle measure producing a congruent figure.

2. After a reflection over a line, $\Delta A'B'C'$ is the image of ΔABC. Explain why triangle ABC is congruent to triangle $\Delta A'B'C'$.

A reflection is a rigid motion. A rigid motion preserves size and angle measure producing a congruent figure.

3. In the diagram below, parallelogram $EFGH$ is mapped onto parallelogram $IJKH$ after a reflection over line ℓ. Use the properties of rigid motions to explain why parallelogram $EFGH$ is congruent to parallelogram $IJKH$.



A reflection is a rigid motion. A rigid motion preserves size and angle measure producing a congruent figure.

4. The image of triangle ABC after a rotation of 200 degrees clockwise centered at the point (3,-1) is triangle DEF. Are the triangles congruent? Use the properties of rigid motions to explain your answer.

Yes, a rotation is a rigid motion. A rigid motion preserves size and angle measure producing a congruent figure.

Regular Polygon Rotations

To determine the minimum number of degrees a regular polygon must be rotated to be mapped onto itself:

1) The minimum rotation is $\frac{360}{n}$.
2) **Any multiple of that will also map the regular polygon onto itself!**

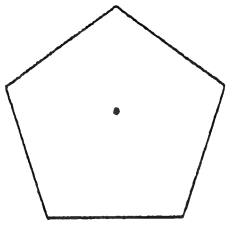
1. What is the minimum number of degrees a regular decagon must be rotated to be mapped onto itself?

$\frac{360}{n}$ $\frac{360}{10} = 36°$

2. What is the minimum number of degrees a regular hexagon must be rotated to be carried onto itself?

$\frac{360}{n}$ $\frac{360}{6} = 60°$

3. A regular pentagon is shown in the diagram below.



If the pentagon is rotated clockwise around its center, the minimum number of degrees it must be rotated to carry the pentagon onto itself is

1) 54°
2) 72°
3) 108°
4) 360°

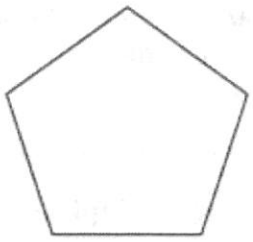
$\frac{360}{n}$ $\frac{360}{5} = 72°$

4. Which regular polygon has a minimum rotation of 45° to carry the polygon onto itself?

(1) octagon $\frac{360}{8} = 45$ (3) hexagon $\frac{360}{6} = 60$

(2) decagon $\frac{360}{10} = 36$ (4) pentagon $\frac{360}{5} = 72$

5. The regular polygon below is rotated about its center.
Which angle of rotation will carry the figure onto itself?



1) 60°
2) 108°
3) 216° $\frac{216}{72}=3$ $\frac{360}{5}=72$
4) 540°

6. Which rotation would map a regular hexagon onto itself?

1) 45°
2) 150°
3) 240° $\frac{240}{60}=4$ $\frac{360}{6}=60$
4) 315°

7. Which rotation about its center will carry a regular decagon onto itself?

1) 54°
2) 162°
3) 198°
4) 252° $\frac{360}{10}=36$ $\frac{252}{36}=7$

8. Which rotation about its center will carry a regular octagon onto itself?

1) 80°
2) 315° $\frac{315}{45}=7$ $\frac{360}{8}=45$
3) 280°
4) 120°

9. Which of the following rotations would not map a regular pentagon onto itself?

1) 144
2) 120
3) 216
4) 720 $\frac{360}{5}=72$

10. Which of the following rotations would not map an equilateral triangle onto itself?

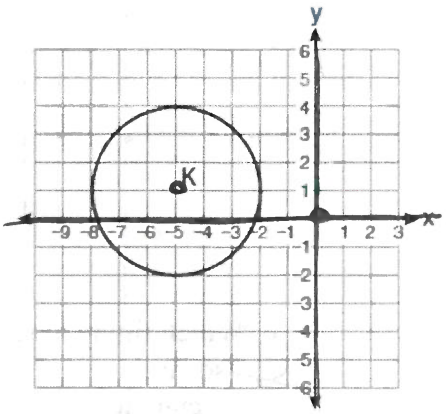
1) 120°
2) 240°
3) 180°
4) 480° $\frac{360}{3}=120$

11. Which figure will not carry onto itself after a 120-degree rotation about its center?

1) equilateral triangle $\frac{360}{3}=120$ ✓
2) regular hexagon $\frac{360}{6}=60(2)=120$
3) regular octagon $\frac{360}{8}=45$ ✗
4) regular nonagon $\frac{360}{9}=40(3)=120$ ✓

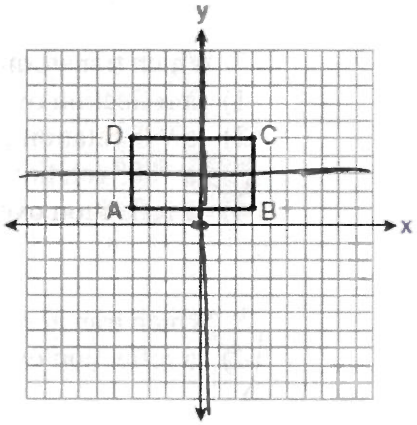
To map a shape onto itself:
Translation/Dilation: Never.
Reflection: **The line of reflection must be a line of symmetry** (cuts shape in half).
Rotation: **Center of rotation must be the center of the shape.** Use common sense for degree measure.

1. Circle K is shown in the graph below.
Which of the following transformations map circle K onto itself?



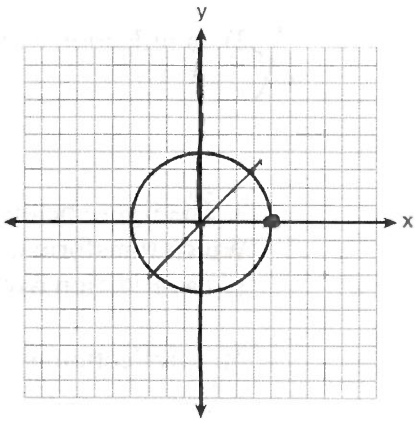
1) Reflection over the line $x-axis$ ✗
2) Reflection over the $y-axis$ ✗
3) Rotation of 90 centered at the origin ✗
4) Rotation of 90 centered at K ✓

2. On the set of axes below, Geoff drew rectangle $ABCD$.



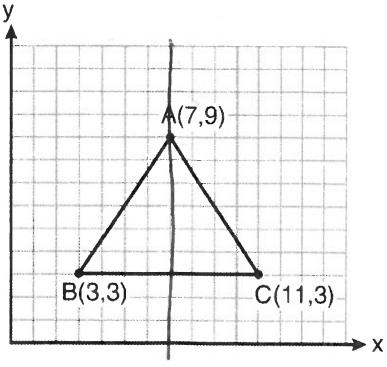
What of the following transformations would map the rectangle onto itself?
1) Reflection over the y axis ✗
2) Reflection over the line $y=3$ ✓
3) Rotation of 180 centered at the origin ✗
4) Translation one unit to the right ✗

3. In the diagram below, which transformation does *not* map the circle onto itself?



1) Rotation of 80 centered at the origin ✓
2) Reflection over the line $y=0$ ✓
3) Rotation of 180 centered at (4,0) ✗
4) Reflection over the line $x=0$ ✓

4. The vertices of the triangle in the diagram below are $A(7,9)$, $B(3,3)$, and $C(11,3)$.
Which transformation will map ΔABC onto itself?

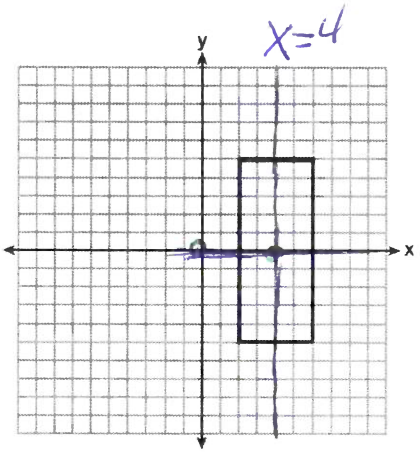


1) Rotation of 60 centered at (3,3)
2) Reflection over the line $y=5$
3) Reflection over the line $x=7$ ✓
4) Translation 3 units up

5. As shown in the graph below, the quadrilateral is a rectangle. Which transformation would *not* map the rectangle onto itself?

1) a reflection over the x-axis
2) a reflection over the line $x = 4$
3) a rotation of 180° about the origin
4) a rotation of 180° about the point $(4, 0)$

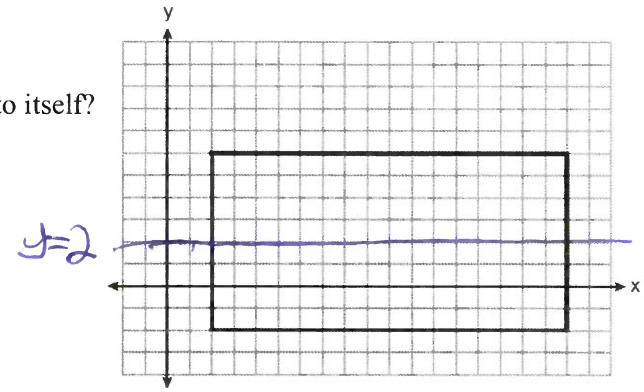
not the center of the shape



6. A rectangle is graphed on the set of axes below.

A reflection over which line would carry the rectangle onto itself?

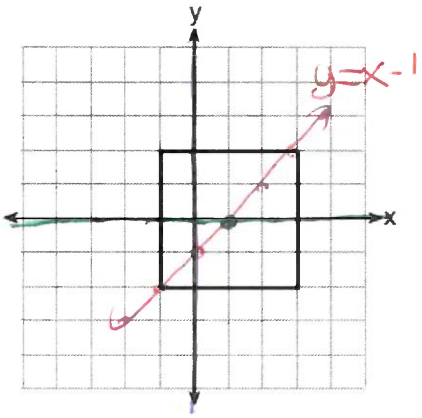
1) $y = 2$
2) $y = 10$
3) $y = \frac{1}{2}x - 3$
4) $y = -\frac{1}{2}x + 7$



7. A square is graphed on the set of axes below, with vertices at $(-1, 2)$, $(-1, -2)$, $(3, -2)$, and $(3, 2)$.

Which transformation would *not* carry the square onto itself?

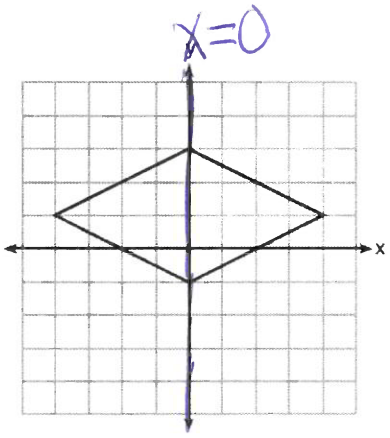
1) reflection over the y-axis
2) reflection over the x-axis
3) rotation of 180 degrees around point $(1, 0)$
4) reflection over the line $y = x - 1$



A rhombus is graphed on the set of axes below.

8. Which transformation would carry the rhombus onto itself?

1) 180° rotation counterclockwise about the origin
2) reflection over the line $y = \frac{1}{2}x + 1$
3) reflection over the line $y = 0$
4) reflection over the line $x = 0$

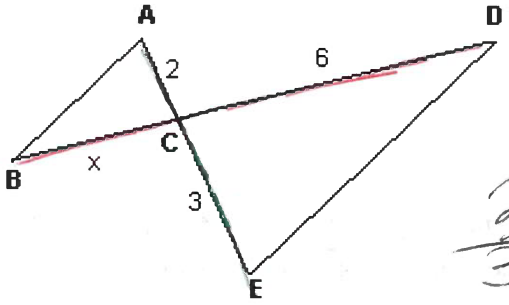


Bow Tie Problems

Corresponding sides of similar triangles are in proportion. Put the corresponding sides on top of each other to create the proportion.

With parallel lines, the corresponding sides are diagonal from each other.

1. In the diagram below, $\overline{AB} \parallel \overline{DE}$. If $AC = 2$, $CD = 6$, and $CE = 3$, what is BC?

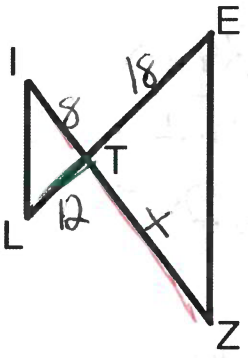


$\frac{2}{3} = \frac{x}{6}$

$\frac{3x}{3} = \frac{12}{3}$

$x = 4$

2. In the diagram below, $\overline{LI} \parallel \overline{ZE}$. If $LT = 12$, $TE = 18$, and $IT = 8$, find TZ.

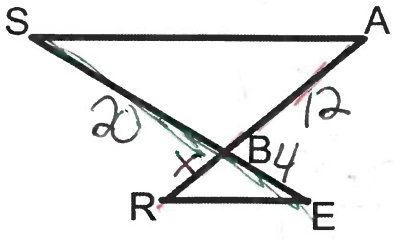


$\frac{8}{x} = \frac{12}{18}$

$\frac{12x}{12} = \frac{144}{12}$

$x = 12$

3. In the diagram below, $\overline{SA} \parallel \overline{RE}$. If $SB = 20$, $BE = 4$, and $BA = 12$, find RB.

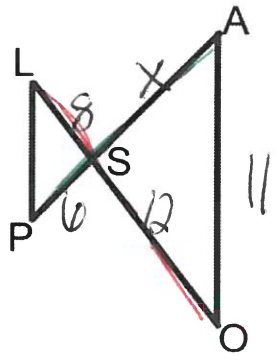


$\frac{20}{4} = \frac{12}{x}$

$\frac{48}{20} = \frac{20x}{20}$

$2.4 = x$

4. In the diagram below, $\overline{LP} \parallel \overline{AO}$. If $LS = 8$, $SO = 12$, $AO = 11$, and $PS = 6$, find SA.

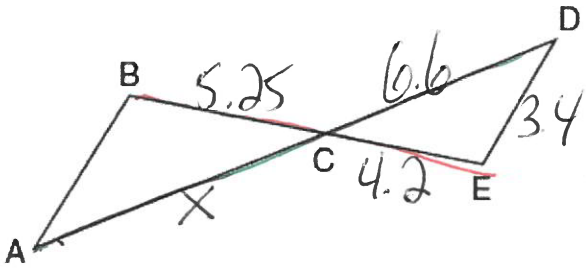


$\frac{6}{x} = \frac{8}{12}$

$\frac{8x}{8} = \frac{72}{8}$

$x = 9$

5. In the diagram below, $\overline{AD}$ intersects $\overline{BE}$ at C, and $\overline{AB} \parallel \overline{DE}$. If $CD = 6.6$ cm, $DE = 3.4$ cm, $CE = 4.2$ cm, and $BC = 5.25$ cm, what is the length of $\overline{AC}$, to the *nearest hundredth of a centimeter*?

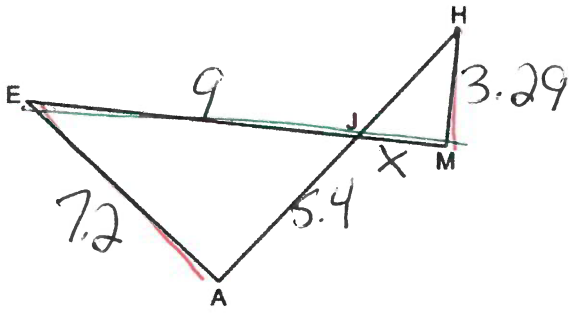


$\frac{x}{6.6} = \frac{5.25}{4.2}$

$\frac{4.2x}{4.2} = \frac{34.65}{4.2}$

$x = 8.25$

6. In the diagram below, $\overline{EM}$ intersects $\overline{HA}$ at J, $\overline{EA} \perp \overline{HA}$, and $\overline{EM} \perp \overline{HM}$. If $EA = 7.2$, $EJ = 9$, $AJ = 5.4$, and $HM = 3.29$, what is the length of $\overline{MJ}$, to the *nearest hundredth*?

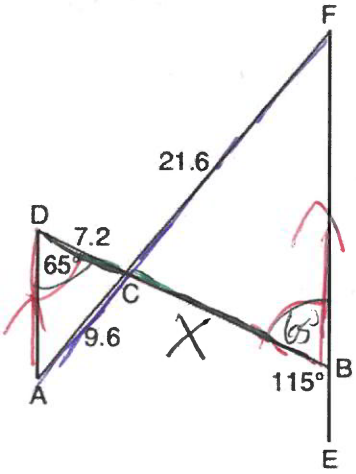


$\frac{9}{x} = \frac{7.2}{3.29}$

$\frac{7.2x}{7.2} = \frac{29.61}{7.2}$

$x = 4.11$

7. In the diagram below, $\overline{AF}$, and $\overline{DB}$ intersect at C, and $\overline{AD}$ and $\overline{FBE}$ are drawn such that $m\angle D = 65°$, $m\angle CBE = 115°$, $DC = 7.2$, $AC = 9.6$, and $FC = 21.6$. What is the length of $\overline{CB}$?

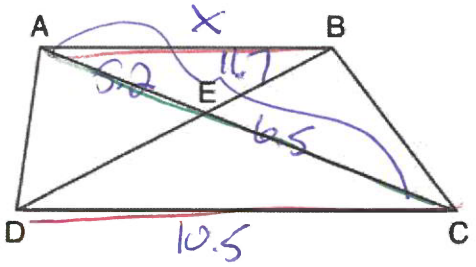


$\frac{7.2}{x} = \frac{9.6}{21.6}$

$\frac{9.6x}{9.6} = \frac{155.52}{9.6}$

$x = 16.2$

8. In trapezoid $ABCD$ below, $\overline{AB} \parallel \overline{CD}$. If $AE = 5.2$, $AC = 11.7$, and $CD = 10.5$, what is the length of $\overline{AB}$, to the *nearest tenth*?

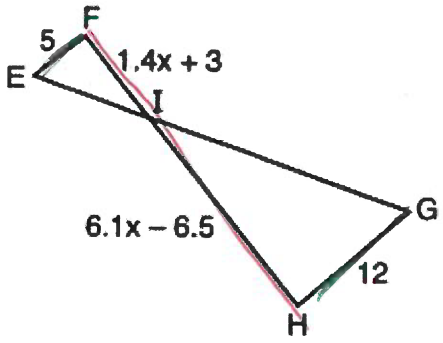


$\frac{5.2}{6.5} = \frac{x}{10.5}$

$\frac{6.5x}{6.5} = \frac{54.6}{6.5}$

$x = 8.4$

9. In the diagram below, $\overline{EF} \parallel \overline{HG}$, $EF = 5$, $HG = 12$, $FI = 1.4x + 3$, and $HI = 6.1x - 6.5$. What is the length of $\overline{HI}$?



$\frac{5}{12} = \frac{1.4x+3}{6.1x-6.5}$ (or Equation Solver)

$5(6.1x-6.5) = 12(1.4x+3)$

$30.5x - 32.5 = 16.8x + 36$

$-16.8x \quad -16.8x$

$13.7x - 32.5 = 36$

$+32.5 \quad +32.5$

$\frac{13.7x}{13.7} = \frac{68.5}{13.7}$

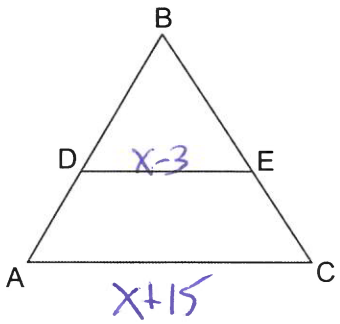
$x = 5$

$HI = 6.1(5) - 6.5$

$HI = 24$

Joining the Midpoints of a Triangle
The midsegments are half of the opposite parallel sides
$2(midsegment) = opposite\ side$

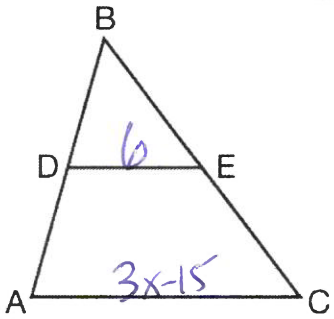
1. D and E are midpoints of $\overline{AB}$ and $\overline{BC}$ respectively. If $\overline{AC} = x + 15$ and $\overline{DE} = x - 3$, find the measure of $\overline{DE}$.



$2(x-3) = x+15$
$2x-6 = x+15$
$-x \quad -x$
$x-6 = 15$
$+6 \quad +6$
$x = 21$

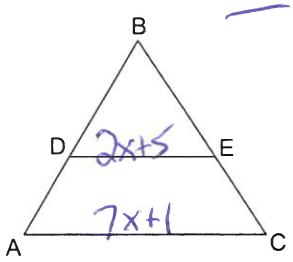
$\overline{DE} = x-3$
$\overline{DE} = 21-3$
$\overline{DE} = 18$

2. In $\triangle ABC$, D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{BC}$. If $AC = 3x - 15$ and $DE = 6$, what is the value of x?



$2(6) = 3x-15$
$12 = 3x-15$
$+15 \quad +15$
$\frac{27}{3} = \frac{3x}{3}$
$9 = x$

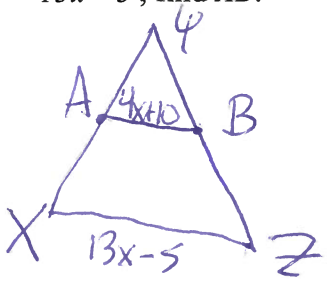
3. D and E are midpoints of $\overline{AB}$ and $\overline{BC}$ respectively. If $\overline{DE} = 2x + 5$ and $\overline{AC} = 7x + 1$, find the measure of $\overline{AC}$.



$2(2x+5) = 7x+1$
$4x+10 = 7x+1$
$-4x \quad -4x$
$10 = 3x+1$
$-1 \quad -1$
$\frac{9}{3} = \frac{3x}{3}$
$3 = x$

$\overline{AC} = 7(3)+1$
$\overline{AC} = 22$

4. In ΔXYZ, A is the midpoint of XY and B is the midpoint of YZ. If $AB = 4x + 10$ and $XZ = 13x - 5$, find AB.



$2(4x+10) = 13x - 5$

$8x + 20 = 13x - 5$

$-8x \quad -8x$

$20 = 5x - 5$

$+5 \quad +5$

$\frac{25}{5} = \frac{5x}{5}$

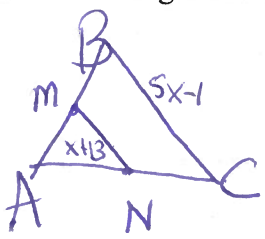
$5 = x$

$AB = 4(5) + 10$

$AB = 30$

5. In $\triangle ABC$, M is the midpoint of $\overline{AB}$ and N is the midpoint of $\overline{AC}$. If $MN = x + 13$ and $BC = 5x - 1$, what is the length of $\overline{MN}$?

1) 3.5
2) 9
3) 16.5
4) 22



$2(x+13) = 5x - 1$

$2x + 26 = 5x - 1$

$-2x \quad -2x$

$26 = 3x - 1$

$+1 \quad +1$

$\frac{27}{3} = \frac{3x}{3}$

$9 = x$

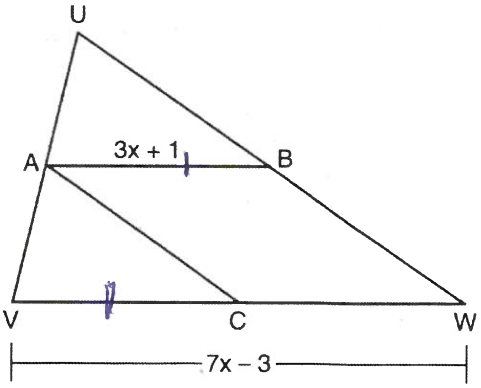
$MN = 9 + 13$

$MN = 22$

6. In the diagram of ΔUVW below, A is the midpoint of $\overline{UV}$, B is the midpoint of $\overline{UW}$, C is the midpoint of $\overline{VW}$, and $\overline{AB}$ and $\overline{AC}$ are drawn.

If $VW = 7x - 3$ and $AB = 3x + 1$, what is the length of $\overline{VC}$?

1) 5
2) 13
3) 16
4) 32



$2(3x+1) = 7x - 3$

$6x + 2 = 7x - 3$

$-6x \quad -6x$

$2 = x - 3$

$+3 \quad +3$

$1x = 5$

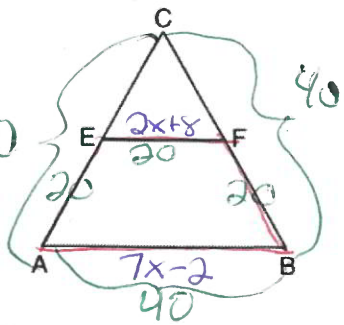
$VC = 3(5) + 1$

$VC = 16$

7. In the diagram of equilateral triangle ABC shown below, E and F are the midpoints of $\overline{AC}$ and $\overline{BC}$, respectively.

If $EF = 2x + 8$ and $AB = 7x - 2$, what is the perimeter of trapezoid $ABFE$?

1) 36
2) 60
3) 100
4) 120



$2(2x+8) = 7x - 2$

$4x + 16 = 7x - 2$

$-4x \quad -4x$

$16 = 3x - 2$

$+2 \quad +2$

$\frac{18}{3} = \frac{3x}{3}$

$6 = x$

$2(6) + 8$

20

$7(6) - 2$

40

$20 + 20 + 20 + 40 = 100$

Candy Corn Problems

If the bases are not involved: $\frac{top}{top} = \frac{bottom}{bottom} = \frac{side}{side}$

If bases are involved: separate your triangles!

1. In the diagram of ΔADC below, $\overline{EB} \parallel \overline{DC}$, $AE = 9$, $ED = 5$, and $AB = 9.2$.

What is the length of $\overline{AC}$, to the *nearest tenth*?

1) 5.1
2) 5.2
3) 14.3
4) 14.4

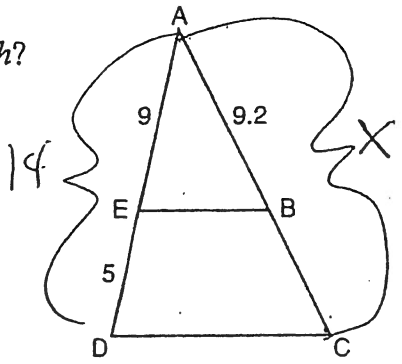
bases not involved

$\frac{top}{top} = \frac{side}{side}$

$\frac{9}{9.2} = \frac{14}{x}$

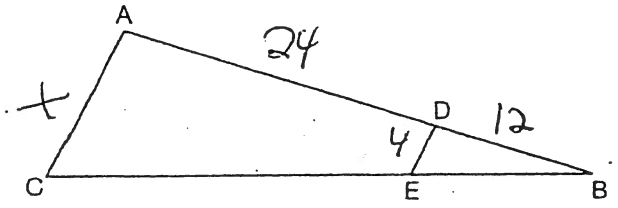
$\frac{9x}{9} = \frac{128.8}{9}$

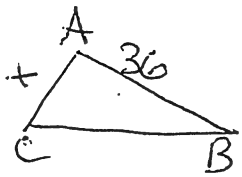
$x = 14.3$

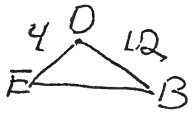


2. In the diagram of ΔABC, points D and E are on $\overline{AB}$ and $\overline{CB}$, respectively, such that $\overline{AC} \parallel \overline{DE}$.

bases involved
separate







If $AD = 24$, $DB = 12$, and $DE = 4$, what is the length of $\overline{AC}$?

1) 8
2) 12
3) 16
4) 72

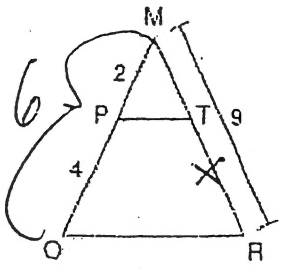
$\frac{x}{4} = \frac{36}{12}$

$\frac{12x}{12} = \frac{144}{12}$

$x = 12$

3. Given ΔMRO shown below, with trapezoid $PTRO$, $MR = 9$, $MP = 2$, and $PO = 4$.

bases not involved



$\frac{bottom}{bottom} = \frac{side}{side}$

$\frac{6}{x} = \frac{9}{4}$

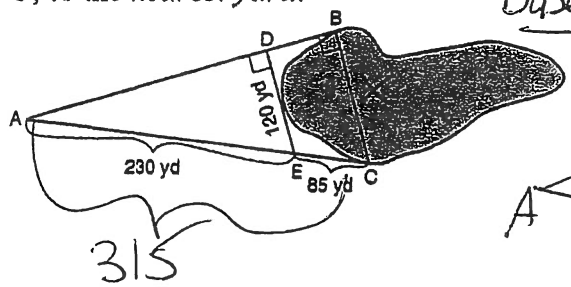
What is the length of $\overline{TR}$?

1) 4.5
2) 5
3) 3
4) 6

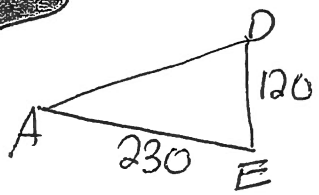
$\frac{6x}{6} = \frac{36}{6}$

$x = 6$

4. To find the distance across a pond from point B to point C, a surveyor drew the diagram below. The measurements he made are indicated on his diagram.
Use the surveyor's information to determine and state the distance from point B to point C, to the *nearest yard*.



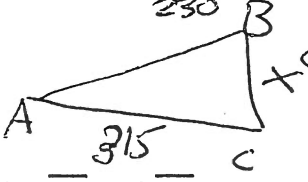
Bases involved
Separate



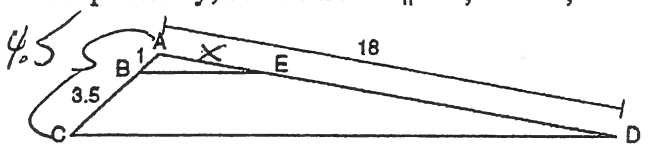
$\frac{230}{315} = \frac{120}{x}$

$\frac{230x}{230} = \frac{37800}{230}$

$x = 164$



5. In the diagram below, triangle ACD has points B and E on sides $\overline{AC}$ and $\overline{AD}$, respectively, such that $\overline{BE} \parallel \overline{CD}$, $AB = 1$, $BC = 3.5$, and $AD = 18$.



What is the length of $\overline{AE}$, to the *nearest tenth*?

bases not involved

$\frac{\text{top}}{\text{top}} = \frac{\text{side}}{\text{side}}$

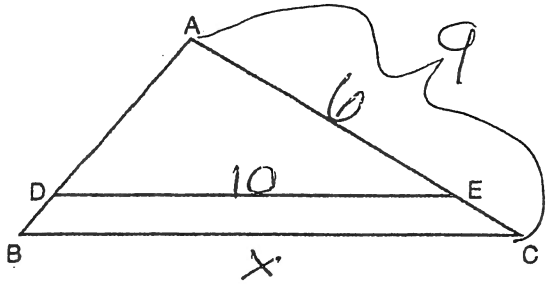
$\frac{1}{x} = \frac{4.5}{18}$

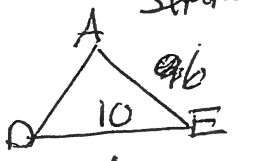
$\frac{4.5x}{4.5} = \frac{18}{4.5}$

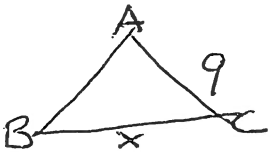
$x = 4.0$

6. In the diagram of $\triangle ABC$ shown below, $\overline{DE} \parallel \overline{BC}$.
If $\overline{AE} = 6$, $\overline{DE} = 10$, and $\overline{AC} = 9$, find $\overline{BC}$

bases involved
Separate





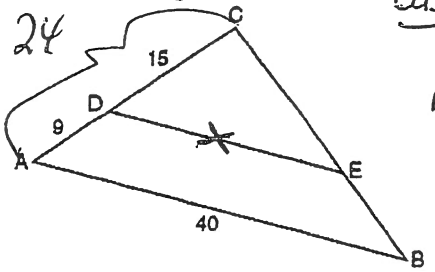


$\frac{10}{x} = \frac{6}{9}$

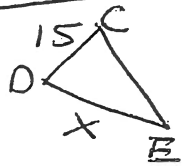
$\frac{6x}{6} = \frac{90}{6}$

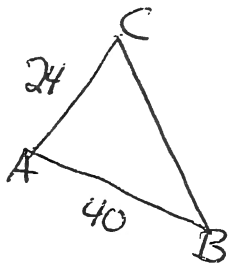
$x = 15$

7. In the diagram of $\triangle ABC$ below, $\overline{DE}$ is parallel to $\overline{AB}$, $CD = 15$, $AD = 9$, and $AB = 40$.
Find the length of $\overline{DE}$.



bases involved



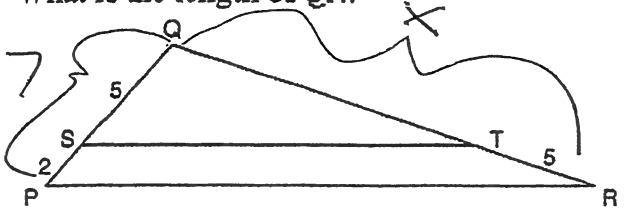


$\frac{15}{24} = \frac{x}{40}$

$\frac{24x}{24} = \frac{600}{24}$

$x = 25$

8. In the diagram below of $\triangle PQR$, $\overline{ST}$ is drawn parallel to $\overline{PR}$, $PS = 2$, $SQ = 5$, and $TR = 5$
What is the length of $\overline{QR}$?



bases not involved

$\frac{\text{side}}{\text{side}} = \frac{\text{bottom}}{\text{bottom}}$

$\frac{7}{x} = \frac{2}{5}$

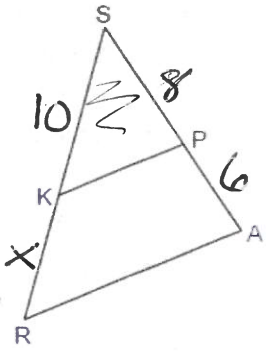
$\frac{2x}{2} = \frac{35}{2}$

$x = 17.5$

9. In the diagram of $\triangle SRA$ below, $\overline{KP}$ is drawn such that $\angle SKP \cong \angle SRA$.

If $SK = 10$, $SP = 8$, and $PA = 6$, what is the length of $\overline{KR}$, to the *nearest tenth*?

1) 4.8
2) 7.5
3) 8.0
4) 13.3



top/top = bottom/bottom

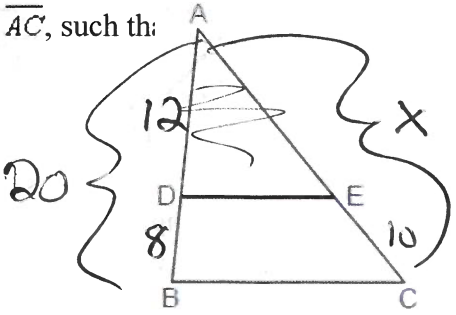
$\frac{10}{8} = \frac{x}{6}$ $\quad \frac{8x}{8} = \frac{60}{8}$

$x = 7.5$

10. In triangle ABC below, D is a point on $\overline{AB}$ and E is a point on $\overline{AC}$, such th

If $AD = 12$, $DB = 8$, and $EC = 10$, what is the length of $\overline{AC}$?

1) 15
2) 22
3) 24
4) 25



side/side = bottom/bottom

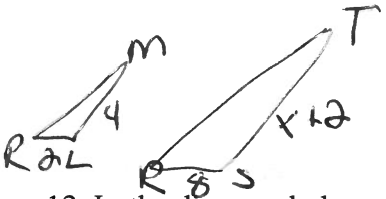
$\frac{20}{x} = \frac{8}{10}$ $\quad \frac{8x}{8} = \frac{200}{8}$

$x = 25$

11. In the diagram below of $\triangle RST$, L is a point on $\overline{RS}$, and M is a point on $\overline{RT}$, such that $LM \parallel ST$.

If $RL = 2$, $LS = 6$, $LM = 4$, and $ST = x + 2$, what is the length of $\overline{ST}$?

1) 10 2) 12 3) 14 4) 16



bases inuled

$\frac{4}{x+2} = \frac{2}{8}$

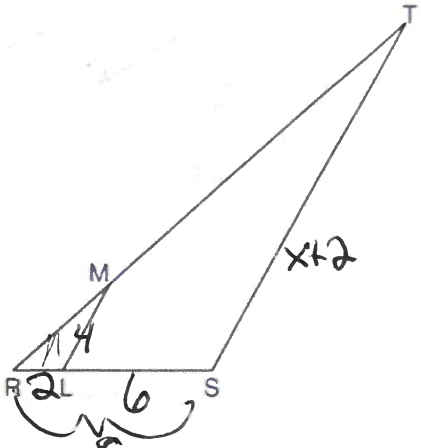
$2(x+2) = 32$

$2x + 4 = 32$

$-4 \quad -4$

$\frac{2x}{2} = \frac{28}{2}$

$x = 14$



12. In the diagram below of $\triangle CER$, $\overline{LA} \parallel \overline{CR}$.

If $CL = 3.5$, $LE = 7.5$, and $EA = 9.5$, what is the length of $\overline{AR}$, to the *nearest tenth*?

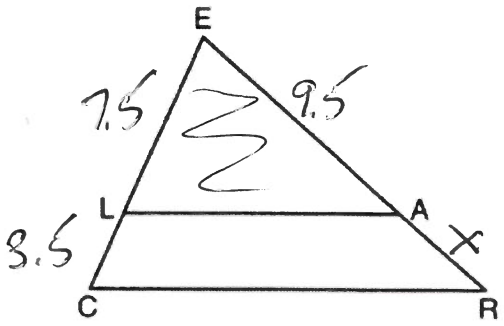
1) 5.5
2) 4.4
3) 3.0
4) 2.8

top/top = bottom/bottom

$\frac{7.5}{9.5} = \frac{3.5}{x}$

$\frac{7.5x}{7.5} = \frac{33.25}{7.5}$

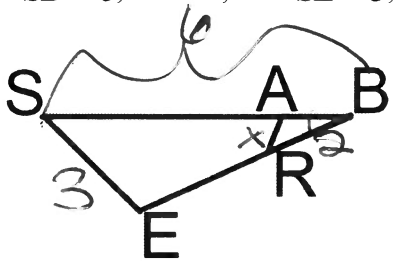
$x = 4.4$

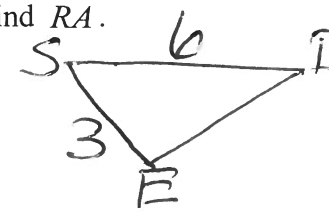


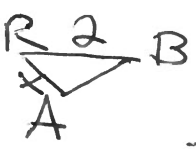
Overlapping Similar Triangles

1) Separate the triangles and draw them with the same orientation
2) Match up the corresponding letters (use reflexive property)
3) Create a proportion and solve

1. In triangle SEB, A is on $\overline{SB}$, and E is on $\overline{EB}$ so that $\angle E \cong \angle BAR$.
If $\overline{SB} = 6$, $\overline{RB} = 2$, and $\overline{SE} = 3$, find $\overline{RA}$.





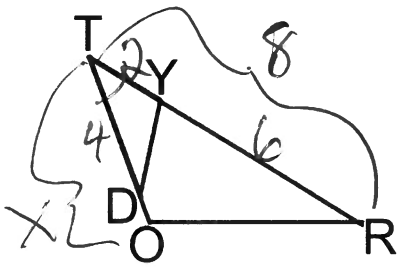


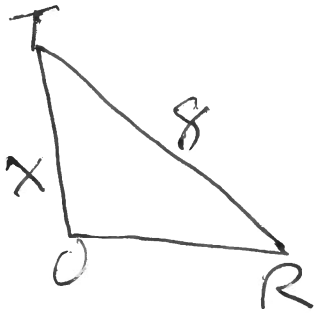
$\frac{6}{2} = \frac{3}{x}$

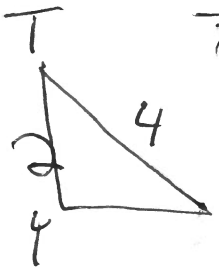
$\frac{6}{6}x = \frac{6}{6}$

$x = 1$

2. In triangle TOR, Y is on $\overline{TR}$, and D is on $\overline{TO}$ so that $\angle TYD \cong \angle ROT$.
If $\overline{TY} = 2$, $\overline{YR} = 6$, and $\overline{TD} = 4$, find $\overline{TO}$.





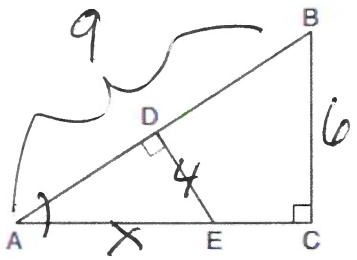


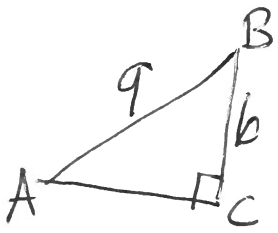
$\frac{x}{2} = \frac{8}{4}$

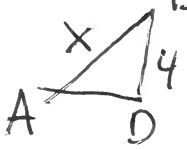
$\frac{4}{4}x = \frac{16}{4}$

$x = 4$

3. In $\triangle ABC$ shown below, $\angle ACB$ is a right angle, E is a point on $\overline{AC}$, and $\overline{ED}$ is drawn perpendicular to hypotenuse $\overline{AB}$. If $AB = 9$, $BC = 6$, and $DE = 4$, what is the length of $\overline{AE}$?





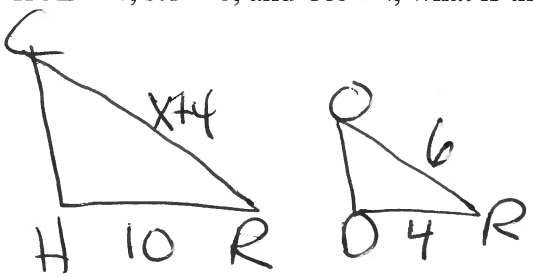


$\frac{9}{x} = \frac{6}{4}$

$\frac{6}{6}x = \frac{36}{6}$

$x = 6$

4. In triangle CHR, O is on $\overline{HR}$, and D is on $\overline{CR}$ so that $\angle H \cong RDO$. If $RD = 4$, $RO = 6$, and $OH = 4$, what is the length of $\overline{CD}$?



$\frac{X+4}{6} = \frac{10}{4}$

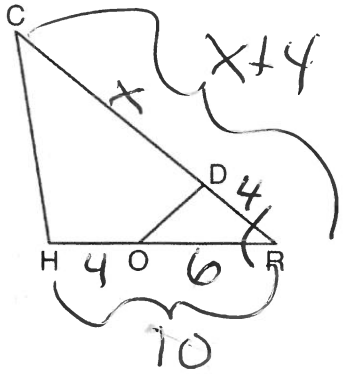
$4(X+4) = 60$

$4X+16 = 60$

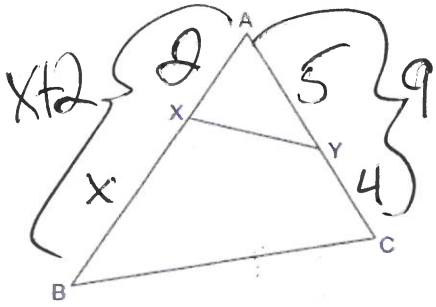
$-16 \quad -16$

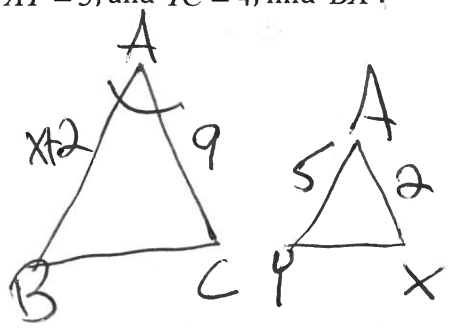
$\frac{4X}{4} = \frac{44}{4}$

$X = 11$



5. In the diagram below of $\triangle ABC$, X and Y are points on $\overline{AB}$ and $\overline{AC}$, respectively, such that $m\angle AYX = m\angle B$. If $\overline{AX} = 2$, $\overline{AY} = 5$, and $\overline{YC} = 4$, find $\overline{BX}$.





$\frac{X+2}{5} = \frac{9}{2}$

$2(X+2) = 45$

$2X+4 = 45$

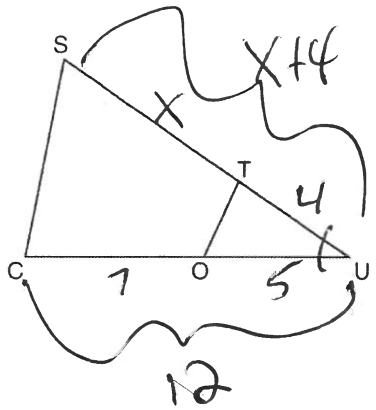
$-4 \quad -4$

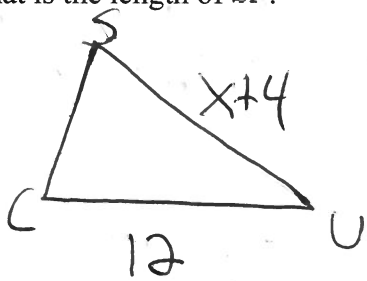
$\frac{2X}{2} = \frac{41}{2}$

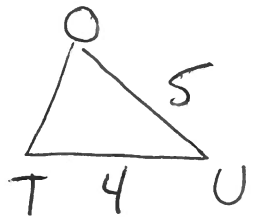
$X = 20.5$

6. In $\triangle SCU$ shown below, points T and O are on $\overline{SU}$ and $\overline{CU}$, respectively. Segment OT is drawn so that $\angle C \cong \angle OTU$.

If $TU = 4$, $OU = 5$, and $OC = 7$, what is the length of $\overline{ST}$?







$\frac{X+4}{5} = \frac{12}{4}$

$4(X+4) = 60$

$4X+16 = 60$

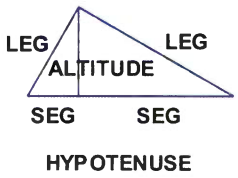
$-16 \quad -16$

$\frac{4X}{4} = \frac{44}{4}$

$X = 11$

When an altitude is drawn to a right triangle
HLLS and SAAS

$\frac{H}{L}=\frac{L}{S}$ $\frac{S}{A}=\frac{A}{S}$

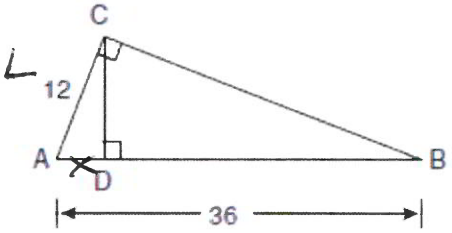


If L is involved, use HLLS
If A is involved, use SAAS

Know how to reduce radicals:

1) Separate into perfect square and non perfect square
2) Take the square root of the perfect square

1. In the diagram below of right triangle ACB, altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$.



If $AB = 36$ and $AC = 12$, what is the length of $\overline{AD}$?

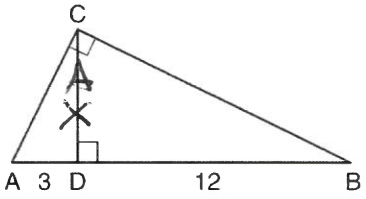
1) 32
2) 6
3) 3
4) 4

$\frac{H}{L}=\frac{L}{S}$ $\frac{36}{12}=\frac{12}{x}$ $36x=144$ $\frac{36x}{36}=\frac{144}{36}$ $x=4$

2. In the diagram below of right triangle ABC, altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$.

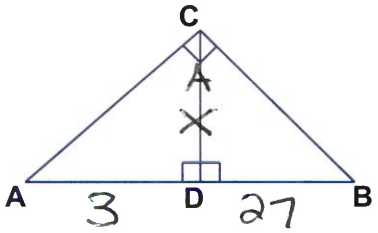
If $AD = 3$ and $DB = 12$, what is the length of altitude $\overline{CD}$?

1) 6
2) $6\sqrt{5}$
3) 3
4) $3\sqrt{5}$



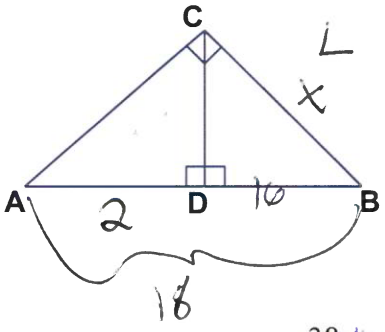
$\frac{S}{A}=\frac{A}{S}$ $\frac{3}{x}=\frac{x}{12}$ $\sqrt{x^2}=\sqrt{36}$ $x=6$

3. If $\overline{AD}=3$ and $\overline{DB}=27$, find $\overline{CD}$



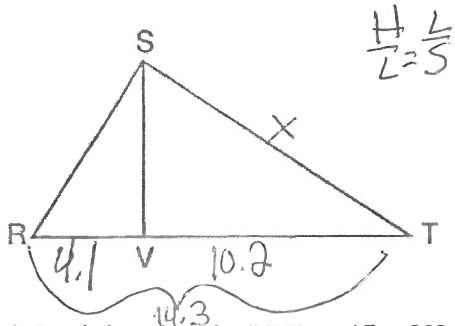
$\frac{S}{A}=\frac{A}{S}$ $\frac{3}{x}=\frac{x}{27}$ $\sqrt{x^2}=\sqrt{81}$ $x=9$

4. If $\overline{AD}=2$ and $\overline{AB}=18$, find $\overline{BC}$ to the nearest tenth



$\frac{H}{L}=\frac{L}{S}$ $\frac{18}{x}=\frac{x}{16}$ $\sqrt{x^2}=\sqrt{288}$ $x=17.0$

5. In right triangle RST below, altitude $\overline{SV}$ is drawn to hypotenuse $\overline{RT}$. If $RV = 4.1$ and $TV = 10.2$, what is the length of $\overline{ST}$, to the *nearest tenth*?

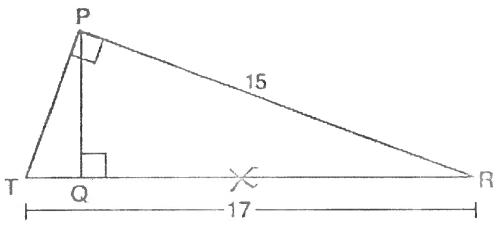


$\frac{H}{L} = \frac{L}{S}$ $\frac{14.3}{x} = \frac{x}{10.2}$

$\sqrt{x^2} = \sqrt{145.86}$

$x = 12.1$

6. In right triangle PRT, $m\angle P = 90°$, altitude $\overline{PQ}$ is drawn to hypotenuse $\overline{RT}$, $RT = 17$, and $PR = 15$. Determine and state, to the *nearest tenth*, the length of $\overline{RQ}$.

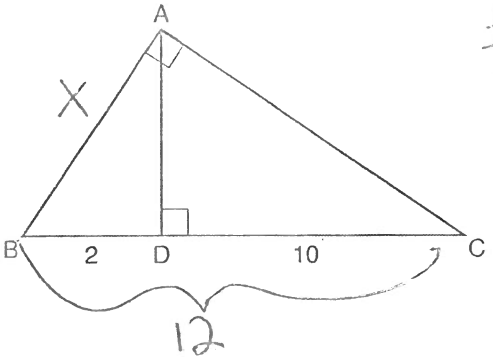


$\frac{H}{L} = \frac{L}{S}$ $\frac{17}{15} = \frac{15}{x}$

$\frac{17x}{17} = \frac{225}{17}$

$x = 13.2$

8. Triangle ABC shown below is a right triangle with altitude $\overline{AD}$ drawn to the hypotenuse $\overline{BC}$. If $BD = 2$ and $DC = 10$, what is the length of $\overline{AB}$ to the *nearest tenth*?

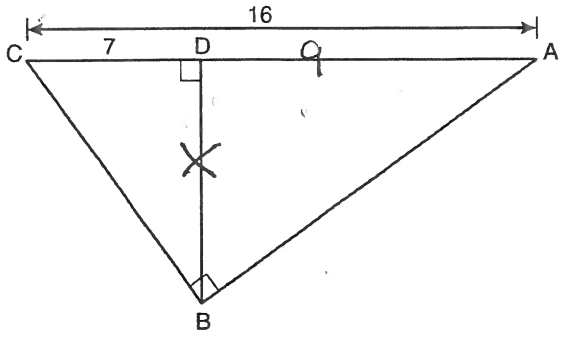


$\frac{H}{L} = \frac{L}{S}$ $\frac{12}{x} = \frac{x}{2}$

$\sqrt{x^2} = \sqrt{24}$

$x = 4.9$

9. In the diagram below of right triangle ABC, altitude $\overline{BD}$ is drawn to hypotenuse $\overline{AC}$, $AC = 16$, and $CD = 7$. What is the length of $\overline{BD}$ to the *nearest tenth*?



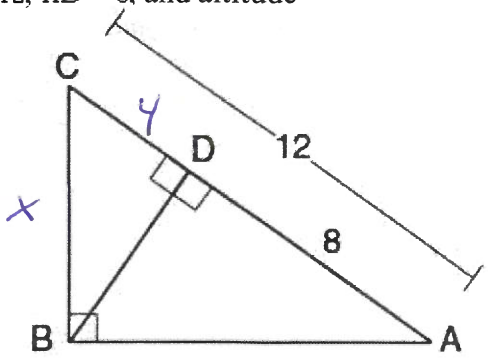
$\frac{S}{A} = \frac{A}{S}$

$\frac{7}{x} = \frac{x}{9}$

$\sqrt{x^2} = \sqrt{63}$

$x = 7.9$

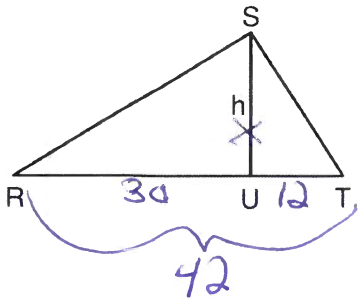
10. In the diagram below of $\triangle ABC$, $\angle ABC$ is a right angle, $AC = 12$, $AD = 8$, and altitude $\overline{BD}$ is drawn. What is the length of $\overline{BC}$ to the *nearest tenth*?



$\frac{H}{L} = \frac{L}{S}$ $\sqrt{x^2} = \sqrt{48}$

$\frac{12}{x} = \frac{x}{4}$ $x = 6.9$

11. In $\triangle RST$ shown below, altitude $\overline{SU}$ is drawn to $\overline{RT}$ at U. If $SU = h$, $UT = 12$, and $RT = 42$, which value of h, to the *nearest tenth*, will make $\triangle RST$ a right triangle with $\angle RST$ as a right angle?

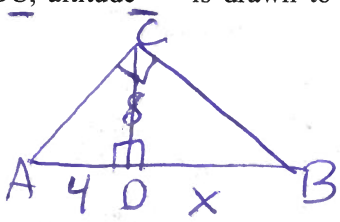


$\frac{S}{A} = \frac{A}{S}$ $\sqrt{x^2} = \sqrt{360}$

$\frac{30}{x} = \frac{x}{12}$ $x = 19.0$

12. In right triangle ABC, altitude $\overline{CD}$ is drawn to hypotenuse $\overline{AB}$. If $AD = 4$ and $CD = 8$, the length of $\overline{BD}$ is

1) $\sqrt{48}$ 3) 12
2) $\sqrt{80}$ 4) 16



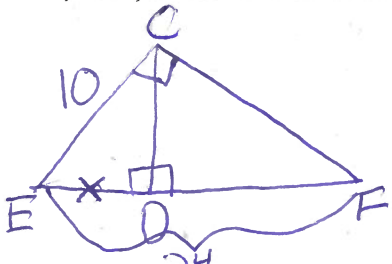
$\frac{S}{A} = \frac{A}{S}$ $\frac{4}{8} = \frac{8}{x}$

$\frac{4x}{4} = \frac{64}{4}$

$x = 16$

13. Line segment CD is the altitude drawn to hypotenuse $\overline{EF}$ in right triangle ECF. If $EC = 10$ and $EF = 24$, then, to the *nearest tenth*, ED is

1) 4.2
2) 5.4
3) 15.5
4) 21.8



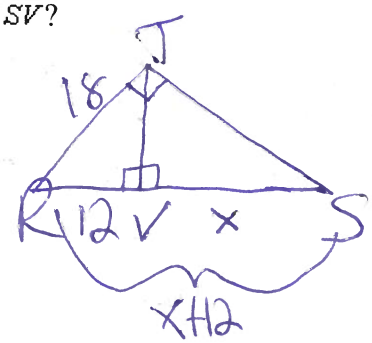
$\frac{H}{L} = \frac{L}{S}$ $\frac{24}{10} = \frac{10}{x}$

$\frac{24x}{24} = \frac{100}{24}$

$x = 4.2$

14. In right triangle RST, altitude $\overline{TV}$ is drawn to hypotenuse $\overline{RS}$. If $RV = 12$ and $RT = 18$, what is the length of $\overline{SV}$?

1) $6\sqrt{5}$
2) 15
3) $6\sqrt{6}$
4) 27



$\frac{H}{L} = \frac{L}{S}$ $\frac{x+12}{18} = \frac{18}{12}$

$12(x+12) = 324$

$12x + 144 = 324$

$-144 \quad -144$

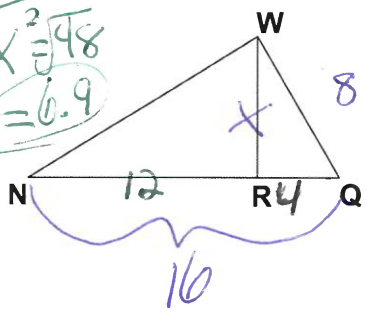
$\frac{12x}{12} = \frac{180}{12}$

$x = 15$

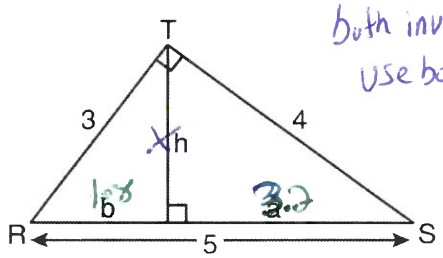
15. Altitude $\overline{WR}$ is drawn to right triangle NWQ. If $\overline{QW} = 8$ and $\overline{NQ} = 16$, find $\overline{WR}$ to the nearest tenth.

$\frac{H}{L}=\frac{L}{S}$ $\quad \frac{16}{8}=\frac{8}{y}$ $\quad 16y=64$ $\quad y=4$ $\quad \frac{S}{A}=\frac{A}{S}$ $\quad \frac{12}{x}=\frac{x}{4}$ $\quad \sqrt{x^2}=\sqrt{48}$ $\quad x=6.9$

both involved use both



16. In the diagram below, $\triangle RST$ is a $3-4-5$ right triangle. The altitude, h, to the hypotenuse has been drawn. Determine the length of h.

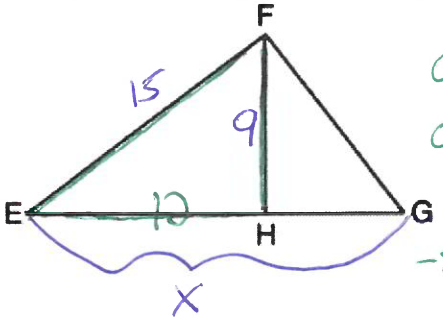


both involved use both

$\frac{H}{L}=\frac{L}{S}$ $\quad \frac{5}{4}=\frac{4}{a}$ $\quad 5a=16$ $\quad a=3.2$ $\quad \frac{S}{A}=\frac{A}{S}$ $\quad \frac{1.8}{x}=\frac{x}{3.2}$ $\quad \sqrt{x^2}=\sqrt{5.76}$ $\quad x=2.4$

17. In the diagram below of right triangle EFG, altitude $\overline{FH}$ intersects hypotenuse $\overline{EG}$ at H. If $FH = 9$ and $EF = 15$, what is EG?

not enough info for HLS or SAAS: $a^2+b^2=c^2$

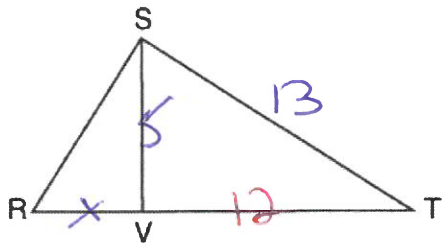


$a^2+b^2=c^2$ $\quad 9^2+b^2=c^2$ $\quad 81+b^2=225$ $\quad -81 \quad -81$ $\quad \sqrt{b^2}=\sqrt{144}$ $\quad b=12$

$\frac{H}{L}=\frac{L}{S}$ $\quad \frac{x}{15}=\frac{15}{12}$ $\quad 12x=225$ $\quad \frac{12}{12}=\frac{225}{12}$ $\quad x=18.75$

18. In right triangle RST below, altitude $\overline{SV}$ is drawn to hypotenuse $\overline{RT}$. If $SV = 5$, and $ST = 13$ find RV.

not enough info for HLS or SAAS: $a^2+b^2=c^2$



$a^2+b^2=c^2$ $\quad 5^2+b^2=13^2$ $\quad 25+b^2=169$ $\quad -25 \quad -25$ $\quad \sqrt{b^2}=\sqrt{144}$ $\quad b=12$

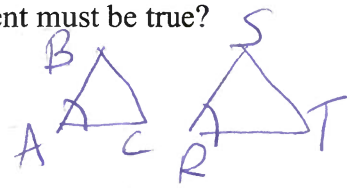
$\frac{S}{A}=\frac{A}{S}$ $\quad \frac{x}{5}=\frac{5}{12}$ $\quad 12x=25$ $\quad \frac{12}{12}=\frac{25}{12}$ $\quad x=\frac{25}{12}$

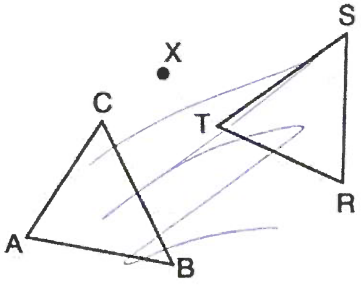
Corresponding Parts of Congruent Triangles are Congruent
Redraw the shapes so it is more clear to see what parts correspond to each other

1. After a counterclockwise rotation about point X, scalene triangle ABC maps onto $\triangle RST$, as shown in the diagram below.

Which statement must be true?

1) $\angle A \cong \angle R$
2) $\angle A \cong \angle S$
3) $\overline{CB} \cong \overline{TR}$
4) $\overline{CA} \cong \overline{TS}$

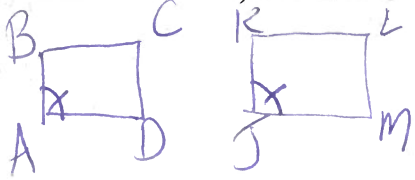


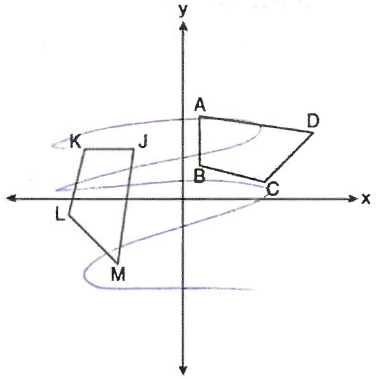


2. In the diagram below, a sequence of rigid motions maps $ABCD$ onto $JKLM$.

Which of the following statements must be true?

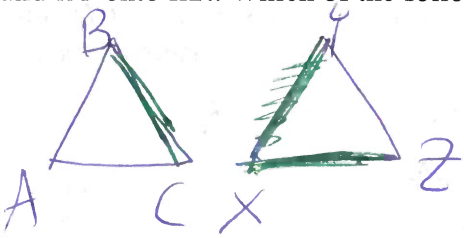
1) $\angle L \cong \angle B$
2) $\angle A \cong \angle J$
3) $\overline{JK} \cong \overline{AC}$
4) $\overline{JM} \cong \overline{AB}$

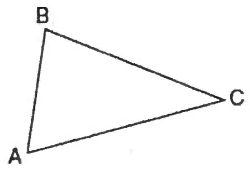


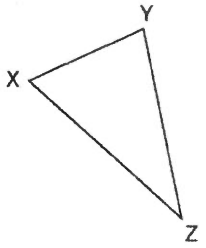


3. In the diagram below of $\triangle ABC$ and $\triangle XYZ$, a sequence of rigid motions maps $\angle A$ onto $\angle X$, $\angle C$ onto $\angle Z$, and $\overline{AC}$ onto $\overline{XZ}$.. Which of the following statements is *not* true?

1) $\overline{AB} \cong \overline{XY}$
2) $\overline{BC} \cong \overline{XZ}$
3) $\angle B \cong \angle Y$
4) $\angle C \cong \angle Z$



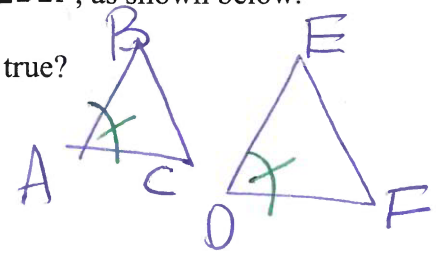


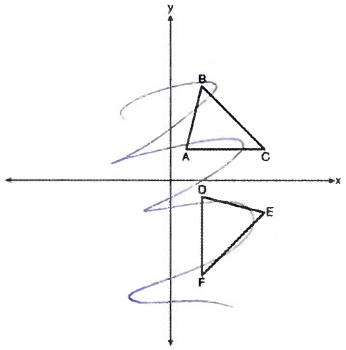


4. The image of $\triangle ABC$ after a rotation of 90° clockwise about the origin is $\triangle DEF$, as shown below.

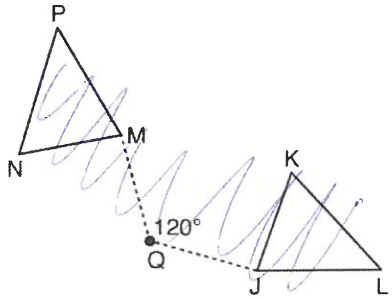
Which statement is true?

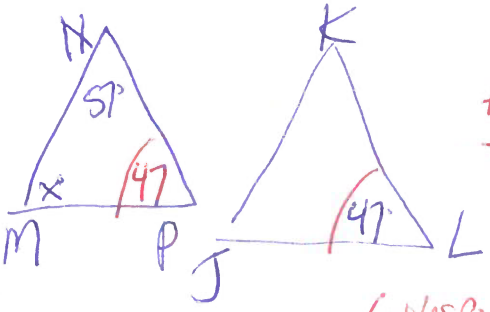
1) $\overline{BC} \cong \overline{DE}$
2) $\overline{AB} \cong \overline{DF}$
3) $\angle C \cong \angle E$
4) $\angle A \cong \angle D$





5. Triangle *MNP* is the image of triangle *JKL* after a 120° counterclockwise rotation about point *Q*. If the measure of angle *L* is 47° and the measure of angle *N* is 57°, determine the measure of angle *M*. Explain how you arrived at your answer.



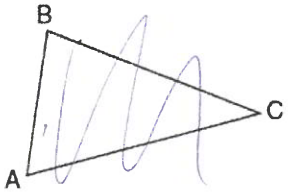


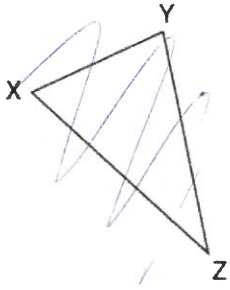
47 180
+57 −104
104 76

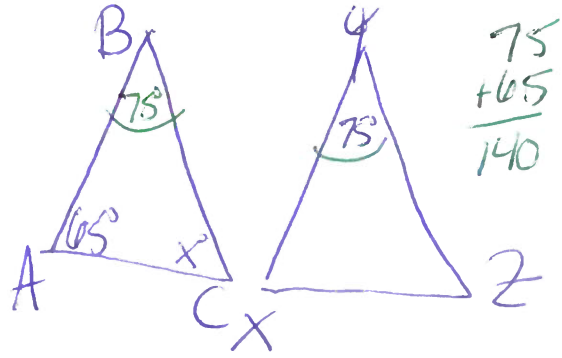
∠M = 76°

Corresponding parts of congruent triangles are congruent.

6. In the diagram below of $\triangle ABC$ and $\triangle XYZ$, a sequence of rigid motions maps $\angle A$ onto $\angle X$, $\angle C$ onto $\angle Z$, and $\overline{AC}$ onto $\overline{XZ}$.. If $\angle A = 65$ and $\angle Y = 75$, find the measure of $\angle C$.

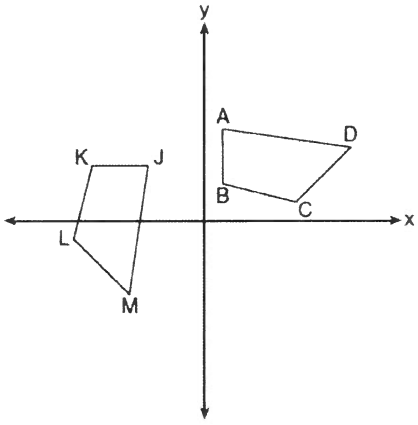


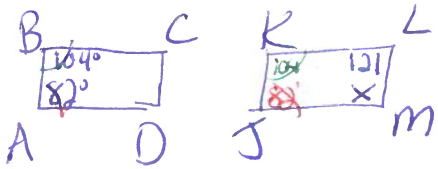




75 180
+65 −140
140 40°

7. In the diagram below, a sequence of rigid motions maps *ABCD* onto *JKLM*. If m∠*A* = 82°, m∠*B* = 104°, and m∠*L* = 121°, find the measure of ∠*M*.

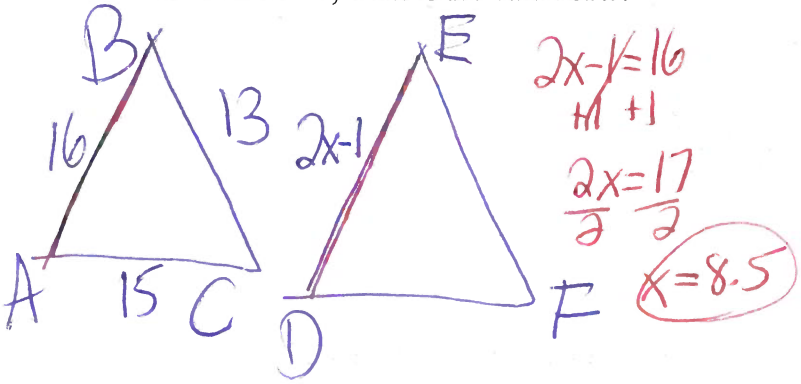




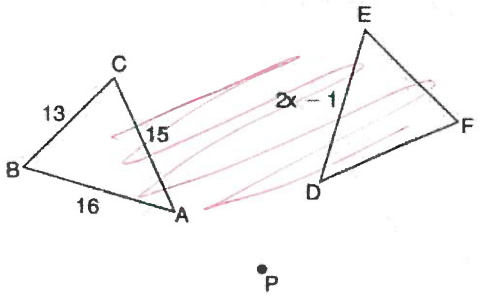
104
+121
+82
307

360
−307
53°

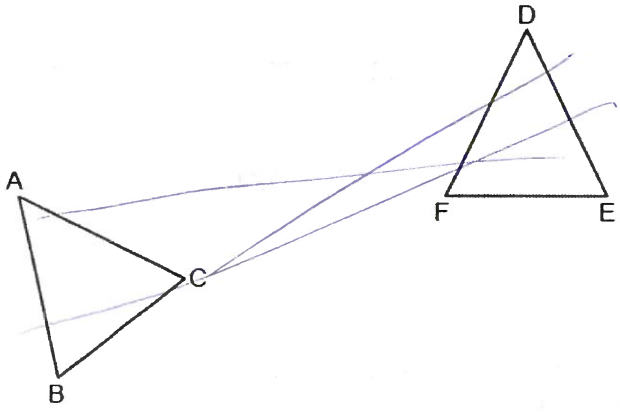
7. In the diagram below, $\triangle ABC$ with sides 13, 15, and 16, is mapped onto $\triangle DEF$ after a clockwise rotation of 90° about point P.
If $DE = 2x - 1$, what is the value of x?

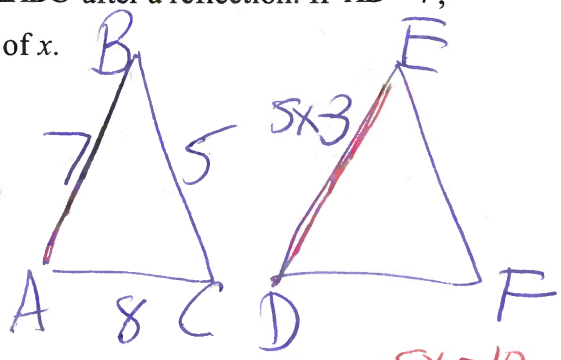


$2x - 1 = 16$
$+1 \quad +1$
$\frac{2x}{2} = \frac{17}{2}$
$x = 8.5$



8. In the diagram below, ΔDEF is the image of ΔABC after a reflection. If $\overline{AB} = 7$, $\overline{CB} = 5$, $\overline{AC} = 8$, and $\overline{DE} = 5x - 3$, find the value of x.

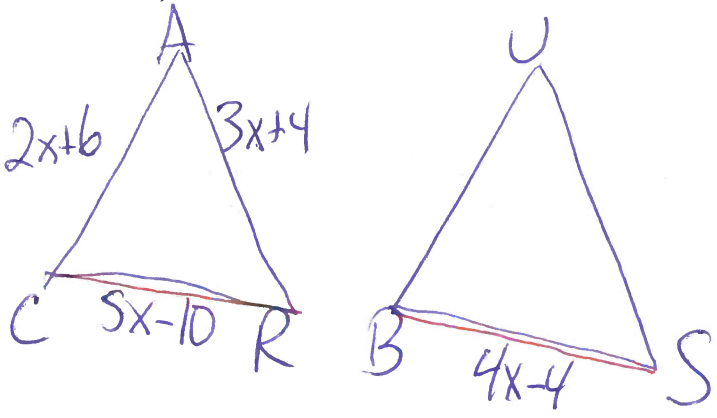


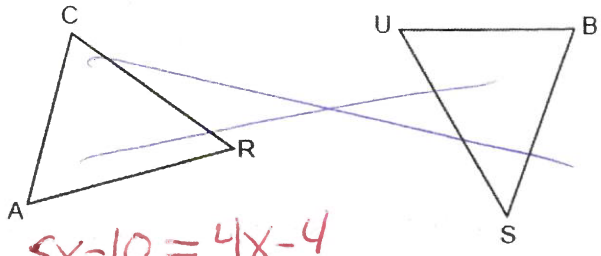


$5x - 3 = 7$
$+3 \quad +3$
$\frac{5x}{5} = \frac{10}{5}$
$x = 2$

9. In the diagram below, $\triangle CAR$ is mapped onto $\triangle BUS$ after a sequence of rigid motions.
If $AR = 3x + 4$, $RC = 5x - 10$, $CA = 2x + 6$, and $SB = 4x - 4$, what is the length of $\overline{SB}$?

1) 6
2) 16
3) 20
4) 28



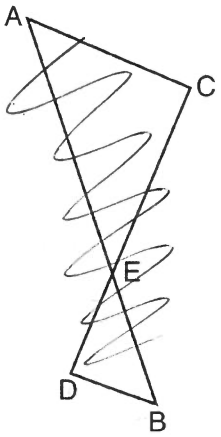


$5x - 10 = 4x - 4$
$-4x \quad -4x$
$x - 10 = -4$
$+10 \quad +10$
$x = 6$

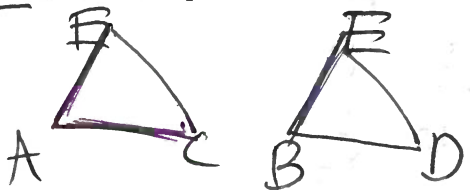
$SB = 4(6) - 4$
$SB = 20$

To determine if a proportion is correct
Look at the letters vertically and horizontally
One direction, the letters should correspond
Second direction, the letters should be in the same triangle
*It does not matter which direction does which

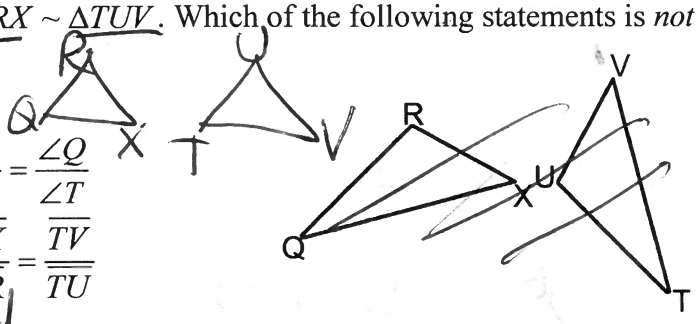
1. As shown in the diagram below, $\overline{AB}$ and $\overline{CD}$ intersect at E, and $\overline{AC} \parallel \overline{BD}$. Given $\triangle AEC \sim \triangle BED$, which equation is true?



1) $\frac{CE}{DE} = \frac{EB}{EA}$
2) $\frac{AE}{BE} = \frac{AC}{BD}$
3) $\frac{EC}{AE} = \frac{BE}{ED}$
4) $\frac{ED}{EC} = \frac{AC}{BD}$



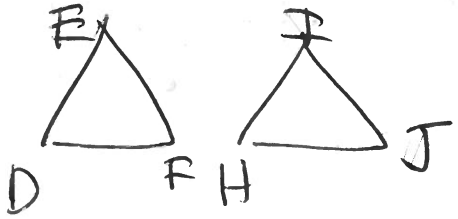
2. In the diagram below, $\Delta QRX \sim \Delta TUV$. Which of the following statements is *not* true?



1) $\frac{QR}{TU} = \frac{QX}{TV}$
2) $\frac{\angle X}{\angle V} = \frac{\angle Q}{\angle T}$
3) $\frac{RX}{UV} = \frac{VT}{XQ}$
4) $\frac{QX}{QR} = \frac{TV}{TU}$

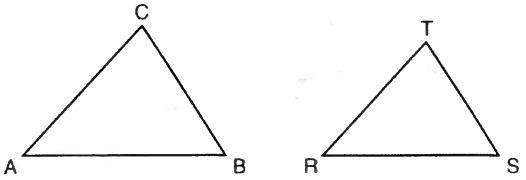
doesn't correspond

3. Given that $\Delta DEF \sim \Delta HIJ$, which is the correct statement about their corresponding sides?



1) $\frac{EF}{IJ} = \frac{DE}{HI}$
2) $\frac{EF}{HI} = \frac{IJ}{DE}$
3) $\frac{DE}{HJ} = \frac{EF}{HI}$
4) $\frac{DE}{JI} = \frac{EF}{HJ}$

4. In the diagram below, $\triangle ABC \sim \triangle RST$.



Which statement is *not* true?
1) $\angle A \cong \angle R$ ✓
2) $\frac{AB}{RS} = \frac{BC}{ST}$
3) $\frac{AB}{BC} = \frac{ST}{RS}$ ✗
4) $\angle B \cong \angle S$ ✓

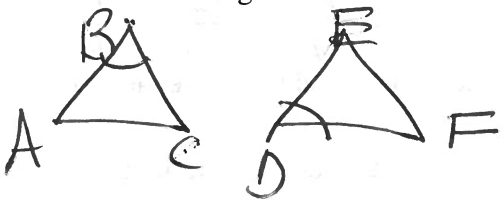
5. Scalene triangle ABC is similar to triangle DEF. Which statement is *false*?

1) $\frac{\overline{AB}}{\overline{BC}} = \frac{\overline{DE}}{\overline{EF}}$

2) $\frac{\overline{AC}}{\overline{DF}} = \frac{\overline{BC}}{\overline{EF}}$

3) $\angle ACB \cong \angle DFE$

4) $\angle ABC \cong \angle EDF$



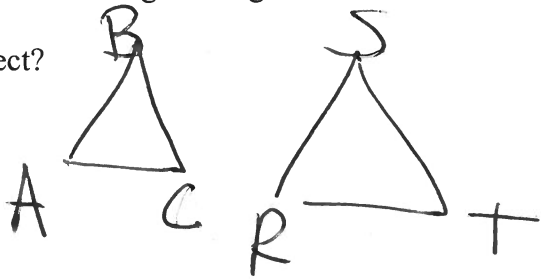
6. Given right triangle ABC with a right angle at C, $m\angle B = 61°$. Given right triangle RST with a right angle at T, $m\angle R = 29°$.
Which proportion in relation to $\triangle ABC$ and $\triangle RST$ is *not* correct?

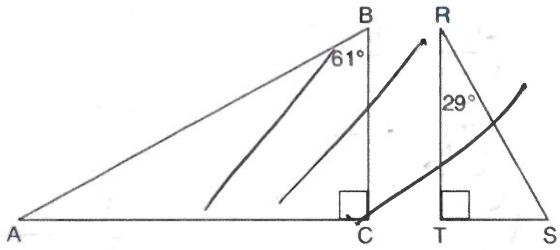
1) $\frac{AB}{RS} = \frac{RT}{AC}$

2) $\frac{BC}{ST} = \frac{AB}{RS}$

3) $\frac{BC}{ST} = \frac{AC}{RT}$

4) $\frac{AB}{AC} = \frac{RS}{RT}$





7. In the diagram below, $\triangle DEF$ is the image of $\triangle ABC$ after a clockwise rotation of 180° and a dilation where $AB = 3$, $BC = 5.5$, $AC = 4.5$, $DE = 6$, $FD = 9$, and $EF = 11$.

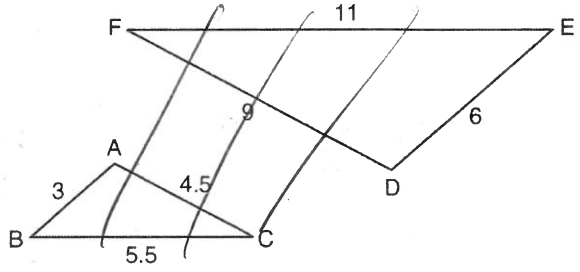
Which relationship must always be true?

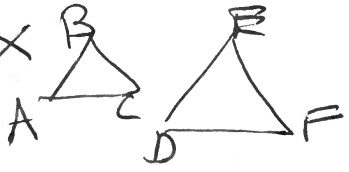
1) $\frac{m\angle A}{m\angle D} = \frac{1}{2}$ X angles are always in 1:1 ratio

2) $\frac{m\angle C}{m\angle F} = \frac{2}{1}$ X

3) $\frac{m\angle A}{m\angle C} = \frac{m\angle F}{m\angle D}$ X

4) $\frac{m\angle B}{m\angle E} = \frac{m\angle C}{m\angle F}$ ✓





8. In the diagram below of isosceles triangle AHE with the vertex angle at H, $\overline{CB} \perp \overline{AE}$ and $\overline{FD} \perp \overline{AE}$.

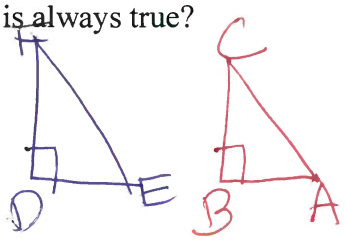
Which statement is always true?

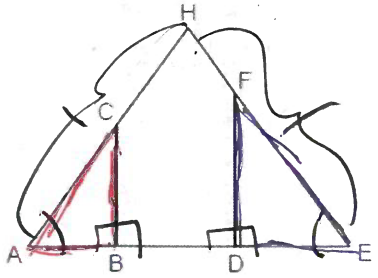
1) $\frac{AH}{AC} = \frac{EH}{EF}$

2) $\frac{AC}{EF} = \frac{AB}{ED}$

3) $\frac{AB}{ED} = \frac{CB}{FE}$

4) $\frac{AD}{AB} = \frac{BE}{DE}$





Candy Corn Problems: Is the Proportion True?

Have a picture of the original problem and the triangles separated.

If bases are not involved, see if it satisfies $\frac{top}{top} = \frac{bottom}{bottom} = \frac{side}{side}$

If bases are involved, separate the triangles and follow the same procedure from previous lesson.

1. In the diagram below of $\triangle ACT$, $\overleftrightarrow{ES}$ is drawn parallel to $\overline{AT}$ such that E is on $\overline{CA}$ and S is on $\overline{CT}$.

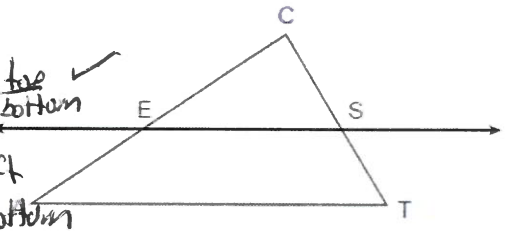
Which statement is always true?

1) $\frac{CE}{CA} = \frac{CS}{ST}$ ~~top/bottom~~ top/side = top/bottom

2) $\frac{CE}{ES} = \frac{EA}{AT}$

3) $\frac{CE}{EA} = \frac{CS}{ST}$ top/bottom = top/bottom ✓

4) $\frac{CE}{ST} = \frac{EA}{CS}$ top left / right bottom ✗



2. In $\triangle ABC$ below, $\overline{DE}$ is drawn such that D and E are on $\overline{AB}$ and $\overline{AC}$, respectively.

If $\overline{DE} \parallel \overline{BC}$, which equation will always be true?

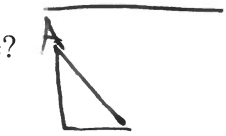
1) $\frac{AD}{DE} = \frac{DB}{BC}$

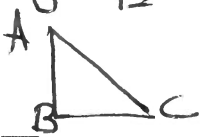
2) ~~$\frac{AD}{DE} = \frac{AB}{BC}$~~

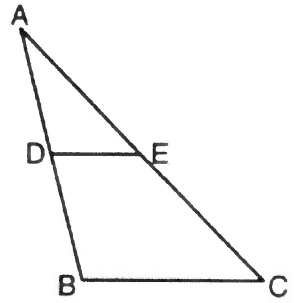
3) $\frac{AD}{BC} = \frac{DE}{DB}$

4) $\frac{AD}{BC} = \frac{DE}{AB}$

bases involved







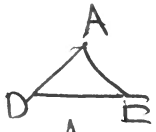
3. In the diagram of $\triangle ABC$ shown below, $\overline{DE} \parallel \overline{BC}$. Which of the following statements is *not* true?

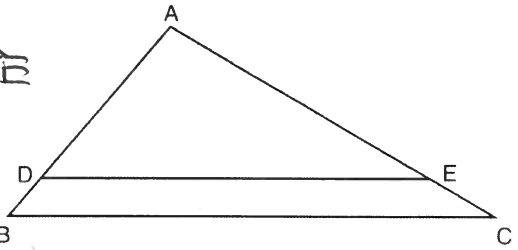
1) $\frac{AD}{DE} = \frac{AB}{BC}$

2) $\frac{BC}{DE} = \frac{CA}{EA}$

3) $\frac{AD}{AE} = \frac{DB}{AC}$ top/top = bottom/side ✗

4) $\frac{DB}{EC} = \frac{AB}{AC}$ bottom/bottom = side/side ✓





4. In the diagram below of right triangle AED, $\overline{BC} \parallel \overline{DE}$.

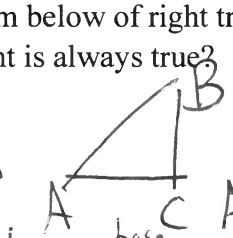
Which statement is always true?

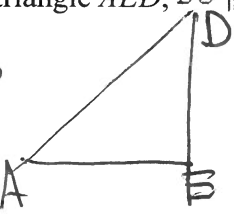
1) $\frac{AC}{BC} = \frac{DE}{AE}$ ✗

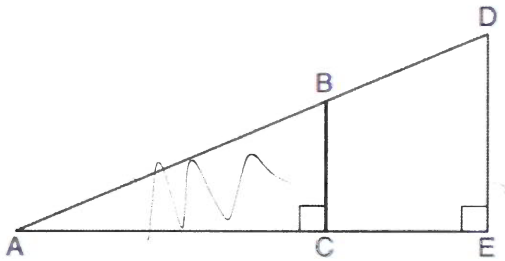
2) $\frac{AB}{AD} = \frac{BC}{DE}$ ✓

3) $\frac{AC}{CE} = \frac{BC}{DE}$ top/bottom = base/base ✗

4) $\frac{DE}{BC} = \frac{DB}{AB}$ base/bottom = bottom/top ✗







HLLS SAAS Problems: Is the Proportion True?

See if each proportion satisfies $\frac{H}{L}=\frac{L}{S}$ or $\frac{S}{A}=\frac{A}{S}$.

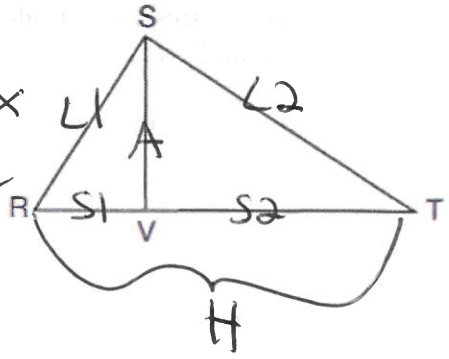
1. In right triangle *RST* below, altitude $\overline{SV}$ is drawn to hypotenuse $\overline{RT}$. Which of the following proportions is true?

1) $\frac{\overline{RV}}{\overline{VS}}=\frac{\overline{VT}}{\overline{VS}}$ $\frac{S_1}{A}=\frac{S_2}{A}$ ✗

2) $\frac{\overline{RT}}{\overline{RS}}=\frac{\overline{RS}}{\overline{VT}}$ $\frac{H}{L_1}=\frac{L_1}{S_2}$ ✗

3) $\frac{\overline{RT}}{\overline{SV}}=\frac{\overline{SV}}{\overline{VT}}$ $\frac{H}{A}=\frac{A}{S_2}$ ✗

4) $\frac{\overline{RT}}{\overline{ST}}=\frac{\overline{ST}}{\overline{VT}}$ $\frac{H}{L_2}=\frac{L_2}{S_2}$ ✓



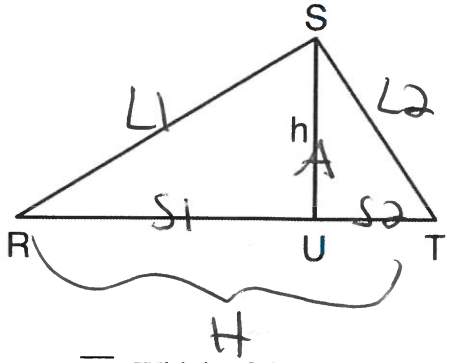
2. In right triangle *RST* below, altitude $\overline{SU}$ is drawn to hypotenuse $\overline{RT}$. Which of the following proportions is *not* true?

1) $\frac{\overline{RU}}{\overline{SU}}=\frac{\overline{SU}}{\overline{UT}}$ $\frac{S_1}{A}=\frac{A}{S_2}$ ✓

2) $\frac{\overline{SU}}{\overline{RU}}=\frac{\overline{RU}}{\overline{UT}}$ $\frac{A}{S_1}=\frac{S_1}{S_2}$ ✗

3) $\frac{\overline{RT}}{\overline{RS}}=\frac{\overline{RS}}{\overline{RU}}$ $\frac{H}{L_1}=\frac{L_1}{S_1}$ ✓

4) $\frac{\overline{TR}}{\overline{ST}}=\frac{\overline{ST}}{\overline{UT}}$ $\frac{H}{L_2}=\frac{L_2}{S_2}$ ✓



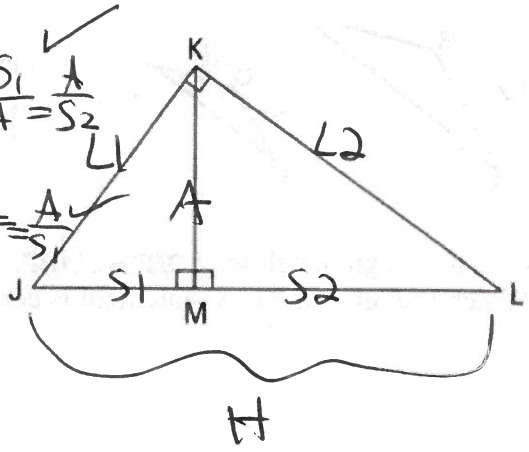
3. In right triangle *JKL* below, altitude $\overline{KM}$ is drawn to hypotenuse $\overline{JL}$. Which of the following proportions is *not* true?

1) $\frac{\overline{JL}}{\overline{JK}}=\frac{\overline{JK}}{\overline{KM}}$ $\frac{H}{L_1}=\frac{L_1}{S_1}$ ✓

2) $\frac{\overline{JM}}{\overline{KM}}=\frac{\overline{KM}}{\overline{ML}}$ $\frac{S_1}{A}=\frac{A}{S_2}$ ✓

3) $\frac{\overline{JL}}{\overline{KL}}=\frac{\overline{KL}}{\overline{JM}}$ $\frac{H}{L_2}=\frac{L_2}{S_1}$ ✗

4) $\frac{\overline{ML}}{\overline{MK}}=\frac{\overline{MK}}{\overline{MJ}}$ $\frac{S_2}{A}=\frac{A}{S_1}$ ✓

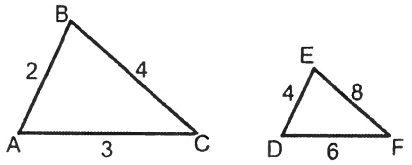


Name Schlansky
Mr. Schlansky

Date _____
Geometry

Determining Whether Triangles are Similar

1. Determine whether the following triangles are similar. Explain your answer.

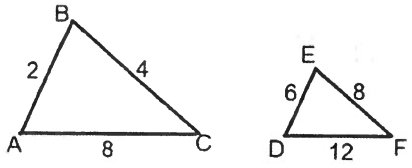


$\frac{2}{4}=\frac{4}{8}=\frac{3}{6}$

$\frac{1}{2}=\frac{1}{2}=\frac{1}{2}$ ✓

Yes, SSS
Three pairs of corresponding sides are in proportion

2. Determine whether the following triangles are similar. Explain your answer.



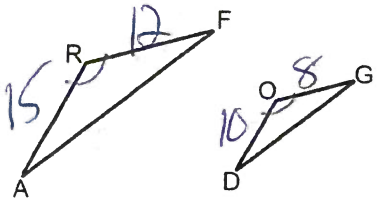
$\frac{2}{6}=\frac{4}{8}\neq\frac{8}{12}$

$\frac{1}{3}\neq\frac{1}{2}\neq\frac{2}{3}$

No, the sides are not in proportion.

✗

3. In the diagram below, $\overline{AR}=15$, $\overline{RF}=12$, $\overline{DO}=10$, $\overline{OG}=8$, and $\angle ARF \cong \angle DOG$. Must $\Delta ARF \sim \Delta DOG$? Explain your answer.



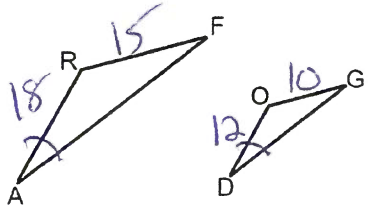
$\frac{15}{10}=\frac{12}{8}$

$\frac{3}{2}=\frac{3}{2}$ ✓

Yes, SAS.
Two pairs of corresponding sides are in proportion and the angle between them is congruent.

4. In the diagram below, $\overline{AR}=18$, $\overline{RF}=15$, $\overline{DO}=12$, $\overline{OG}=10$, and $\angle RAF \cong \angle ODG$. Must $\Delta ARF \sim \Delta DOG$? Explain your answer.

No!
The angle is not inbetween the two sides so it is not SAS!

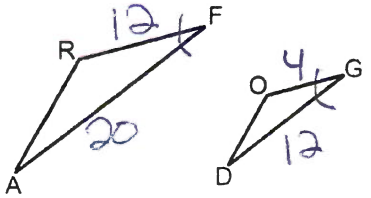


$\frac{18}{12}=\frac{15}{10}$

$\frac{3}{2}=\frac{3}{2}$ but it doesn't matter

5. In the diagram below, $\overline{AF}=20$, $\overline{RF}=12$, $\overline{DG}=12$, $\overline{OG}=4$, and $\angle F \cong \angle G$. Must $\Delta ARF \sim \Delta DOG$? Explain your answer.

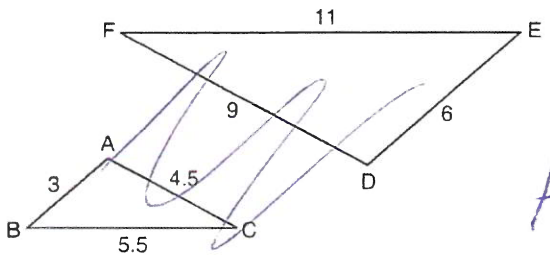
No!
The angle is inbetween the 2 sides but the sides are not in proportion.

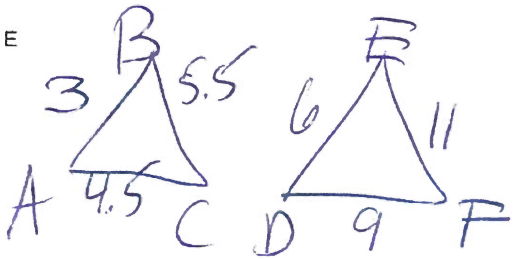


$\frac{12}{4}=\frac{20}{12}$

$3 \neq \frac{5}{3}$ X

6. In the diagram below, $\triangle DEF$ is the image of $\triangle ABC$ after a clockwise rotation of 180° and a dilation where $AB = 3$, $BC = 5.5$, $AC = 4.5$, $DE = 6$, $FD = 9$, and $EF = 11$.





Show that $\Delta ABC \sim \Delta DEF$

$\frac{3}{6}=\frac{5.5}{11}=\frac{4.5}{9}$

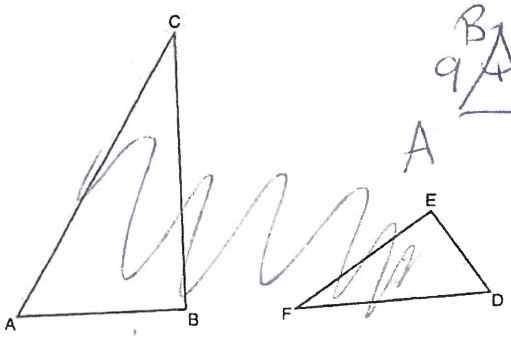
Yes, SSS.

$\frac{1}{2}=\frac{1}{2}=\frac{1}{2}$

The pairs of corresponding sides are in proportion

7. Triangles ABC and DEF are drawn below.

If $AB = 9$, $BC = 15$, $DE = 6$, $EF = 10$, and $\angle B \cong \angle E$, are the triangles similar? Explain your answer.



B E
9 15 6 10
A C D F

$\frac{9}{6} = \frac{15}{10}$ Yes, SAS

$\frac{3}{2} = \frac{3}{2}$ ✓

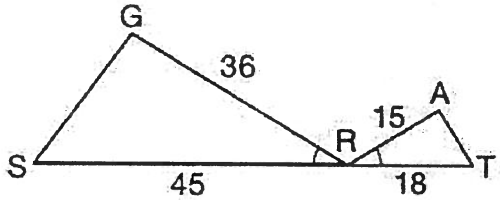
8. Triangles RST and XYZ are drawn below. If $RS = 6$, $ST = 14$, $XY = 9$, $YZ = 21$, and $\angle S \cong \angle Y$, is $\triangle RST$ similar to $\triangle XYZ$? Justify your answer.

S Y
6 14 9 21
R T X Z

$\frac{6}{9} = \frac{14}{21}$ Yes, SAS

$\frac{2}{3} = \frac{2}{3}$ ✓

9. In the diagram below, $\angle GRS \cong \angle ART$, $GR = 36$, $SR = 45$, $AR = 15$, and $RT = 18$.



It's either SAS or not similar

$\frac{36}{15} = \frac{45}{18}$

Which triangle similarity statement is correct?

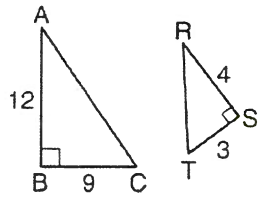
1) ~~$\triangle GRS \sim \triangle ART$ by AA.~~
2) $\triangle GRS \sim \triangle ART$ by SAS.
3) ~~$\triangle GRS \sim \triangle ART$ by SSS.~~
4) $\triangle GRS$ is not similar to $\triangle ART$.

$\frac{12}{5} \neq \frac{15}{6}$

Not Similar
Not in proportion

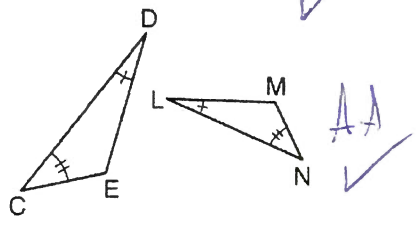
10. Using the information given below, which set of triangles can *not* be proven similar?

1)



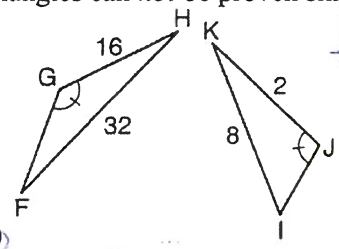
$\frac{12}{4}=\frac{9}{3}$ $\frac{3}{1}=\frac{3}{1}$ SAS ✓

2)



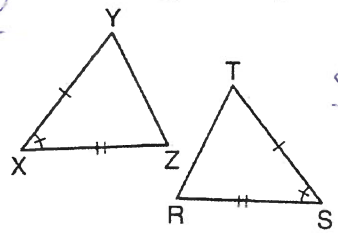
AA ✓

3)



$\frac{16}{2}=\frac{32}{8}$ also, the angle $8 \neq 4$ is not in between the two sides ✗

4)



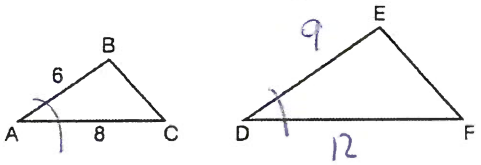
SAS ✓

11. In the diagram below, $\triangle ABC \sim \triangle DEF$.

the angle must be inbetween the two sides

If $AB = 6$ and $AC = 8$, which statement will justify similarity by SAS?

1) $DE = 9$, $DF = 12$, and $\angle A \cong \angle D$
2) $DE = 8$, $DF = 10$, and $\angle A \cong \angle D$
3) $DE = 36$, $DF = 64$, and $\angle C \cong \angle F$
4) $DE = 15$, $DF = 20$, and $\angle C \cong \angle F$



1) $\frac{6}{9}=\frac{8}{12}$ 10 $\frac{2}{3}=\frac{2}{3}$ ✓

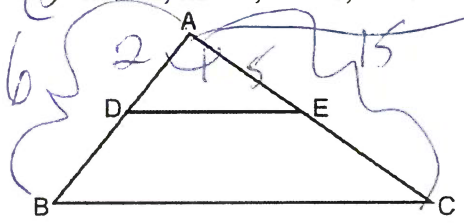
2) $\frac{6}{8}=\frac{8}{10}$ $\frac{3}{4} \neq \frac{4}{5}$

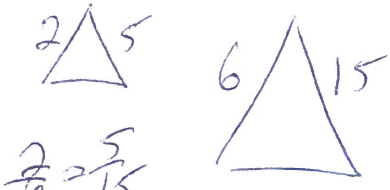
12. In the diagram below, $\triangle ABC \sim \triangle ADE$.

Which measurements are justified by this similarity?

1) $AD = 3$, $AB = 6$, $AE = 4$, and $AC = 12$
2) $AD = 5$, $AB = 8$, $AE = 7$, and $AC = 10$
3) $AD = 3$, $AB = 9$, $AE = 5$, and $AC = 10$
4) $AD = 2$, $AB = 6$, $AE = 5$, and $AC = 15$

reflexive

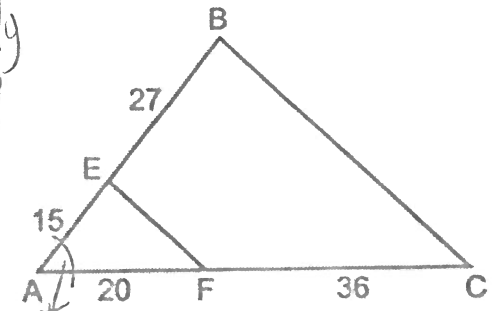




$\frac{2}{6}=\frac{5}{15}$ $\frac{1}{3}=\frac{1}{3}$ ✓ SAS

13. In the diagram below, $AE = 15$, $EB = 27$, $AF = 20$, and $FC = 36$. Is $\Delta ABC \sim \Delta AEF$. Explain your answer.

Candy Corn



reflexive

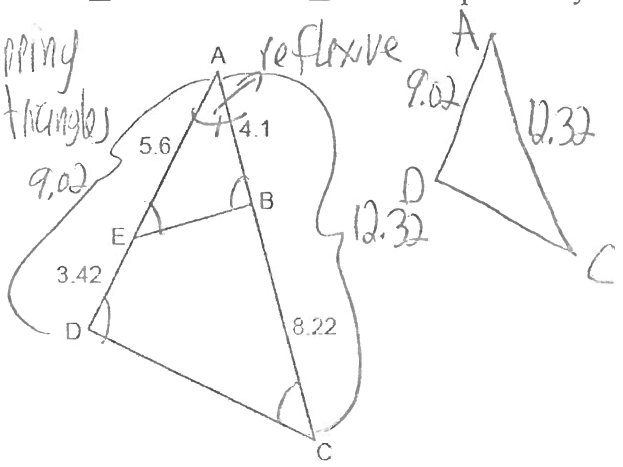
$\frac{15}{20} = \frac{27}{36}$

$\frac{3}{4} = \frac{3}{4}$

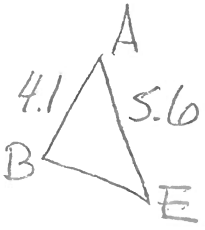
Yes, SAS

14. In ΔADC below, $\overline{EB}$ is drawn such that $AB = 4.1$, $AE = 5.6$, $BC = 8.22$, and $ED = 3.42$. Is ΔABE similar to ΔADC? Explain why.

Overlapping Similar triangles



reflexive

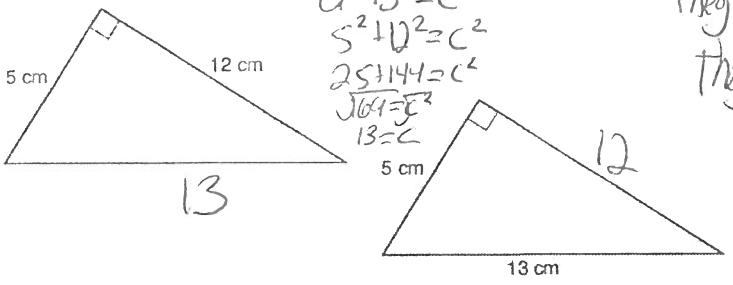


$\frac{9.02}{4.1} = \frac{12.32}{5.6}$

$2.2 = 2.2$

Yes, SAS

15. Skye says that the two triangles below are congruent. Margaret says that the two triangles are similar. Are Skye and Margaret both correct? Explain why.



$a^2 + b^2 = c^2$

$5^2 + 12^2 = c^2$

$25 + 144 = c^2$

$\sqrt{169} = \sqrt{c^2}$

$13 = c$

They are congruent.

They are similar.

Congruent figures are always similar

16. If ΔABC is mapped onto ΔDEF after a line reflection and ΔDEF is mapped onto ΔXYZ after a translation, the relationship between ΔABC and ΔXYZ is that they are always

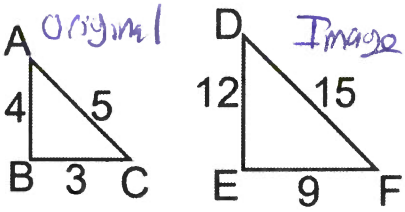
1) congruent and similar
2) congruent but not similar
3) similar but not congruent
4) neither similar nor congruent

Scale factor = $\frac{image}{original}$

1. In the diagram below, ΔDEF is the image of ΔABC after a dilation.

What is the scale factor of the dilation?

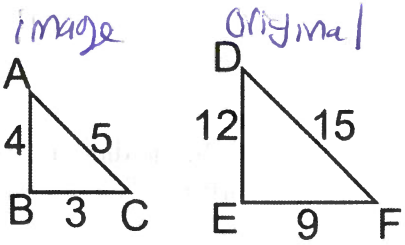
$\frac{15}{5}=3$



2. In the diagram below, ΔABC is the image of ΔDEF after a dilation.

What is the scale factor of the dilation?

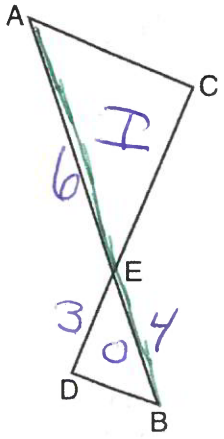
$\frac{5}{15}=\frac{1}{3}$



3. In the diagram below, ΔACE is the image of ΔBDE after a sequence of transformations. If $\overline{AE}=6$, $\overline{DE}=3$, and $\overline{EB}=4$, what is the scale factor?

$\frac{6}{4}=\frac{3}{2}$

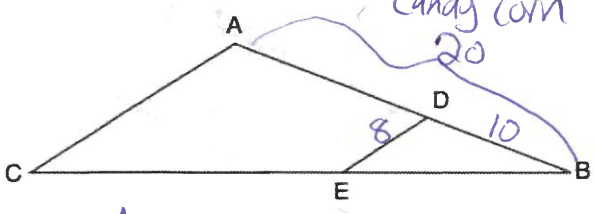
Bow Tie

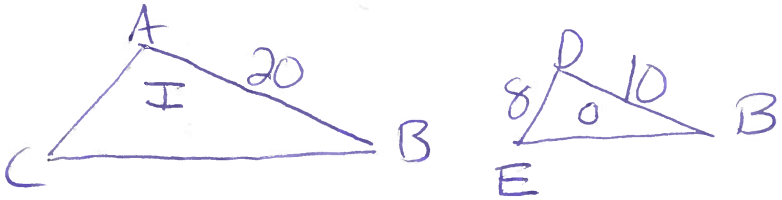


4. In the diagram below, ΔABC is the image of ΔDBE after a dilation centered at point A. If $\overline{AB}=20$, $\overline{DE}=8$, and $\overline{DB}=10$, what is the scale factor?

Candy corn

$\frac{20}{10}=2$





5. After a dilation with center (0, 0), the image of $\overline{DB}$ is $\overline{D'B'}$. If $DB = 4.5$ and $D'B' = 18$, what is the scale factor of this dilation?

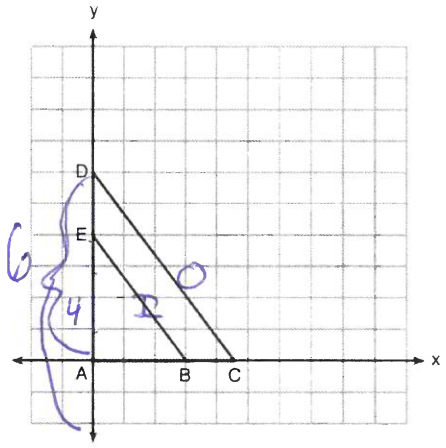
$\frac{18}{4.5} = 4$

6. In the diagram below, $\triangle ABE$ is the image of $\triangle ACD$ after a dilation centered at the origin. The coordinates of the vertices are $A(0, 0)$, $B(3, 0)$, $C(4.5, 0)$, $D(0, 6)$, and $E(0, 4)$.

The scale factor of dilation is

1) $\frac{2}{3}$
2) $\frac{3}{2}$
3) $\frac{3}{4}$
4) $\frac{4}{3}$

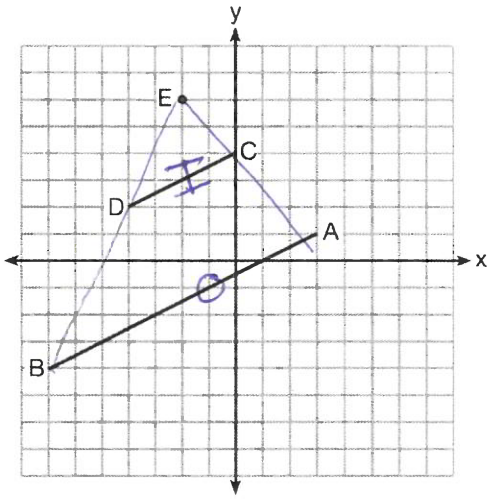
$\frac{4}{6} = \frac{2}{3}$



7. In the diagram below, $\overline{CD}$ is the image of $\overline{AB}$ after a dilation of scale factor k with center E.

Which ratio is equal to the scale factor k of the dilation?

1) $\frac{EC}{EA}$
2) $\frac{BA}{EA}$
3) $\frac{EA}{BA}$
4) $\frac{EA}{EC}$



Dilating Segments with Perimeter and Area

Multiply the original segment and scale factor to find the image.
Multiply the original perimeter and scale factor to find the image perimeter.
Multiply the original area and the $(scale\ factor)^2$ to find the image area.
*You may have to use distance formula to find original segment.
*The center of dilation does not effect the size of the image
*When the midpoints are joined, the scale factor is $\frac{1}{2}$

1. A line segment with a length of 5 is dilated by a scale factor of 4. What is the length of its image?

$5(4)=20$

2. A line segment has a length of 12 and is dilated by $\frac{1}{2}$. What is the length of its image?

$12(\frac{1}{2})=6$

3. A three-inch line segment is dilated by a scale factor of 6 and centered at its midpoint. What is the length of its image?

1) 9 inches
2) 2 inches
3) 15 inches
4) 18 inches

$3(6)=18$

4. Triangle JOY has a perimeter of 10 and an area of 12. What is the perimeter and area of triangle JOY after a dilation by a scale factor of 2?

perimeter image = scale factor(perimeter original)
perimeter = $2(10)=20$

area image = area original(scale factor)^2
area = $12(2)^2=48$

5. Quadrilateral CAMI has a perimeter of 20 and an area of 15. What is the perimeter and area of quadrilateral CAMI after a dilation by a scale factor of 4?

perimeter image = perimeter original(scale factor)
perimeter = $20(4)=80$

area image = area original(scale factor)^2
area = $15(4)^2=240$

Perimeter RSTV = 4(9) = 36

6. Given square *RSTV*, where *RS* = 9 cm. If square *RSTV* is dilated by a scale factor of 3 about a given center, what is the perimeter, in centimeters, of the image of *RSTV* after the dilation?

1) 12
2) 27
3) 36
4) 108

Perimeter image = perimeter original (scale factor)
Perimeter = 36(3) = 108

7. Triangle *RJM* has an area of 6 and a perimeter of 12. If the triangle is dilated by a scale factor of 3 centered at the origin, what are the area and perimeter of its image, triangle *R'J'M'*?

1) area of 9 and perimeter of 15
2) area of 18 and perimeter of 36
3) area of 54 and perimeter of 36
4) area of 54 and perimeter of 108

P image = P original (scale factor)
P = 12(3) = 36
A image = A original (scale factor)^2
A = 6(3)^2 = 54

8. Rectangle *A'B'C'D'* is the image of rectangle *ABCD* after a dilation centered at point *A* by a scale factor of $\frac{2}{3}$. Which statement is correct?

P image = $\frac{2}{3}$ P original

1) Rectangle *A'B'C'D'* has a perimeter that is $\frac{2}{3}$ the perimeter of rectangle *ABCD*.
2) Rectangle *A'B'C'D'* has a perimeter that is $\frac{3}{2}$ the perimeter of rectangle *ABCD*.
3) Rectangle *A'B'C'D'* has an area that is $\frac{2}{3}$ the area of rectangle *ABCD*.
4) Rectangle *A'B'C'D'* has an area that is $\frac{3}{2}$ the area of rectangle *ABCD*.

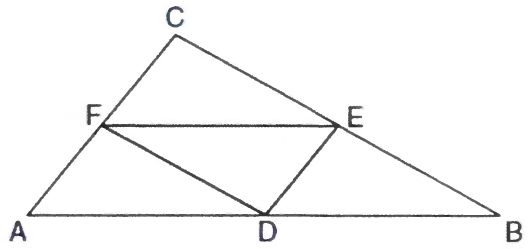
1:2 ratio

9. In the diagram below of $\triangle ABC$, *D*, *E*, and *F* are the midpoints of $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$, respectively.

What is the ratio of the area of $\triangle CFE$ to the area of $\triangle CAB$?

1) 1:1
2) 1:2
3) 1:3
4) 1:4

(scale factor/ratio)^2
(1:2)^2 = 1^2:2^2
1:4



10. The area of $\triangle TAP$ is 36 cm^2. A second triangle, *JOE*, is formed by connecting the midpoints of each side of $\triangle TAP$. What is the area of *JOE*, in square centimeters?

1) 9
2) 12
3) 18
4) 27

1:2 scale factor = $\frac{1}{2}$

area image = area original (scale factor)^2
= 36$(\frac{1}{2})^2$
= 9

Trigonometric Ratios (SOHCAHTOA)

1) Label each side with H, A, and O

2) Use SOHCAHTOA ($\sin\theta = \frac{O}{H}$, $\cos\theta = \frac{A}{H}$, $\tan\theta = \frac{O}{A}$)

1. Find the following trig ratios for the given triangle.

$\sin A$ $\frac{O}{H} = \frac{8}{17}$

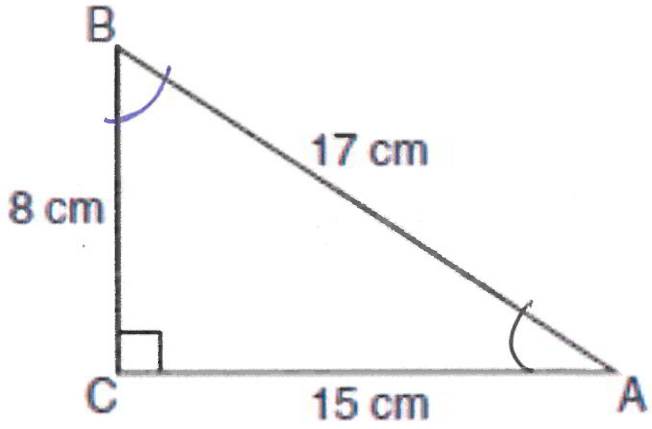
$\cos A$ $\frac{A}{H} = \frac{15}{17}$

$\tan A$ $\frac{O}{A} = \frac{8}{15}$

$\sin B$ $\frac{O}{H} = \frac{15}{17}$

$\cos B$ $\frac{A}{H} = \frac{8}{17}$

$\tan B$ $\frac{O}{A} = \frac{15}{8}$



2. Find the following trig ratios for the given triangle.

$\sin J$ $\frac{O}{H} = \frac{7}{25}$

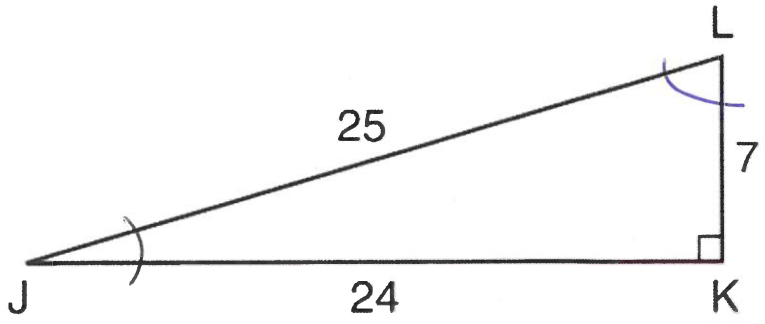
$\cos J$ $\frac{A}{H} = \frac{24}{25}$

$\tan J$ $\frac{O}{A} = \frac{7}{24}$

$\sin L = \frac{O}{H} = \frac{24}{25}$

$\cos L = \frac{A}{H} = \frac{7}{25}$

$\tan L = \frac{O}{A} = \frac{24}{7}$



3. In ΔABC below, the measure of $\angle A = 90°$, $AB = 6$, $AC = 8$, and $BC = 10$.

Which ratio represents the cosine of $\angle B$?

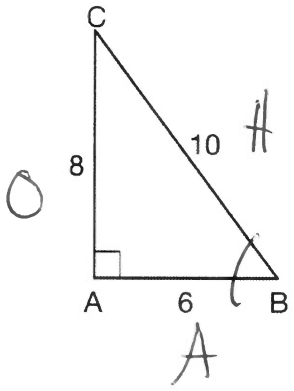
1) $\frac{10}{8}$

2) $\frac{8}{6}$

3) $\frac{6}{10}$

4) $\frac{8}{10}$

$\frac{A}{H} = \frac{6}{10}$



4. In triangle MCT, the measure of $\angle T = 90°$, $MC = 85$ cm, $CT = 84$ cm, and $TM = 13$cm. Which ratio represents the sine of $\angle C$?

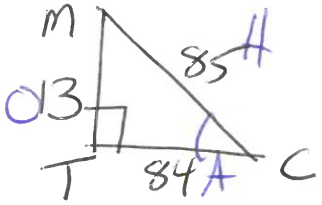
1) $\frac{13}{85}$

2) $\frac{84}{85}$

3) $\frac{13}{84}$

4) $\frac{84}{13}$

$\frac{O}{H} = \frac{13}{85}$



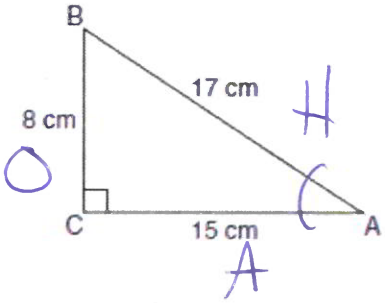
5. Which equation shows a correct trigonometric ratio for angle A in the right triangle below?

1) $\sin A = \frac{15}{17}$

2) $\tan A = \frac{8}{17}$

3) $\cos A = \frac{15}{17}$

4) $\tan A = \frac{5}{8}$



6. In right triangle JKL in the diagram below, $KL = 7$, $JK = 24$, $JL = 25$, and $\angle K = 90°$.

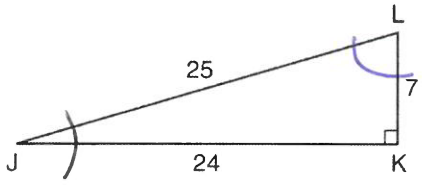
Which statement is *not* true?

1) $\tan L = \frac{24}{7}$

2) $\cos L = \frac{24}{25}$

3) $\tan J = \frac{7}{24}$

4) $\sin J = \frac{7}{25}$



~~Finding Sides and Angles with~~ Trig

Finding Sides and Angles with Trig (SOHCAHTOA)

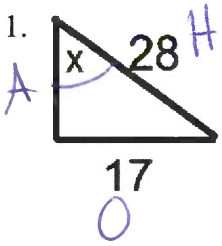
1) Label each side with H, A, and O
2) Determine whether to use sine, cosine, or tangent (Which two are involved?)
3) Substitute into appropriate formula

*If finding a side, cross multiply and solve

*If finding an angle, use $\sin^{-1}$, $\cos^{-1}$, *or* $\tan^{-1}$

*angle of depression = angle of elevation

Find x in each of the following pictures rounding to the *nearest integer*

1. 

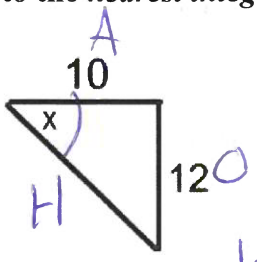
H A O

$\sin\theta = \frac{O}{H}$

$\sin^{-1}\sin X = \frac{17}{28}$

$X = \sin^{-1}\left(\frac{17}{28}\right)$

$X = 37°$

2. 

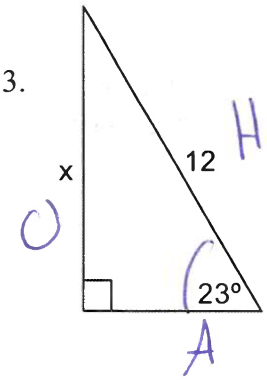
A H O

$\tan\theta = \frac{O}{A}$

$\tan^{-1}\tan X = \frac{12}{10}$

$X = \tan^{-1}\left(\frac{12}{10}\right)$

$X = 50°$

3. 

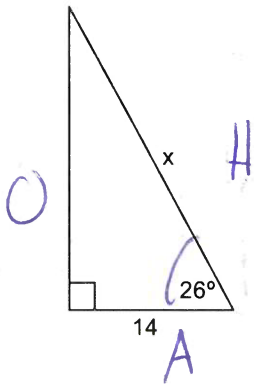
O H A

$\sin\theta = \frac{O}{H}$

$\frac{\sin 23}{1} = \frac{x}{12}$

$X = 12\sin 23$

$X = 5$

4. 

O H A

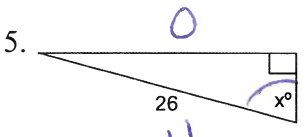
$\cos\theta = \frac{A}{H}$

$\frac{\cos 26}{1} = \frac{14}{x}$

$x\cos 26 = 14$

$\frac{x\cos 26}{\cos 26} = \frac{14}{\cos 26}$

$X = 16$

5. 

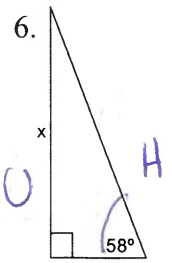
O A H

$\cos\theta = \frac{A}{H}$

$\cos^{-1}\cos X = \frac{10}{26}$

$X = \cos^{-1}\left(\frac{10}{26}\right)$

$X = 67$

6. 

O H A

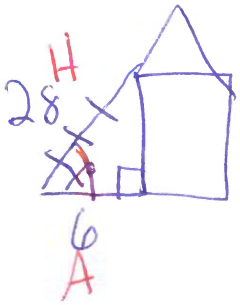
$\tan\theta = \frac{O}{A}$

$\frac{\tan 58}{1} = \frac{x}{12}$

$X = 12\tan 58$

$X = 19$

6. A 28-foot ladder is leaning against a house. The bottom of the ladder is 6 feet from the base of the house. Find the measure of the angle formed by the ladder and the ground, to the *nearest degree*.



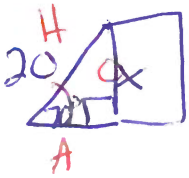
$\cos\theta = \frac{A}{H}$

$\cos^{-1}\cos X = \cos^{-1}\frac{6}{28}$

$X = \cos^{-1}\left(\frac{6}{28}\right)$

$X = 78°$

7. A 20-foot support post leans against a wall, making a 70° angle with the ground. To the *nearest tenth of a foot*, how far up the wall will the support post reach?



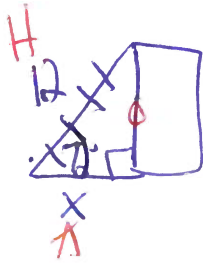
$\sin\theta = \frac{O}{H}$

$\sin 70 = \frac{X}{20}$

$X = 20\sin 70$

$X = 18.8$

8. A ladder 12 feet long leans against a wall and makes an angle of 72° with the ground. Find, to the *nearest tenth of a foot,* the distance from the wall to the base of the ladder.



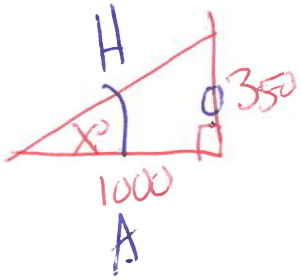
$\cos\theta = \frac{A}{H}$

$\cos 72 = \frac{X}{12}$

$X = 12\cos 72$

$X = 3.7$

9. A man standing on level ground is 1000 feet away from the base of a 350-foot-tall building. Find, to the *nearest degree*, the measure of the angle of elevation to the top of the building from the point on the ground where the man is standing.



$\tan\theta = \frac{O}{A}$

$\tan X = \frac{350}{1000}$

$X = \tan^{-1}\frac{350}{1000}$

$X = 19°$

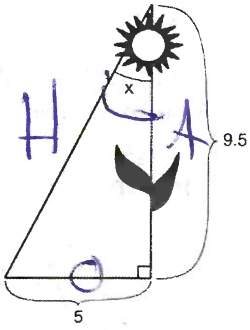
10. The diagram below shows the path a bird flies from the top of a 9.5-foot-tall sunflower to a point on the ground 5 feet from the base of the sunflower. To the *nearest tenth of a degree*, what is the measure of angle x?

$\tan\theta = \frac{O}{A}$

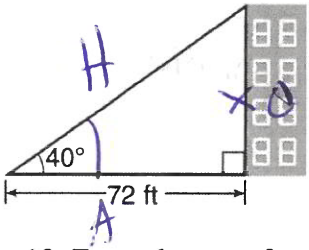
$\tan^{-1}\tan x = \tan^{-1}\frac{5}{9.5}$

$x = \tan^{-1}\left(\frac{5}{9.5}\right)$

$x = 27.8$



11. As shown in the diagram below, a building casts a 72-foot shadow on the ground when the angle of elevation of the Sun is 40°. How tall is the building, to the *nearest foot*?



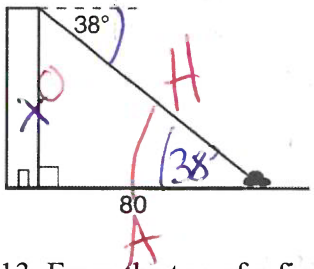
$\tan\theta = \frac{O}{A}$

$\tan 40 = \frac{x}{72}$

$x = 72\tan 40$

$x = 60$

12. From the top of an apartment building, the angle of depression to a car parked on the street below is 38 degrees, as shown in the diagram below. The car is parked 80 feet from the base of the building. Find the height of the building, to the *nearest tenth of a foot*.



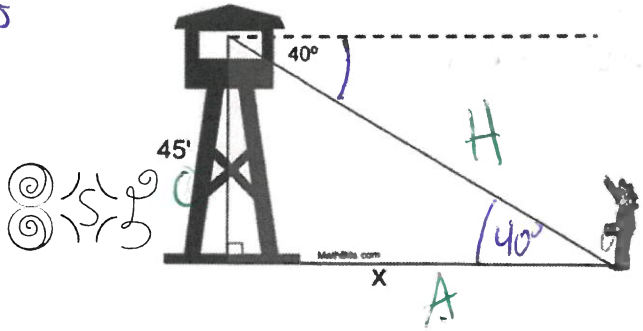
$\tan\theta = \frac{O}{A}$

$\tan 38 = \frac{x}{80}$

$x = 80\tan 38$

$x = 62.5$

13. From the top of a fire tower, a forest ranger sees his partner on the ground at an angle of depression of 40. If the tower is 45 feet in height, how far is the partner from te base of the tower to the *nearest tenth of a foot*.



$\tan\theta = \frac{O}{A}$

$\tan 40 = \frac{45}{x}$

$\frac{x\tan 40}{\tan 40} = \frac{45}{\tan 40}$

$x = 53.6$

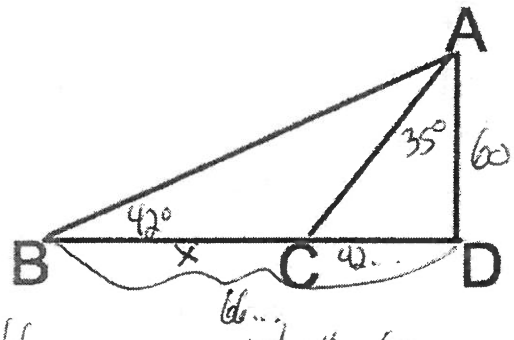
angle of elevation = angle of depression

Name Schlansky
Mr. Schlansky

Date _____
Geometry

Compound Right Triangles (Subtraction)

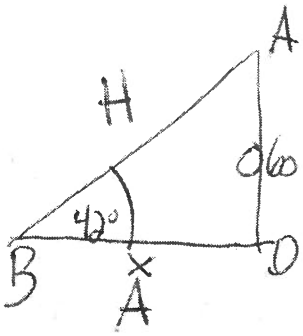
1. In the diagram below, $m\angle CAD = 35$, $m\angle ABD = 42$, and $m\overline{AD} = 60$. Find to the nearest tenth, $m\overline{BC}$.



66...
−42..
24.6

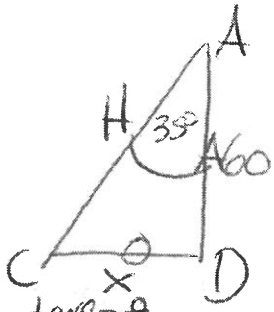
$\frac{x \tan 42}{\tan 42} = \frac{60}{\tan 42}$

$x = 66...$



$\tan\theta = \frac{O}{A}$

$\frac{\tan 42}{1} = \frac{60}{x}$



$\tan\theta = \frac{O}{A}$

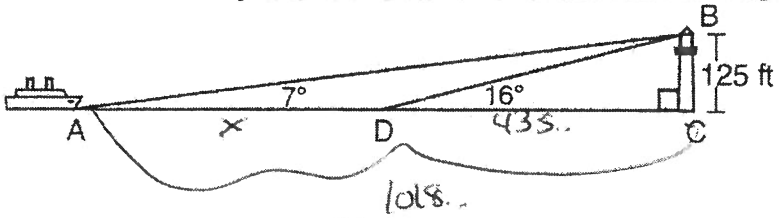
$\frac{\tan 35}{1} = \frac{x}{60}$

$x = 60 \tan 35$

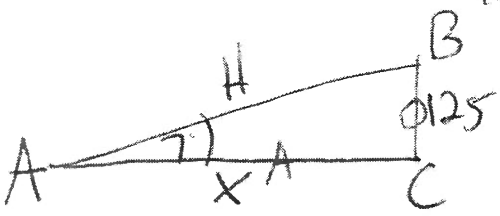
$x = 42...$

2. As shown in the diagram below, a ship is heading directly toward a lighthouse whose beacon is 125 feet above sea level. At the first sighting, point *A*, the angle of elevation from the ship to the light was 7°. A short time later, at point *D*, the angle of elevation was 16°.

To the *nearest foot*, determine and state how far the ship traveled from point *A* to point *D*.



1018..
−435..
582

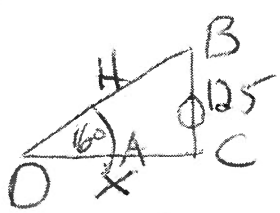


$\tan\theta = \frac{O}{A}$

$\frac{\tan 7}{1} = \frac{125}{x}$

$\frac{x \tan 7}{\tan 7} = \frac{125}{\tan 7}$

$x = 1018.$



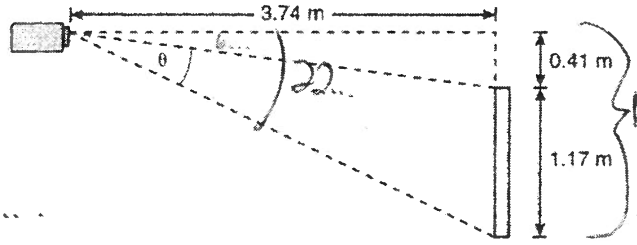
$\tan\theta = \frac{O}{A}$

$\frac{\tan 16}{1} = \frac{125}{x}$

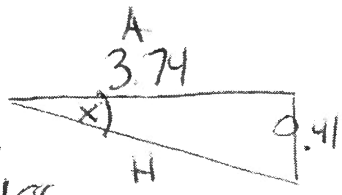
$\frac{x \tan 16}{\tan 16} = \frac{125}{\tan 16}$

$x = 435...$

3. As modeled below, a projector mounted on a ceiling is 3.74 m from a wall, where a whiteboard is displayed. The vertical distance from the ceiling to the top of the whiteboard is 0.41 m, and the height of the whiteboard is 1.17 m. Determine and state the projection angle, θ, to the *nearest tenth of a degree*.



22...
− 6...
16.6°

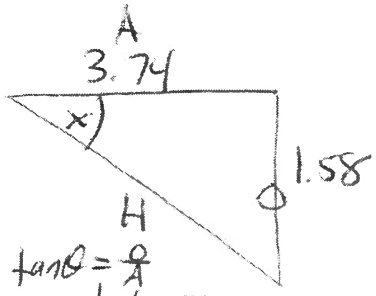


$\tan\theta = \frac{O}{A}$

$\tan^{-1}\tan x = \tan^{-1}\frac{.41}{3.74}$

$x = \tan^{-1}\left(\frac{.41}{3.74}\right)$

$x = 6...$



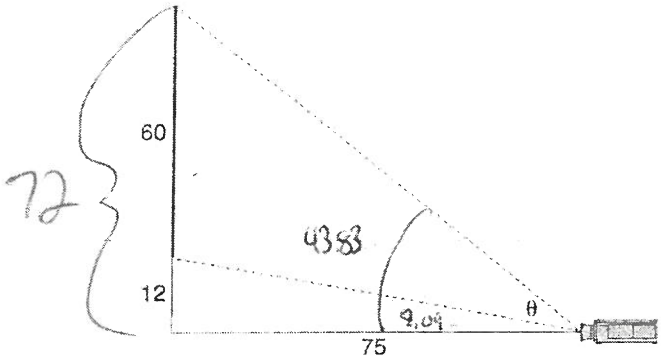
$\tan\theta = \frac{O}{A}$

$\tan^{-1}\tan x = \tan^{-1}\frac{1.58}{3.74}$

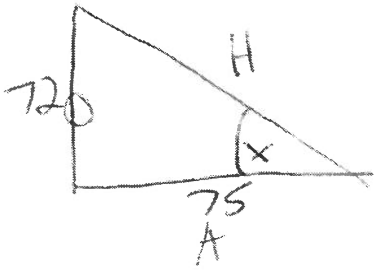
$x = \tan^{-1}\left(\frac{1.58}{3.74}\right)$

$x = 22...$

4. As modeled below, a movie is projected onto a large outdoor screen. The bottom of the 60-foot-tall screen is 12 feet off the ground. The projector sits on the ground at a horizontal distance of 75 feet from the screen.
Determine and state, to the *nearest tenth of a degree*, the measure of θ, the projection angle.



43.83...
− 9.09...
34.7°

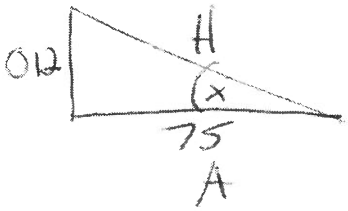


$\tan\theta = \frac{O}{A}$

$\tan^{-1}\tan x = \tan^{-1}\frac{72}{75}$

$x = \tan^{-1}\left(\frac{72}{75}\right)$

$x = 43.83...$



$\tan\theta = \frac{O}{A}$

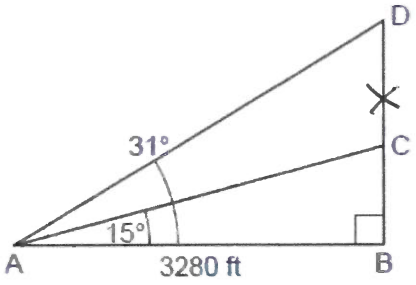
$\tan^{-1}\tan x = \tan^{-1}\frac{12}{75}$

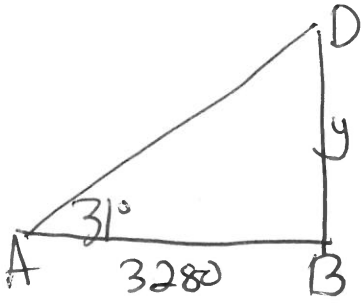
$x = \tan^{-1}\left(\frac{12}{75}\right)$

$x = 9.09...$

72
87

5. Cape Canaveral, Florida is where NASA launches rockets into space. As modeled in the diagram below, a person views the launch of a rocket from observation area *A*, 3280 feet away from launch pad *B*. After launch, the rocket was sighted at *C* with an angle of elevation of 15°. The rocket was later sighted at *D* with an angle of elevation of 31°. Determine and state, to the *nearest foot*, the distance the rocket traveled between the two sightings, *C* and *D*.

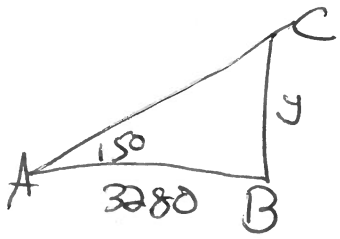




$\tan 31 = \frac{y}{3280}$

$y = 3280 \tan 31$

$y = 1970..$



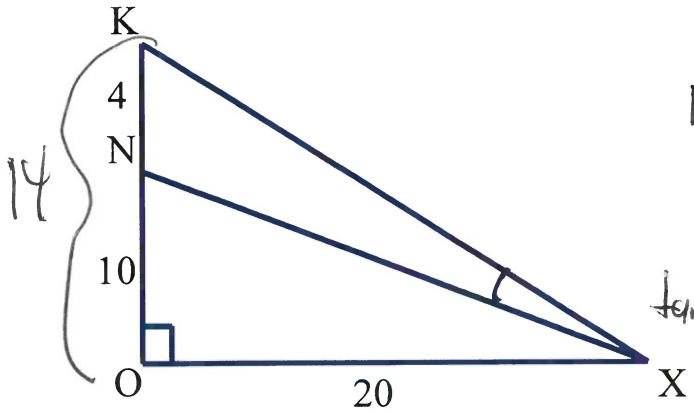
$\tan 15 = \frac{y}{3280}$

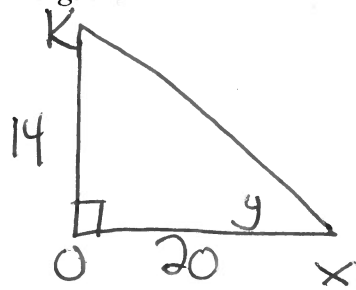
$y = 3280 \tan 15$

$y = 878..$

$1970.. - 878.. = 1092$

6. Find the measure of $\angle KXN$ below the *nearest degree*.

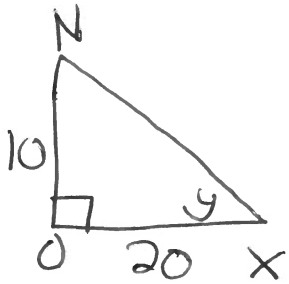




$\tan^{-1} \tan y = \tan^{-1} \frac{14}{20}$

$y = \tan^{-1} \frac{14}{20}$

$y = 34..$



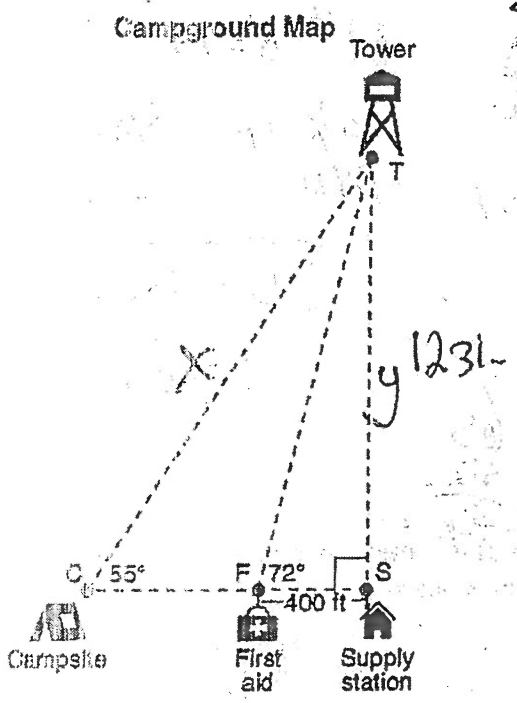
$\tan^{-1} \tan y = \tan^{-1} \frac{10}{20}$

$y = \tan^{-1} \frac{10}{20}$

$y = 26..$

$34.. - 26.. = 8°$

1. The map of a campground is shown below. Campsite C, first aid station F, and supply station S lie along a straight path. The path from the supply station to the tower, T, is perpendicular to the path from the supply station to the campsite. The length of path $\overline{FS}$ is 400 feet. The angle formed by path $\overline{TF}$ and path $\overline{FS}$ is 72°. The angle formed by path $\overline{TC}$ and path $\overline{CS}$ is 55°. Determine and state, to the *nearest foot*, the distance from the campsite to the tower.



ST is a side in both triangles

$\tan 72 = \frac{y}{400}$

$3.0777 = \frac{y}{400}$

$y = 1231'$

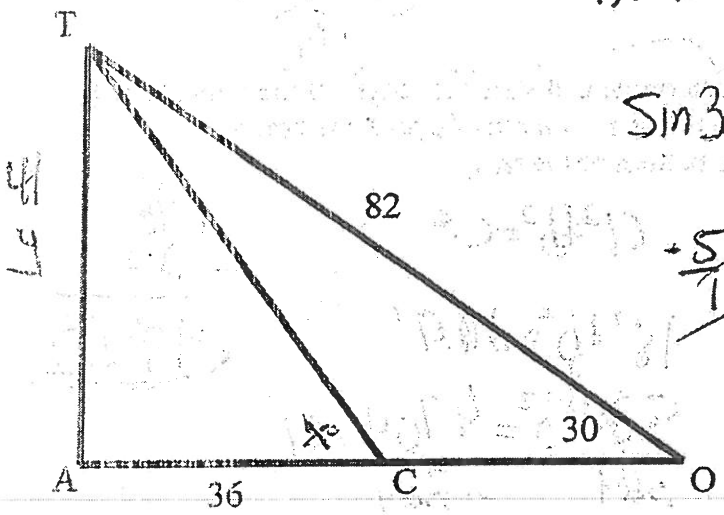
$\sin 55 = \frac{1231'}{x}$

$.8192 = \frac{1231'}{x}$

$\frac{.8192x}{.8192} = \frac{1231'}{.8192}$

$x = 1503$

2. Find the measure of $\angle TCA$ in the diagram of right triangle TAO below to the nearest tenth of a degree.



TA is a side in both triangles

$\sin 30 = \frac{y}{82}$

$\frac{.5}{1} = \frac{y}{82}$

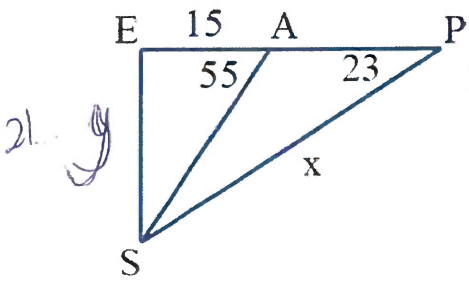
$y = 41$

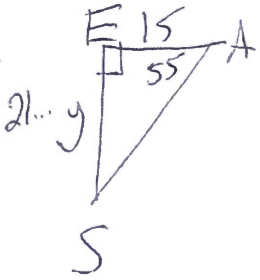
$\tan x = \frac{41}{36}$

$x = \tan^{-1}\frac{41}{36}$

$x = 48.7°$

3. Find the measure of $\overline{SP}$ in the diagram of right triangle SEP below to the nearest unit.

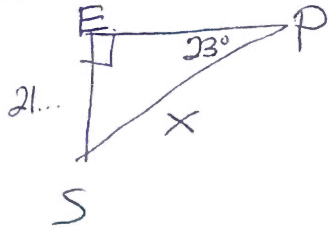




$\tan 55 = \frac{y}{15}$

$y = 15 \tan 55$

$y = 21...$

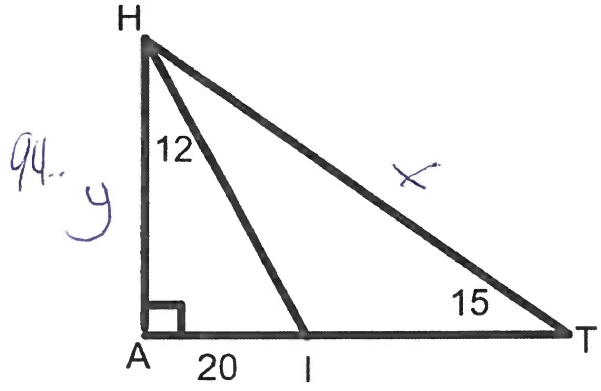


$\sin 23 = \frac{21...}{x}$

$\frac{x \sin 23}{\sin 23} = \frac{21...}{\sin 23}$

$x = 55$

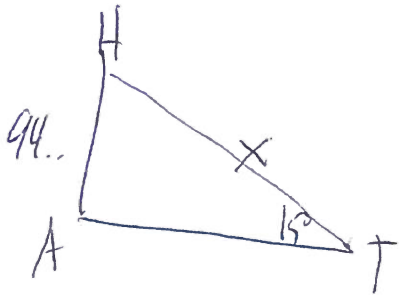
4. Find the measure of $\overline{HT}$ in the diagram of right triangle HAT below to the nearest unit.



$\tan 12 = \frac{20}{y}$

$\frac{y \tan 12}{\tan 12} = \frac{20}{\tan 12}$

$y = 94...$



$\sin 15 = \frac{94...}{x}$

$\frac{x \sin 15}{\sin 15} = \frac{94...}{\sin 15}$

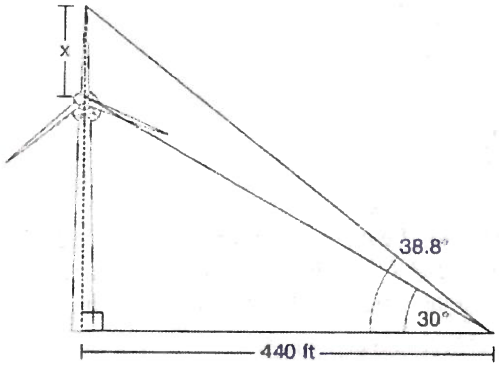
$x = 364$

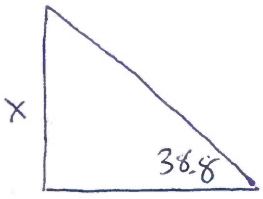
Name Schlansky
Mr. Schlansky

Date _____
Geometry

Compound Right Triangle Problems (Subtraction/Reflexive) Practice

1. Nick wanted to determine the length of one blade of the windmill pictured below. He stood at a point on the ground 440 feet from the windmill's base. Using surveyor's tools, Nick measured the angle between the ground and the highest point reached by the top blade and found it was 38.8°. He also measured the angle between the ground and the lowest point of the top blade, and found it was 30°. Determine and state a blade's length, x, to the *nearest foot*.

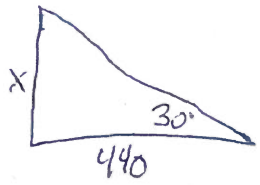




$\tan 38.8 = \frac{x}{440}$

$x = 440 \tan 38.8$

$x = 353...$



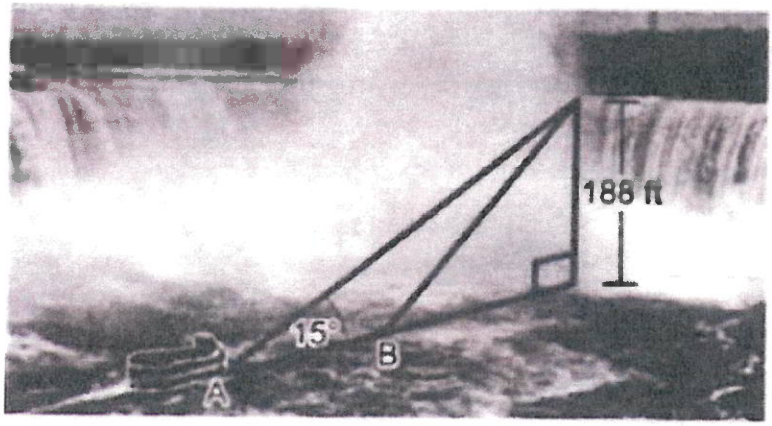
$\tan 30 = \frac{x}{440}$

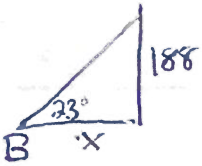
$x = 440 \tan 30$

$x = 254...$

$353... - 254... = (100)$

2. In the diagram below, a boat at point A is traveling toward the most powerful waterfall in North America, the Horseshoe Falls. The Horseshoe Falls has a vertical drop of 188 feet. The angle of elevation from point A to the top of the waterfall is 15°. After the boat travels toward the falls, the angle of elevation at point B to the top of the waterfall is 23°. Determine and state, to the *nearest foot*, the distance the boat traveled from point A to point B.

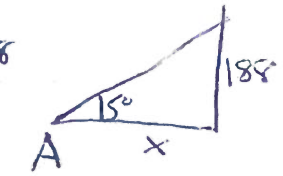




$\tan 23 = \frac{188}{x}$

$x \tan 23 = 188$

$x = 442...$



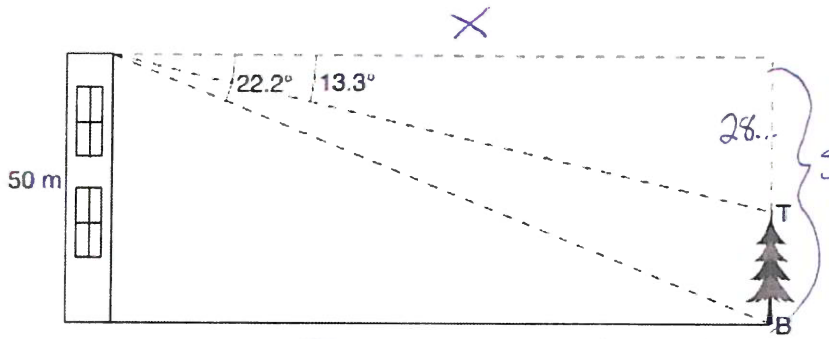
$\tan 15 = \frac{188}{x}$

$x \tan 15 = 188$

$x = 701...$

$701... - 442... = (258)$

3. As modeled in the diagram below, a building has a height of 50 meters. The angle of depression from the top of the building to the top of the tree, T, is 13.3°. The angle of depression from the top of the building to the bottom of the tree, B, is 22.2°. Determine and state, to the *nearest meter*, the height of the tree.



X

28...

50

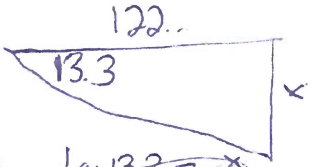
50 − 28... = 21

$\tan 22.2 = \frac{50}{x}$

$x \tan 22.2 = 50$

$\frac{x \tan 22.2}{\tan 22.2} = \frac{50}{\tan 22.2}$

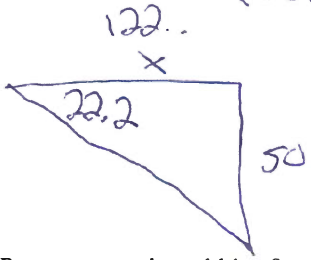
$x = 122..$



$\tan 13.3 = \frac{x}{122..}$

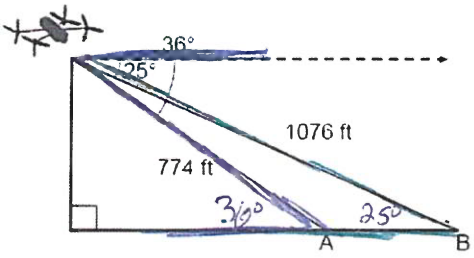
$x = 122.. \tan 13.3$

$x = 28..$

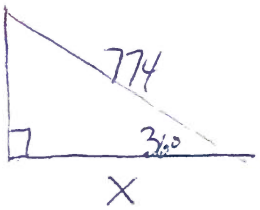


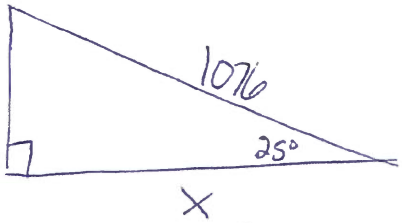
4. A drone is used to measure the size of a brush fire on the ground. Segment AB represents the width of the fire, as shown below. The drone calculates the distance to point B to be 1076 feet at an angle of depression of 25°. At the same point, the drone calculates the distance to point A to be 774 feet at an angle of depression of 36°. Determine and state the width of the fire, $\overline{AB}$, to the *nearest foot*.

angle of depression = angle of elevation



975..
−626..
349





$\cos 36 = \frac{x}{774}$

$\cos 25 = \frac{x}{1076}$

$x = 774 \cos 36$

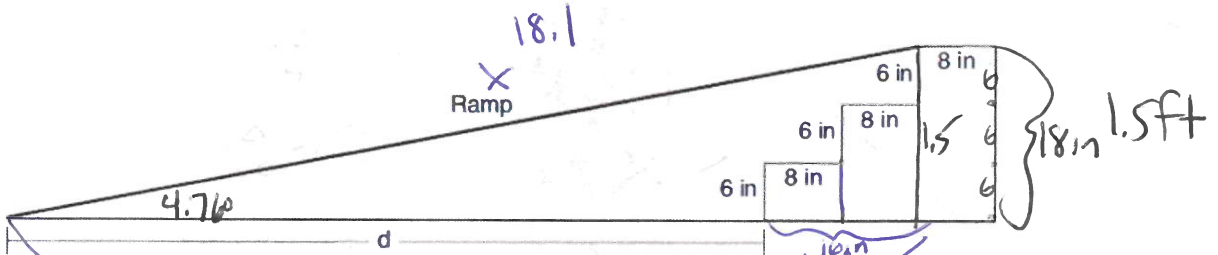
$x = 626..$

$x = 1076 \cos 25$

$x = 975...$

Compound Right Triangle Problems: Other
Problem Solve using SOHCAHTOA and/or Pythagorean Theorem

1. As modeled in the diagram below, an access ramp starts on flat ground and ends at the beginning of the top step. Each step is 6 inches tall and 8 inches deep. If the angle of elevation of the ramp is 4.76°, determine and state the length of the ramp, to the *nearest tenth of a foot*. Determine and state, to the *nearest tenth of a foot*, the horizontal distance, *d*, from the bottom of the stairs to the bottom of the ramp.



18.1
x
18 in 1.5 ft

$\sin\theta = \frac{O}{H}$

$\sin 4.76 = \frac{1.5}{x}$

$x \sin 4.76 = 1.5$

$\frac{x \sin 4.76}{\sin 4.76} = \frac{1.5}{\sin 4.76}$

$x = 18.1$ ft

$\cos\theta = \frac{A}{H}$

$\cos 4.76 = \frac{y}{18.1}$

$y = 18.1 \cos 4.76$

$y = 18...$

18... − 1.3 = 16.7 ft

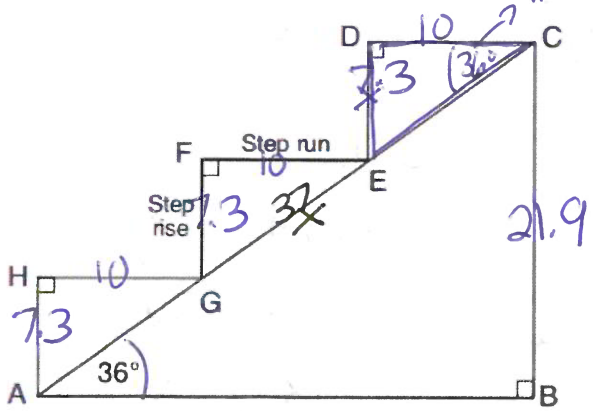
18 in $\times \frac{1 \text{ ft}}{12 \text{ in}} = 1.5$ ft

16 in $\times \frac{1 \text{ ft}}{12 \text{ in}} = 1.\overline{3}$ ft

You could have also used Pythagorean Theorem instead

2. A homeowner is building three steps leading to a deck, as modeled by the diagram below. All three step rises, $\overline{HA}$, $\overline{FG}$, and $\overline{DE}$, are congruent, and all three step runs, $\overline{HG}$, $\overline{FE}$, and $\overline{DC}$, are congruent. Each step rise is perpendicular to the step run it joins. The measure of $\angle CAB = 36°$ and $\angle CBA = 90°$.

If each step run is parallel to $\overline{AB}$ and has a length of 10 inches, determine and state the length of each step rise, to the *nearest tenth of an inch*. Determine and state the length of $\overline{AC}$, to the *nearest inch*.



alternate interior angles

$\tan\theta = \frac{O}{A}$

$\tan 36 = \frac{x}{10}$

$x = 10 \tan 36$

$x = 7.3$

$7.3(3) = 21.9$

$\sin\theta = \frac{O}{H}$

$\sin 36 = \frac{21.9}{x}$

$x \sin 36 = 21.9$

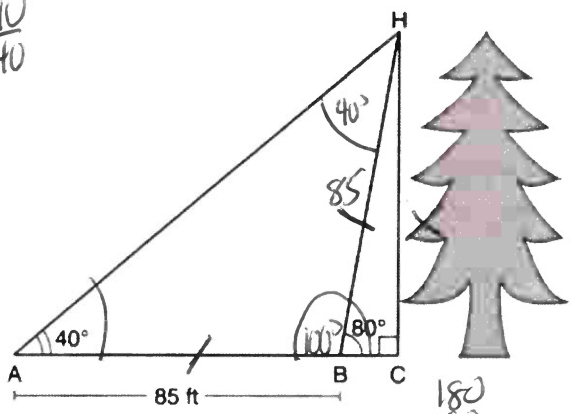
$\frac{x \sin 36}{\sin 36} = \frac{21.9}{\sin 36}$

$x = 37$

You also could have used Pythagorean Theorem instead.

3. Barry wants to find the height of a tree that is modeled in the diagram below, where $\angle C$ is a right angle. The angle of elevation from point A on the ground to the top of the tree, H, is 40°. The angle of elevation from point B on the ground to the top of the tree, H, is 80°. The distance between points A and B is 85 feet. Barry claims that $\triangle ABH$ is isosceles. Explain why Barry is correct. Determine and state, to the *nearest foot*, the height of the tree.

40 180 −140 40
H00 140



180 −80 100

ΔABH is isosceles because it has 2 ≡ angles.

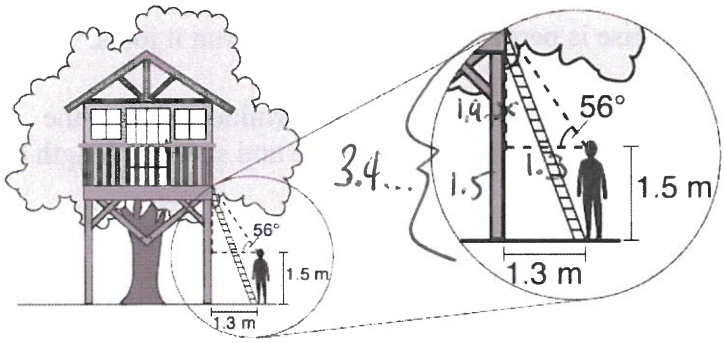
$\sin\theta = \frac{O}{H}$

$\sin 80 = \frac{x}{85}$

$x = 85 \sin 80$

$x = 84$

4. David has just finished building his treehouse and still needs to buy a ladder to be attached to the ledge of the treehouse and anchored at a point on the ground, as modeled below. David is standing 1.3 meters from the stilt supporting the treehouse. This is the point on the ground where he has decided to anchor the ladder. The angle of elevation from his eye level to the bottom of the treehouse is 56 degrees. David's eye level is 1.5 meters above the ground. Determine and state the minimum length of a ladder, to the *nearest tenth of a meter*, that David will need to buy for his treehouse.

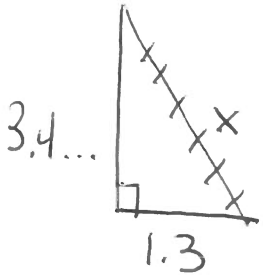


$\tan\theta = \frac{O}{A}$

$\tan 56 = \frac{x}{1.3}$

$x = 1.3 \tan 56$

$x = 1.9...$



$a^2 + b^2 = c^2$

$1.3^2 + 3.4...^2 = x^2$

$\sqrt{13.4...} = \sqrt{x^2}$

$3.7 = x$

Acute Angles in a Right Triangle

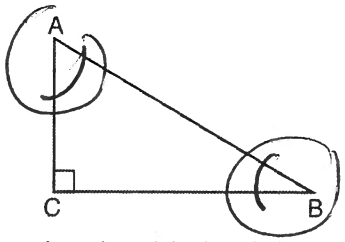
$\sin A = \cos B$**: In a right triangle, the sine of one acute angle is equal to the cosine of the other acute angle**

$A + B = 90$**: The two acute angles in a right triangle are complementary**

1. In scalene triangle ABC shown in the diagram below, $m\angle C = 90°$.

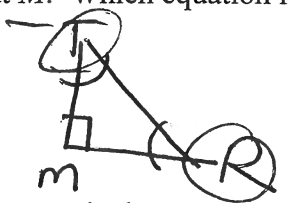
Which equation is always true?

1) $\sin A = \sin B$
2) $\cos A = \cos B$
3) $\cos A = \sin C$
4) $\sin A = \cos B$



2. Right triangle TMR is a scalene triangle with the right angle at M. Which equation is true?

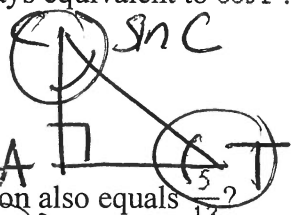
1) ~~$\sin M = \cos T$~~
2) ~~$\sin R = \cos R$~~
3) $\sin T = \cos R$
4) ~~$\sin T = \cos M$~~



3. Right triangle ACT has $m\angle A = 90°$. Which expression is always equivalent to $\cos T$?

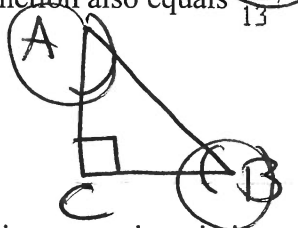
1) $\cos C$
2) $\sin C$
3) $\tan T$
4) $\sin T$

$\sin A = \cos B$



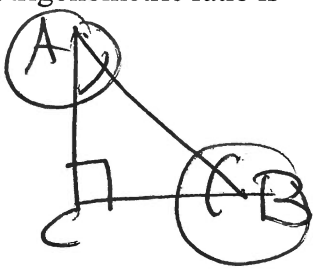
4. In right triangle ABC, $m\angle C = 90°$. If $\cos B = \frac{5}{13}$, which function also equals $\frac{5}{13}$?

1) $\tan A$
2) $\tan B$
3) $\sin A$
4) $\sin B$

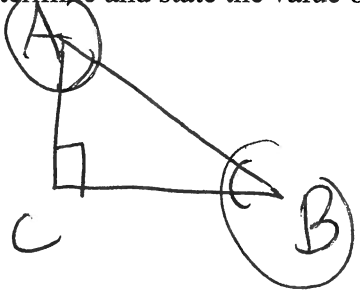


5. In right triangle ABC, $m\angle C = 90°$ and $AC \neq BC$. Which trigonometric ratio is equivalent to $\sin B$?

1) $\cos A$ COSA
2) $\cos B$
3) $\tan A$
4) $\tan B$



6. In right triangle ABC with the right angle at C, $\sin A = 2x + 0.1$ and $\cos B = 4x - 0.7$. Determine and state the value of x. Explain your answer.



$\sin A = \cos B$

$2x + 0.1 = 4x - 0.7$

$-2x \quad -2x$

$0.1 = 2x - 0.7$

$+0.7 \quad +0.7$

$\frac{0.8}{2} = \frac{2x}{2}$

$0.4 = x$

62 64 80

7. If $\sin(3x + 2)^\circ = \cos(4x - 10)^\circ$, what is the value of x to the *nearest tenth*?

(1) 7.6 (2) 12.0 (3) 14.0 (4) 26.9

$\sin A = \cos B$

$A + B = 90$

$3x + 2 + 4x - 10 = 90$

$7x - 8 = 90$

$+8 \quad +8$

$\frac{7x}{7} = \frac{98}{7}$

$x = 14$

8. If $\sin(2x + 7)^\circ = \cos(4x - 7)^\circ$, what is the value of x?

1) 7
2) 15
3) 21
4) 30

$\sin A = \cos B$

$A + B = 90$

$2x + 7 + 4x - 7 = 90$

$\frac{6x}{6} = \frac{90}{6}$

$x = 15$

9. In a right triangle, $\sin(40 - x)^\circ = \cos(3x)^\circ$. What is the value of x?

1) 10
2) 15
3) 20
4) 25

$\sin A = \cos B$

$A + B = 90$

$40 - x + 3x = 90$

$2x + 40 = 90$

$-40 \quad -40$

$\frac{2x}{2} = \frac{50}{2}$

$x = 25$

10. In a right triangle, the acute angles have the relationship $\sin(2x + 4) = \cos(46)$. What is the value of x?

1) 20
2) 21
3) 24
4) 25

$\sin A = \cos B$

$A + B = 90$

$2x + 4 + 46 = 90$

$2x + 50 = 90$

$-50 \quad -50$

$\frac{2x}{2} = \frac{40}{2}$

$x = 20$

11. Which expression is always equivalent to $\sin x$ when $0^\circ < x < 90^\circ$?

1) $\cos(90^\circ - x)$
2) $\cos(45^\circ - x)$
3) $\cos(2x)$
4) $\cos x$

$\sin A = \cos B$

$\sin x = \cos(90 - x)$

$x + (90 - x) = 90$

12. Which of the following is equivalent to $\sin 40$?

1) $\sin 50$
2) $\cos 50$
3) $\cos 40$
4) $\tan 50$

$\sin A = \cos B$

$\sin 40 = \cos 50$

$A + B = 90$

$40 + B = 90$

$-40 \quad -40$

$B = 50$

these must add to 90

13. Which of the following is equivalent to $\cos 57$?

1) $\sin 57$
2) $\sin 33$
3) $\cos 33$
4) $\cos 123$

$\sin 33$

$90 - 57 = 33$

14. Which expression is equal to $\sin 30^\circ$?

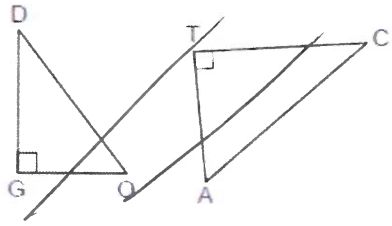
1) $\tan 30^\circ$
2) $\sin 60^\circ$
3) $\cos 60^\circ$
4) $\cos 30^\circ$

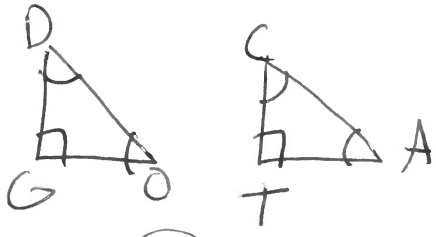
$\cos 60$

$90 - 30 = 60$

Trigonometry with Similar Triangles
Draw your own triangles separately!
Match up the corresponding angles and apply trigonometry rules from there.

1. In the diagram below, $\triangle DOG \sim \triangle CAT$, where $\angle G$ and $\angle T$ are right angles.





Which expression is always equivalent to $\sin D$?

1) $\cos A$
2) $\sin A$
3) $\tan A$
4) $\cos C$

$\cos O = \cos A$

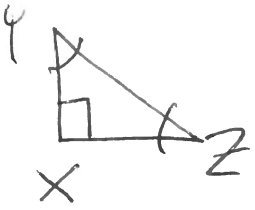
$\sin A = \cos B$

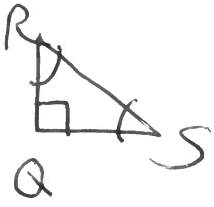
2. If scalene triangle XYZ is similar to triangle QRS and $m\angle X = 90°$, which equation is always true?

1) $\sin Y = \sin S$
2) $\cos R = \cos Z$
3) $\cos Y = \sin Q$
4) $\sin R = \cos Z$

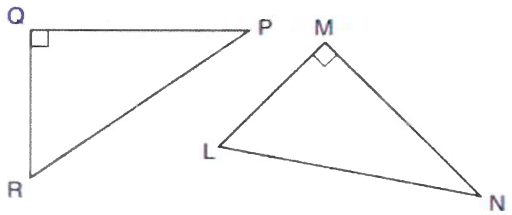
$\sin A = \cos B$

$\sin Y =$

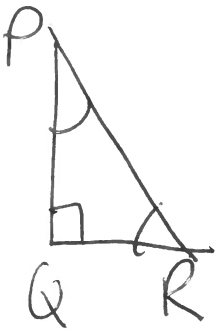




3. In the diagram below, right triangle PQR is transformed by a sequence of rigid motions that maps it onto right triangle NML. What ratio is equal to cos L?



1) $\sin R$
2) $\cos R$
3) $\sin P$
4) $\cos P$



$\sin A = \cos B$

$\cos L = \sin N$

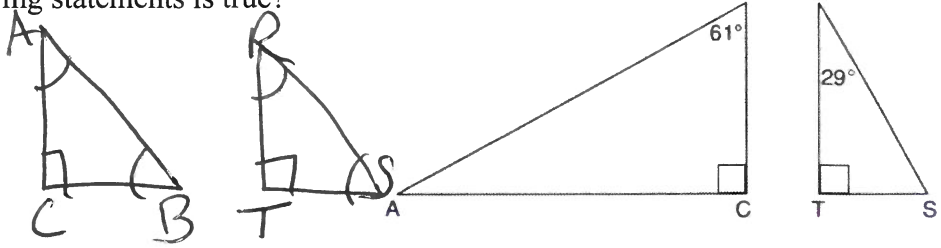
$\sin P$

SOH CAH TOA

4. Given right triangle ABC with a right angle at C, $m\angle B = 61°$. Given right triangle RST with a right angle at T, $m\angle R = 29°$.
Which of the following statements is true?

1) $\sin A = \cos C$
2) $\sin B = \cos R$
3) $\sin S = \cos B$
4) $\sin C = \cos T$

$\sin B = \cos A$ → $\cos R$



5. In the diagram below of $\triangle HAR$ and $\triangle NTY$, angles H and N are right angles, and $\triangle HAR \sim \triangle NTY$. If $AR = 13$ and $HR = 12$, what is the measure of angle Y, to the *nearest degree*?

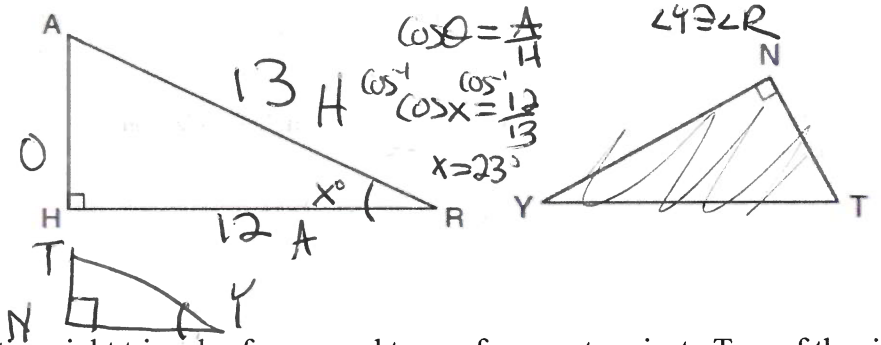
1) 23°
2) 25°
3) 65°
4) 67°

$\cos\theta = \frac{A}{H}$

$\cos^{-1}\cos x = \cos^{-1}\frac{12}{13}$

$x = 23°$

$\angle Y \cong \angle R$



6. Kayla was cutting right triangles from wood to use for an art project. Two of the right triangles she cut are shown below.

If $\triangle ABC \sim \triangle DEF$, with right angles B and E, $BC = 15$ cm, and $AC = 17$ cm, what is the measure of $\angle F$, to the *nearest degree*?

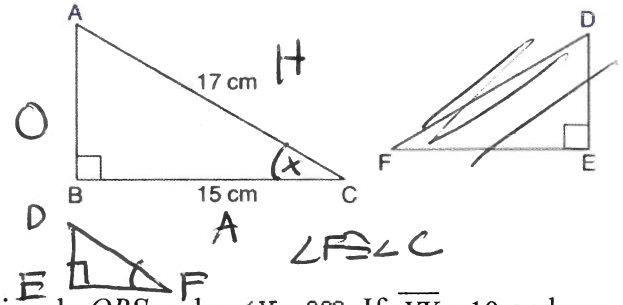
1) 28°
2) 41°
3) 62°
4) 88°

$\cos\theta = \frac{A}{H}$

$\cos^{-1}\cos x = \cos^{-1}\frac{15}{17}$

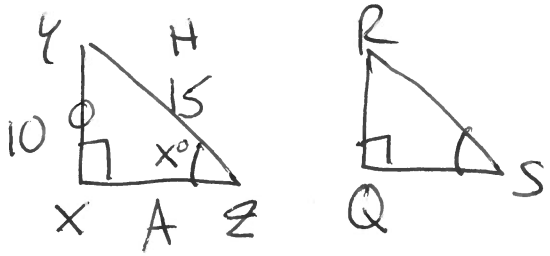
$x = \cos^{-1}\left(\frac{15}{17}\right)$

$x = 28°$



$\angle F \cong \angle C$

7. Scalene triangle XYZ is similar to triangle QRS and $m\angle X = 90°$. If $\overline{XY} = 10$ and $\overline{ZY} = 15$, find the measure of $\angle S$ to the *nearest tenth of a degree*.



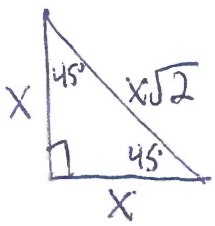
$\sin\theta = \frac{O}{H}$

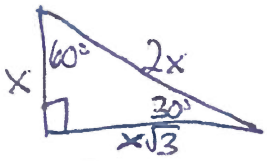
$\sin^{-1}\sin x = \sin^{-1}\frac{10}{15}$

$x = \sin^{-1}\left(\frac{10}{15}\right)$

$x = 41.8°$

Name Schlansky
Mr. Schlansky



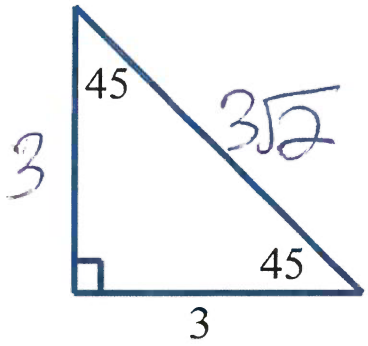


Date _____
Algebra II

Special Right Triangles

Fill in the two missing sides of each of the following triangles.

1.



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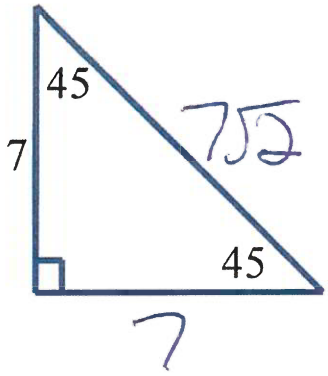

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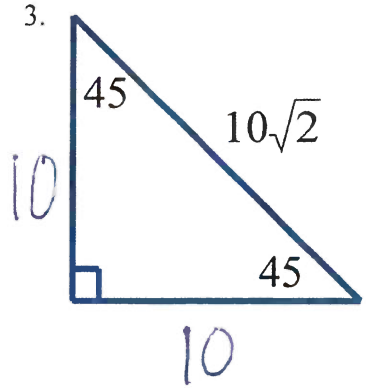

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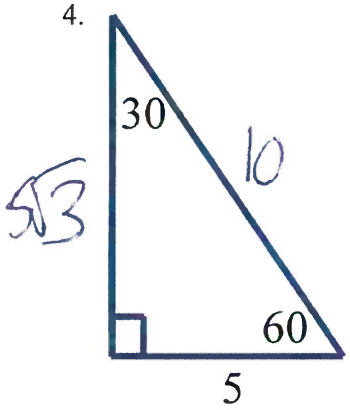

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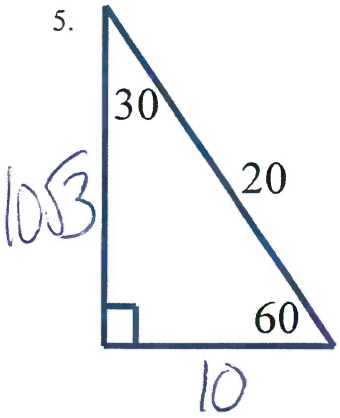

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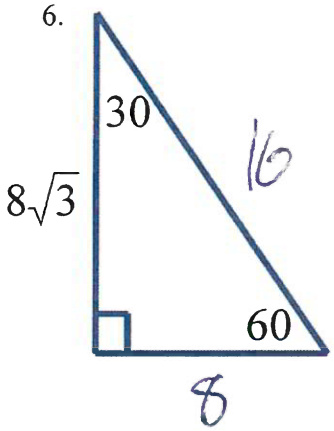

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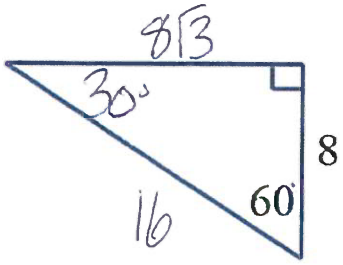
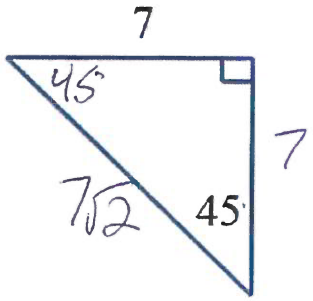
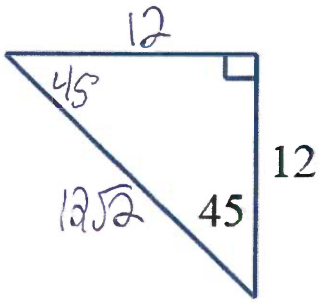

8√3
30°
8
16
60°

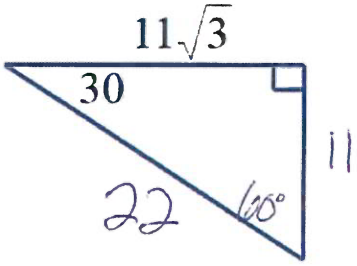
8.


7
45°
7
7√2
45°

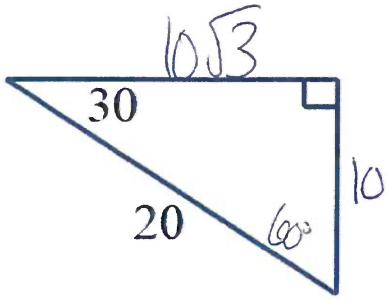
9.


12
45°
12
12√2
45

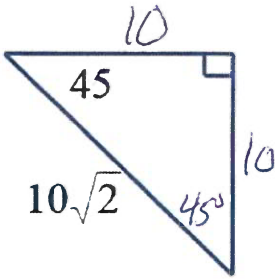
10.

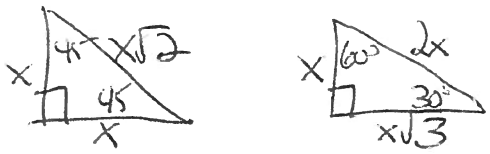

11√3
30
11
22
60°

11.


10√3
30
10
20
60°

12.

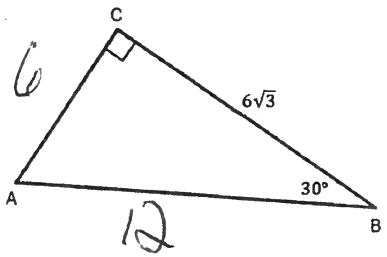

10
45
10
10√2
45°



13 19. In right triangle ABC below, $m\angle C = 90°$, $m\angle B = 30°$, and $CB = 6\sqrt{3}$.

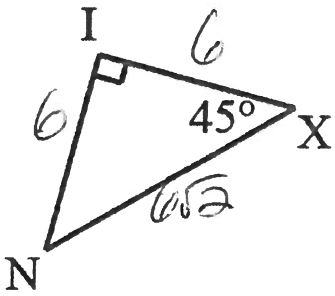
The length of $\overline{AB}$ is

1) $3\sqrt{3}$
2) 9
3) 12
4) $12\sqrt{3}$



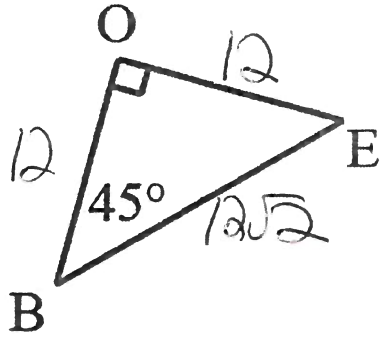
14 20. In right triangle NIX below, $m\angle I = 90°$, $m\angle X = 45°$, and $\overline{NX} = 6\sqrt{2}$. Find $\overline{IX}$.

1) $6\sqrt{2}$
2) 6
3) $12\sqrt{2}$
4) 12



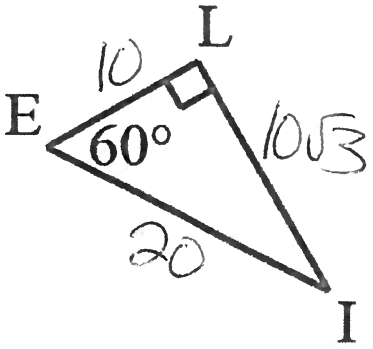
15 21. In right triangle BOE below, $m\angle O = 90°$, $m\angle B = 45°$, and $\overline{OE} = 12$. Find $\overline{BE}$.

1) 12
2) $12\sqrt{3}$
3) $12\sqrt{2}$
4) 24



16 22. In right triangle BOE below, $m\angle L = 90°$, $m\angle E = 60°$, and $\overline{IE} = 20$. Find $\overline{LI}$.

1) $20\sqrt{3}$
2) 10
3) $10\sqrt{3}$
4) 20



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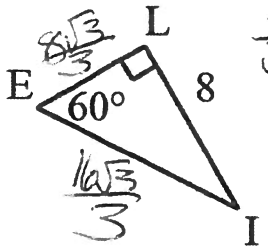
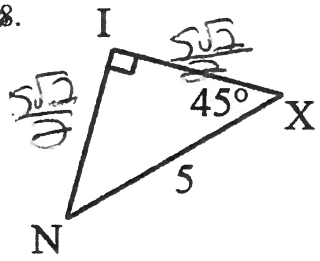
Rationalize the denominator for each of the following

17. $\frac{7}{\sqrt{2}}$ $\frac{\sqrt{2}}{\sqrt{2}}$ $\frac{7\sqrt{2}}{2}$ 18. $\frac{2}{\sqrt{3}}$ $\frac{\sqrt{3}}{\sqrt{3}}$ $\frac{2\sqrt{3}}{3}$ 19. $\frac{6}{\sqrt{2}}$ $\frac{\sqrt{2}}{\sqrt{2}}$ $\frac{6\sqrt{2}}{2}$ $3\sqrt{2}$ 20. $\frac{12}{\sqrt{3}}$ $\frac{\sqrt{3}}{\sqrt{3}}$ $\frac{12\sqrt{3}}{3}$ $4\sqrt{3}$

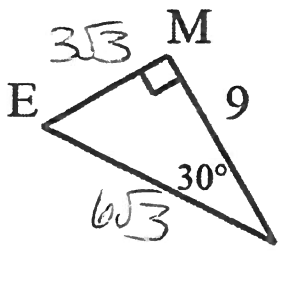
Find the missing sides for each of the triangles below in simplest radical form

21. 

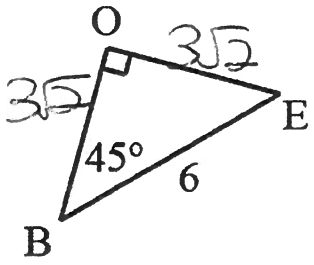
$\frac{8\sqrt{3}}{3}$ $\frac{8\sqrt{3}}{\sqrt{3}\sqrt{3}}$ $\frac{8\sqrt{3}}{3}$ $\frac{16\sqrt{3}}{3}$

22. 

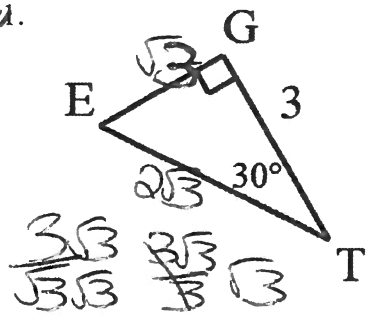
$\frac{5\sqrt{2}}{2}$ $\frac{5}{\sqrt{2}}$ $\frac{\sqrt{2}}{\sqrt{2}}$ $\frac{5\sqrt{2}}{2}$

23. 

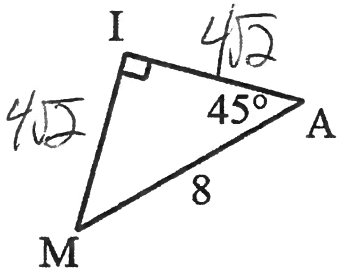
$3\sqrt{3}$ $6\sqrt{3}$ $\frac{9\sqrt{3}}{\sqrt{3}\sqrt{3}}$ $\frac{9\sqrt{3}}{3}$ $3\sqrt{3}$

24. 

$3\sqrt{2}$ $3\sqrt{2}$ $\frac{6}{\sqrt{2}}$ $\frac{\sqrt{2}}{\sqrt{2}}$ $\frac{6\sqrt{2}}{2}$ $3\sqrt{2}$

25. 

$\sqrt{3}$ $2\sqrt{3}$ $\frac{3\sqrt{3}}{\sqrt{3}\sqrt{3}}$ $\frac{3\sqrt{3}}{3}$ $\sqrt{3}$

26. 

$4\sqrt{2}$ $4\sqrt{2}$ $\frac{8}{\sqrt{2}}$ $\frac{\sqrt{2}}{\sqrt{2}}$ $\frac{8\sqrt{2}}{2}$ $4\sqrt{2}$

Pythagorean Theorem

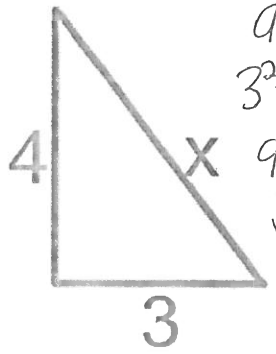
Look out for hidden right triangles where you may need to use $a^2 + b^2 = c^2$

a and b are the legs

c is the hypotenuse

Find the missing side of each right triangle rounding to the nearest tenth if necessary

1.



$a^2+b^2=c^2$

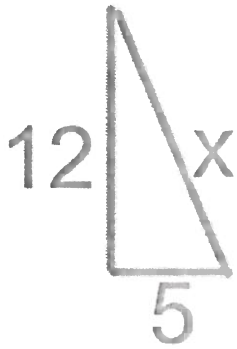
$3^2+4^2=x^2$

$9+16=x^2$

$\sqrt{25}=\sqrt{x^2}$

$5=x$

2.



$a^2+b^2=c^2$

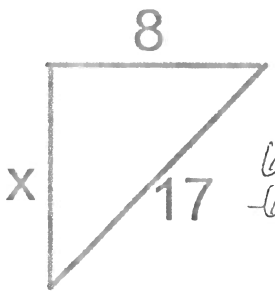
$5^2+12^2=x^2$

$25+144=x^2$

$\sqrt{169}=\sqrt{x^2}$

$13=x$

3.



$a^2+b^2=c^2$

$8^2+x^2=17^2$

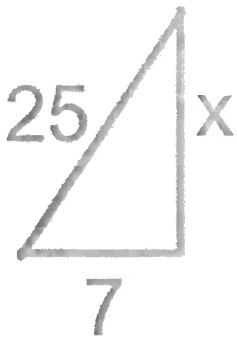
$64+x^2=289$

$-64 \quad -64$

$\sqrt{x^2}=\sqrt{225}$

$x=15$

4.



$a^2+b^2=c^2$

$7^2+x^2=25^2$

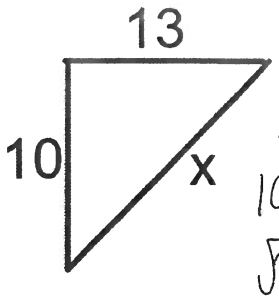
$49+x^2=625$

$-49 \quad -49$

$\sqrt{x^2}=\sqrt{576}$

$x=24$

5.



$a^2+b^2=c^2$

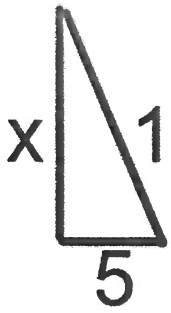
$10^2+13^2=x^2$

$100+169=x^2$

$\sqrt{269}=\sqrt{x^2}$

$16.4=x$

6.



$a^2+b^2=c^2$

$5^2+x^2=15^2$

$25+x^2=225$

$-25 \quad -25$

$\sqrt{x^2}=\sqrt{200}$

$x=14.1$

Prism/cylinder
parallel: base
perpendicular: rectangle

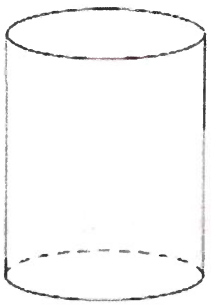
pyramid/cone
parallel: base
perpendicular: triangle

Name Schlansky Date _____
Mr. Schlansky Geometry

Cross Sections

1. A plane intersects a cylinder perpendicular to its bases.

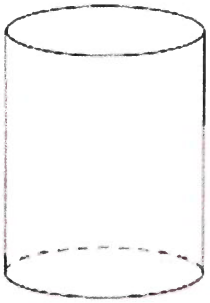
vertical



This cross section can be described as a
1) rectangle
2) parabola
3) triangle
4) circle

2. A plane intersects a cylinder parallel to its bases.

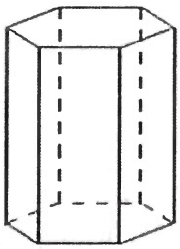
horizontal



This cross section can be described as a
1) rectangle
2) parabola
3) triangle
4) circle

3. A right hexagonal prism is shown below. A two-dimensional cross section that is perpendicular to the base is taken from the prism.

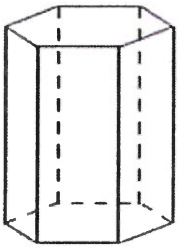
vertical



Which figure describes the two-dimensional cross section?
1) triangle
2) rectangle
3) pentagon
4) hexagon

4. A right hexagonal prism is shown below. A two-dimensional cross section that is parallel to the base is taken from the prism.

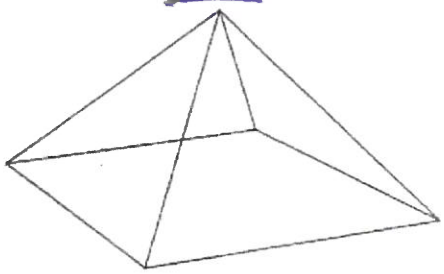
horizontal



Which figure describes the two-dimensional cross section?
1) triangle
2) rectangle
3) pentagon
4) hexagon

horizontal

5. A square pyramid is intersected by a plane passing through the vertex and parallel to the base.

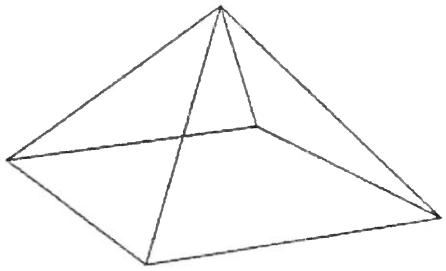


Which two-dimensional shape describes this cross section?

1) square
2) triangle
3) pentagon
4) rectangle

6. A square pyramid is intersected by a plane passing through the vertex and perpendicular to the base.

vertical

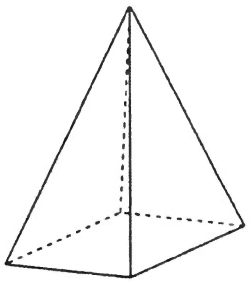


Which two-dimensional shape describes this cross section?

1) square
2) triangle
3) pentagon
4) rectangle

horizontal

7. In the diagram below, a plane intersects a square pyramid parallel to its base.

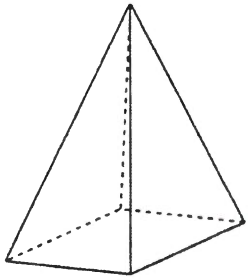


Which two-dimensional shape describes this cross section?

1) circle
2) square
3) triangle
4) pentagon

vertical

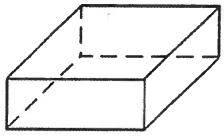
8. In the diagram below, a plane intersects a square pyramid perpendicular to its base.

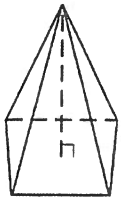


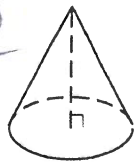
Which two-dimensional shape describes this cross section?

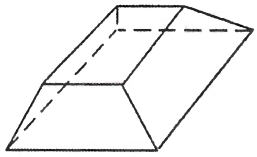
1) circle
2) square
3) triangle
4) pentagon

9. Which figure can have the same cross section as a sphere? circle

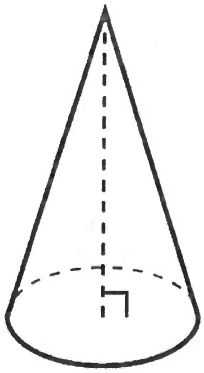
1) 

3) 

2) 

4) 

10. William is drawing pictures of cross sections of the right circular cone below.



Which drawing can *not* be a cross section of a cone?

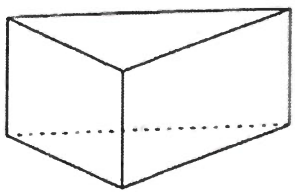
1)

2)

3)

4)

11. The right prism with a triangular base shown below is cut by a plane perpendicular to its bases. Vertical



The two-dimensional shape of the cross section is always a

1) triangle
2) rhombus
3) pentagon
4) rectangle

Vertical

12. A plane intersects a hexagonal prism. The plane is perpendicular to the base of the prism. Which two-dimensional figure is the cross section of the plane intersecting the prism?

1) triangle 3) hexagon
2) trapezoid 4) rectangle

13. A right cylinder is cut perpendicular to its base. The shape of the cross section is a

Vertical

1) circle
2) cylinder
3) rectangle
4) triangular prism

14. The cross section of a regular pyramid contains the altitude of the pyramid. The shape of this cross section is a

Vertical

1) circle
2) square
3) triangle
4) rectangle

15. A two-dimensional cross section is taken of a three-dimensional object. If this cross section is a triangle, what can *not* be the three-dimensional object?

1) cone 3) pyramid
2) cylinder 4) rectangular prism

16. A plane intersects a sphere. Which two-dimensional shape is formed by this cross section?

1) rectangle 3) square
2) triangle 4) circle

17. Which is *not* a possible two-dimensional cross section of a three-dimensional cylinder?

1) circle 2) rectangle 3) ellipse 4) triangle

18. Which figure(s) below can have a triangle as a two-dimensional cross section?

I. cone Vertical
II. cylinder X
III. cube corner
IV. square pyramid Vertical

1) I, only 2) IV, only 3) I, II, and IV, only 4) I, III, and IV, only

Area with Coordinate Geometry
Box Method

1) Build a rectangle around the shape
2) Find the area of the rectangle (*A=lw)*
3) Find the area of the triangles outside of the shape (*A=.5lw)*
4) Subtract the triangle areas from the rectangle area

Find the area of the following shapes

1.

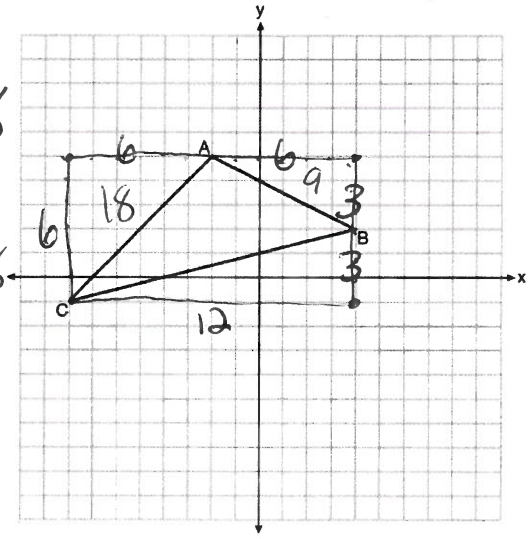
$A_R = 6(12) = 72$

$A_{T1} = \frac{1}{2}(6)(6) = 18$

$A_{T2} = \frac{1}{2}(6)(3) = 9$

$A_{T3} = \frac{1}{2}(12)(3) = 18$

18 72
+9 −45
+18 27
45



2.

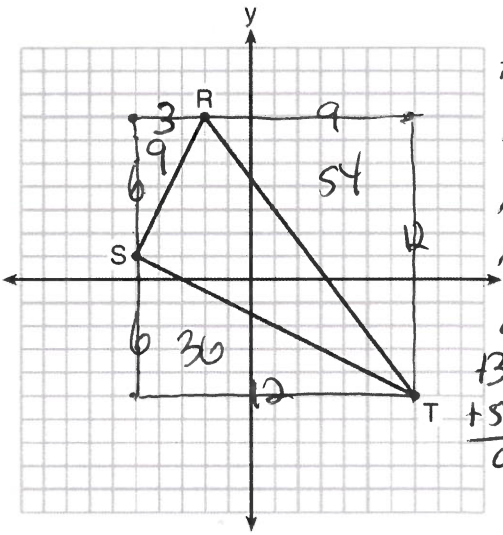
$A_R = 12(12) = 144$

$A_{T1} = \frac{1}{2}(3)(6) = 9$

$A_{T2} = \frac{1}{2}(6)(12) = 36$

$A_{T3} = \frac{1}{2}(9)(12) = 54$

9 144
136 −99
+54 45
99



3.

$A_R = 9(6) = 54$

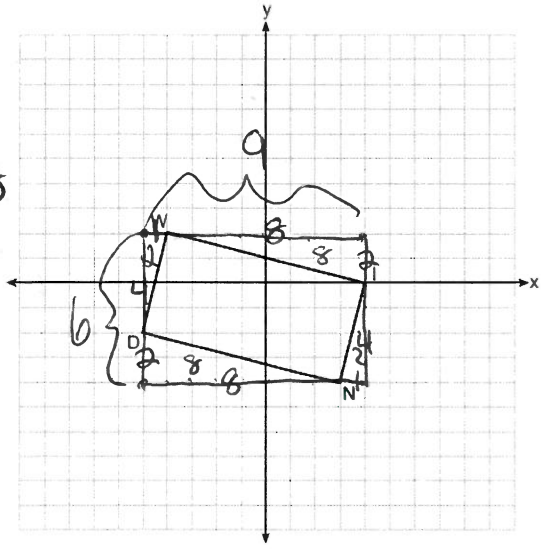
$A_{T1} = \frac{1}{2}(1)(4) = 2$

$A_{T2} = \frac{1}{2}(8)(2) = 8$

$A_{T3} = \frac{1}{2}(4)(1) = 2$

$A_{T4} = \frac{1}{2}(2)(8) = 8$

2 54
+8 −20
+2 34
+8
20



4.

$A_R = 6(8) = 48$

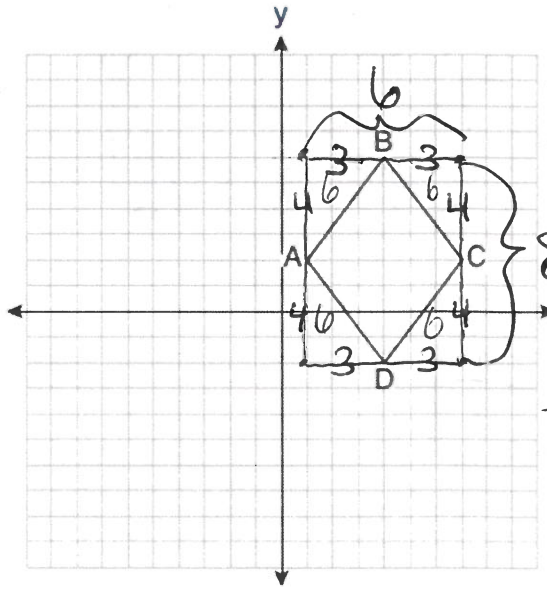
$A_{T1} = \frac{1}{2}(3)(4) = 6$

$A_{T2} = \frac{1}{2}(3)(4) = 6$

$A_{T3} = \frac{1}{2}(3)(4) = 6$

$A_{T4} = \frac{1}{2}(3)(4) = 6$

6 48
+6 −24
+6 24
+6
24



 93

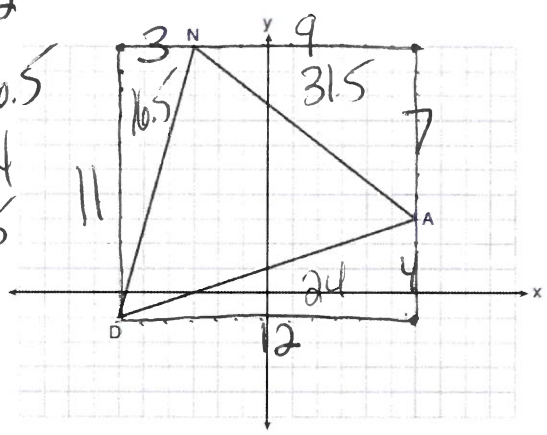
$A_R = 11(12) = 132$

$A_{T1} = \frac{1}{2}(3)(11) = 16.5$

$A_{T2} = \frac{1}{2}(12)(4) = 24$

$A_{T3} = \frac{1}{2}(9)(7) = 31.5$

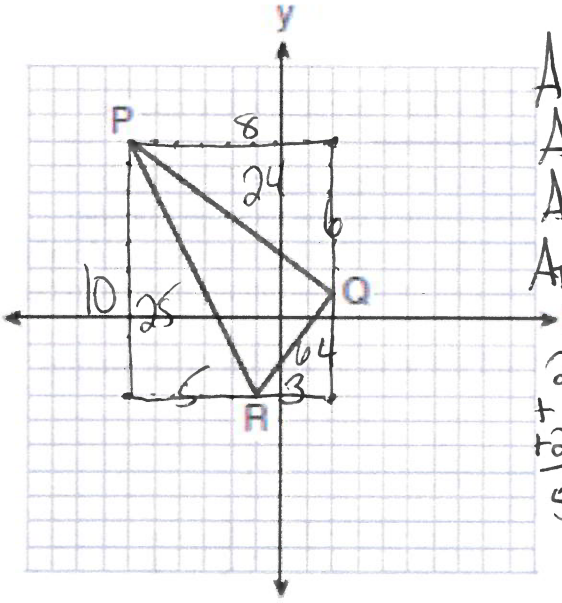
5.



16.5
+31.5
+24
72

132
−72
60

6.



$A_R = 8(10) = 80$

$A_{T1} = \frac{1}{2}(8)(6) = 24$

$A_{T2} = \frac{1}{2}(4)(3) = 6$

$A_{T3} = \frac{1}{2}(10)(5) = 25$

24
+6
+25
55

80
−55
25

$A_R = 8(7) = 56$

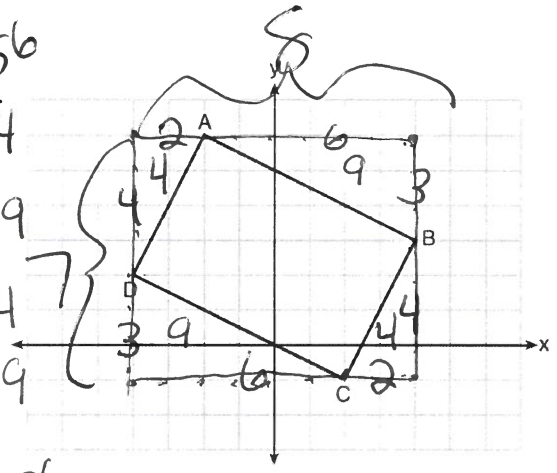
$A_{T1} = \frac{1}{2}(2)(4) = 4$

$A_{T2} = \frac{1}{2}(6)(3) = 9$

$A_{T3} = \frac{1}{2}(9)(8) = 4$

$A_{T4} = \frac{1}{2}(3)(6) = 9$

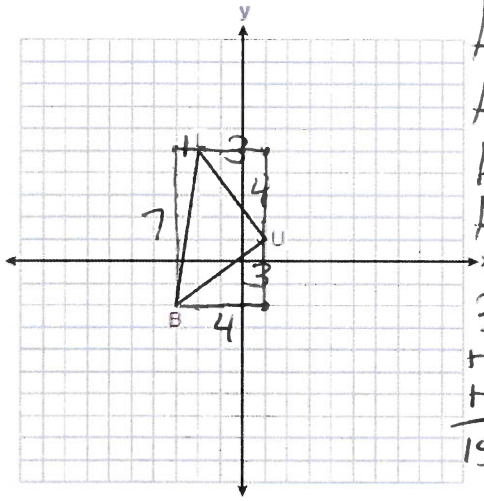
7.



4
+9
+4
+9
26

56
−26
30

8.



$A_R = 7(4) = 28$

$A_{T1} = \frac{1}{2}(1)(7) = 3.5$

$A_{T2} = \frac{1}{2}(3)(4) = 6$

$A_{T3} = \frac{1}{2}(3)(4) = 6$

3.5
+6
+6
15.5

28
−15.5
12.5

94

Triangle Area:

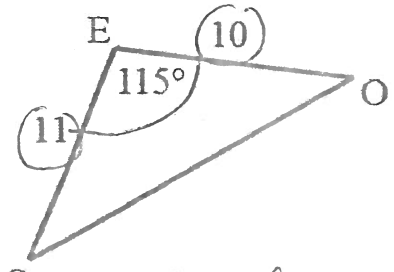
Area of a Triangle: $A = \frac{1}{2}ab\sin C$

a, b are sides and C is the INCLUDED angle

You may need to add and subtract from 180 in order to find the third angle of the triangle.

Find the area of the following triangles to the nearest ~~tenth of a~~ square unit

1.

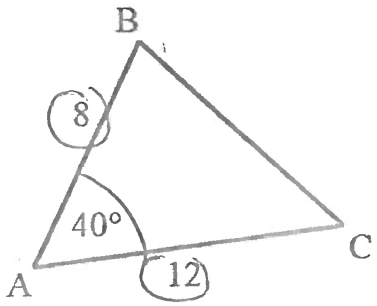


$A = \frac{1}{2}ab\sin C$

$A = \frac{1}{2}(11)(10)\sin 115$

$A = 50$

2.

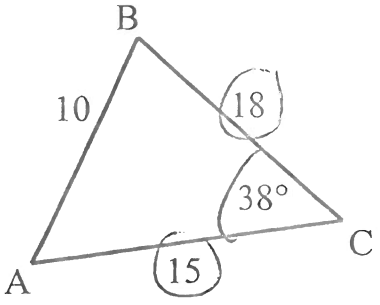


$A = \frac{1}{2}ab\sin C$

$A = \frac{1}{2}(8)(12)\sin 40$

$A = 31$

3.

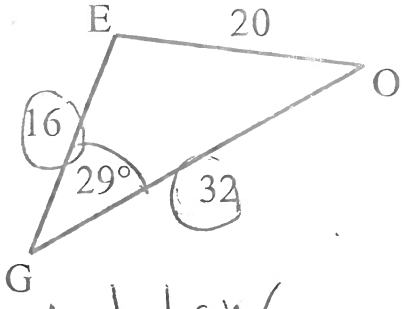


$A = \frac{1}{2}ab\sin C$

$A = \frac{1}{2}(15)(18)\sin 38$

$A = 83$

4.

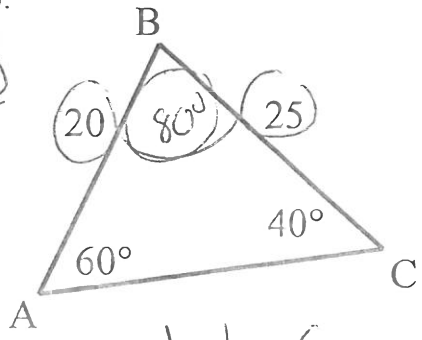


$A = \frac{1}{2}ab\sin C$

$A = \frac{1}{2}(16)(32)\sin 29$

$A = 124$

5.



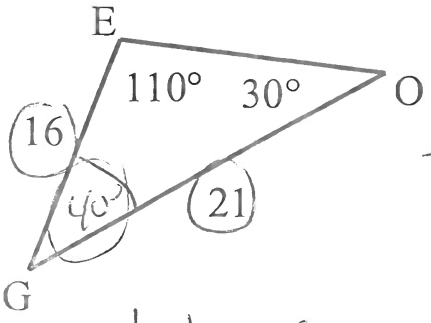
60 180
+40 −100
100 80

$A = \frac{1}{2}ab\sin C$

$A = \frac{1}{2}(20)(25)\sin 80°$

$A = 246$

6.



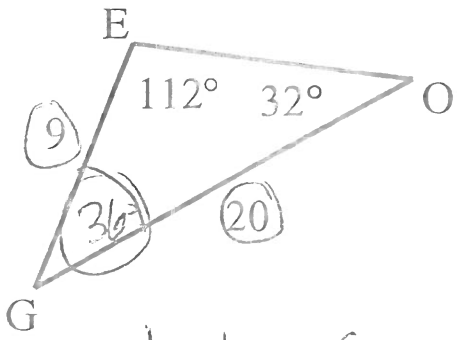
110 180
+30 −140
140 40

$A = \frac{1}{2}ab\sin C$

$A = \frac{1}{2}(16)(21)\sin 40$

$A = 108$

7.



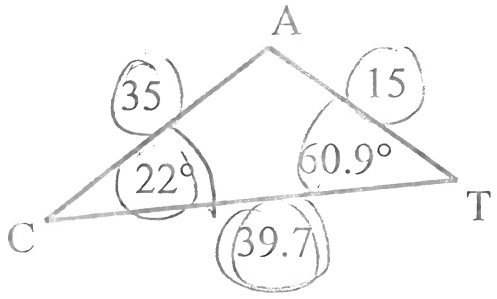
112 180
+32 −144
144 36

$A = \frac{1}{2}ab\sin C$

$A = \frac{1}{2}(9)(20)\sin 36$

$A = 53$

8.



$A = \frac{1}{2}(35)(39.7)\sin 22°$

or

$A = \frac{1}{2}(15)(39.7)\sin 60.9$

$A = 260$

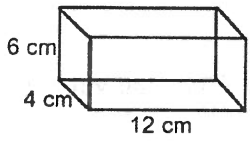
$V = lwh$ $V = \pi r^2 h$

Name Schlansky
Mr. Schlansky

Date _____
Geometry

Calculating Volume

1. Find the volume of the rectangular prism given below

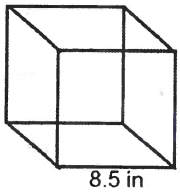


$V = lwh$
$V = 12(4)(6)$
$V = 288 cm^3$

2. Find the volume of a rectangular prism that has dimensions 10 feet by 14 feet by 8 feet.

$V = lwh$
$V = 10(14)(18)$
$V = 2520 ft^3$

3. Find the volume of the cube shown below.



$V = lwh$
$V = 8.5(8.5)(8.5)$
$V = 614.125 in^3$

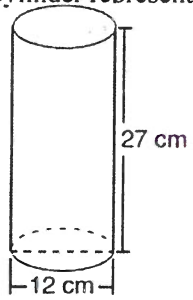
4. Lenny made a cube in technology class. Each edge measured 1.5 cm. What is the volume of the cube in cubic centimeters?

$V = lwh$
$V = 614.125 cm^3$

5. Which expression represents the volume, in cubic centimeters, of the cylinder represented in the diagram below?

1) 162π 3) 972π
2) 324π 4) $3,888\pi$

$V = \pi r^2 h$
$V = \pi(6)^2(27)$
$V = 972\pi$



6. What is the volume, in cubic centimeters, of a cylinder that has a height of 15 cm and a diameter of 12 cm?

1) 180π
2) 540π
3) 675π
4) $2,160\pi$

$V=\pi r^2h$
$V=\pi(6)^2(15)$
$V=540\pi$

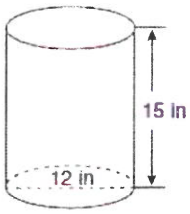
7. A cylinder is 8 *cm* tall and has a base with a radius of 3 *cm*. What is the total volume of the cylinder to the *nearest tenth of a centimeter*?

$V=\pi r^2h$
$V=\pi(3)^2(8)$
$V=226.2\ cm^3$

8. A cylindrical container has a diameter of 12 inches and a height of 15 inches, as illustrated in the diagram below.

What is the volume of this container?

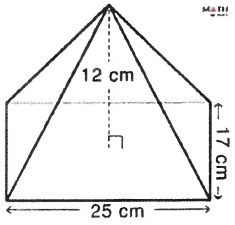
1) 6,785.8
2) 4,241.2
3) 2,160.0
4) 1,696.5



(Not drawn to scale)

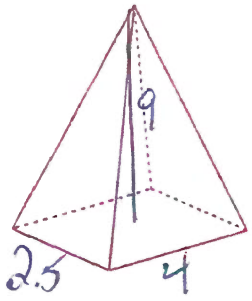
$V=\pi r^2h$
$V=\pi(6)^2(15)$
$V=1696.5$

9. Find the volume of the pyramid below



$V=\frac{1}{3}lwh$
$V=\frac{1}{3}(25)(17)(12)$
$V=1700\ cm^3$

10. The pyramid below has a base with a length of 4 cm, width of 2.5 cm, and an altitude ot 9 cm. Find the volume of the pyramid.



$V = \frac{1}{3}lwh$

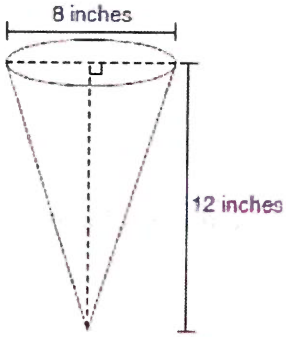
$V = \frac{1}{3}(2.5)(4)(9)$

$V = 30\,cm^3$

11. In the diagram below, a right circular cone has a diameter of 8 inches and a height of 12 inches.

What is the volume of the cone to the *nearest cubic inch*?

1) 201 3) 603
2) 481 4) 804

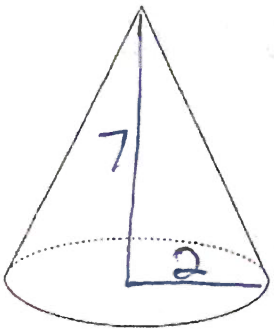


$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(4)^2(12)$

$V = 201$

12. A cone has a base with a radius of 2 and an altitude of 7. Find its volume in terms of π.

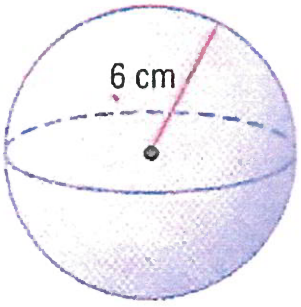


$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(2)^2(7)$

$V = \frac{28}{3}\pi$

13. Find the volume of the sphere shown below in terms of π.



$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(6)^3$

$V = 288\pi$

$r = 5$

14. Find the volume of a sphere with a diameter of 10 inches rounded to the *nearest hundredth* of a cubic inch.

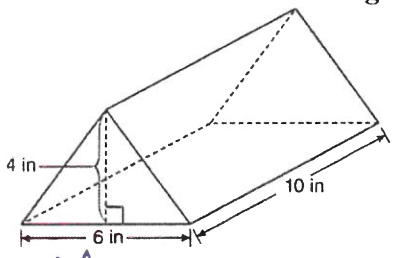
$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(5)^3$

$V = 523.60$

Find the volume of the following triangular prisms

15.

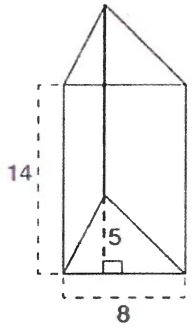


$V = \frac{1}{2}lwh$

$V = \frac{1}{2}(6)(4)(10)$

$V = 120\ in^3$

16.

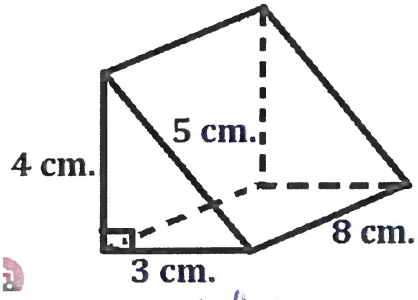


$V = \frac{1}{2}lwh$

$V = \frac{1}{2}(8)(5)(14)$

$V = 280$

17.

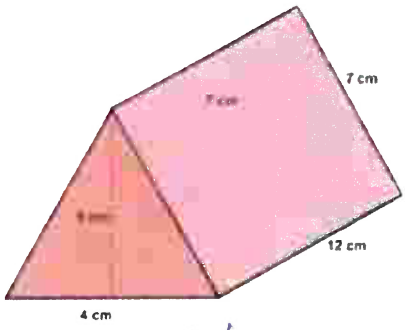


$V = \frac{1}{2}lwh$

$V = \frac{1}{2}(8)(3)(4)$

$V = 48$

18.

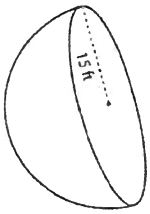


$V = \frac{1}{2}lwh$

$V = \frac{1}{2}(4)(12)(6)$

$V = 144$

19. Find the volume of the shape below in terms of π.



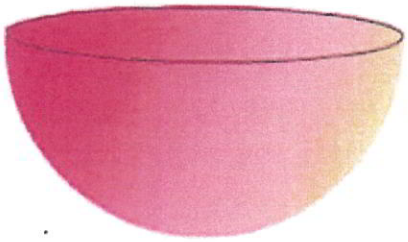
$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$V = \frac{1}{2}\left(\frac{4}{3}\pi(15)^3\right)$

$V = 2250\pi$

$r = 9.1$

20. Find the volume of the object below if the diameter is 18.2 meters. Round your answer to the *nearest cubic meter*.

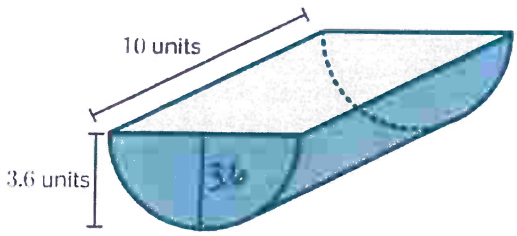


$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$V = \frac{1}{2}\left(\frac{4}{3}\pi(9.1)^3\right)$

$V = 1578$

21. Find the volume of the object below to the *nearest cubic unit*.

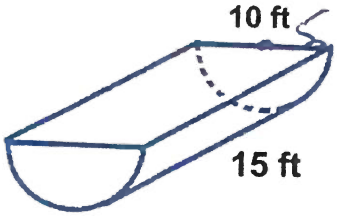


$V = \frac{1}{2}\pi r^2 h$

$V = \frac{1}{2}\pi(3.6)^2(10)$

$V = 204$

22. Find the volume of the object below in terms of π.



$V = \frac{1}{2}\pi r^2 h$

$V = \frac{1}{2}\pi(5)^2(15)$

$V = 187.5\pi$

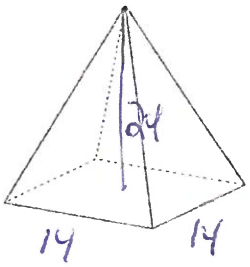
Square pyramid has length=width

23. A regular pyramid has a square base with an edge length of 14 cm and an altitude of 24 cm. Find its volume.

$V = \frac{1}{3}lwh$

$V = \frac{1}{3}(14)(14)(24)$

$V = 1568 cm^3$



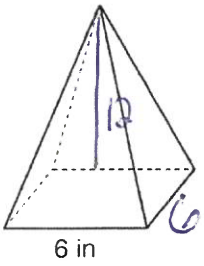
24. Find the volume of a square pyramid with a base with edge length 4 inches and a height of 18 inches.

$V = \frac{1}{3}lwh$

$V = \frac{1}{3}(4)(4)(18)$

$V = 96 in^3$

25. As shown in the diagram below, a regular pyramid has a square base whose side measures 6 inches. If the altitude of the pyramid measures 12 inches, find its volume.

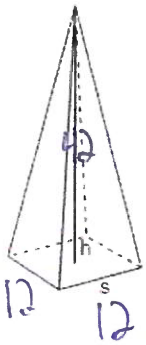


$V = \frac{1}{3}lwh$

$V = \frac{1}{3}(6)(6)(12)$

$V = 144 in^3$

26. A side, *s*, of the base of the pyramid is 12 meters, and the height, *h*, is 42 meters. What is the volume of the pyramid in cubic meters?



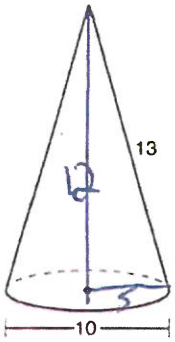
$V = \frac{1}{3}lwh$

$V = \frac{1}{3}(12)(12)(42)$

$V = 2016 m^3$

If given slant height, use $a^2+b^2=c^2$

27. Determine and state the volume of the cone, in terms of π.



$V=\frac{1}{3}\pi r^2h$

$V=\frac{1}{3}\pi(5)^2(12)$

$V=100\pi$

$a^2+b^2=c^2$

$5^2+b^2=13^2$

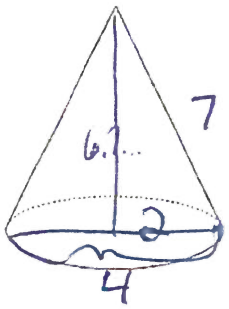
$25+b^2=169$

$-25 \quad -25$

$\sqrt{b^2}=\sqrt{144}$

$b=12$

28. A cone has a base with a diameter of 4 and a slant height of 7.
Find its volume rounded to the *nearest* tenth.



$V=\frac{1}{3}\pi r^2h$

$V=\frac{1}{3}\pi(2)^2(6.7..)$

$V=28.1$

$a^2+b^2=c^2$

$2^2+b^2=7^2$

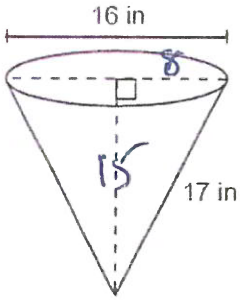
$4+b^2=49$

$-4 \quad -4$

$\sqrt{b^2}=\sqrt{45}$

$b=6.7$

29. In the diagram below, a cone has a diameter of 16 inches and a slant height of 17 inches. What is the volume of the cone, in terms of π, in cubic inches?



$V=\frac{1}{3}\pi r^2h$

$V=\frac{1}{3}\pi(8)^2(15)$

$V=320\pi$

$a^2+b^2=c^2$

$8^2+b^2=17^2$

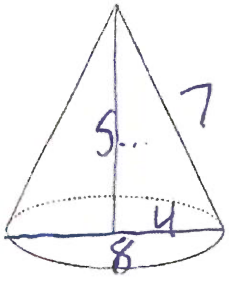
$64+b^2=289$

$-64 \quad -64$

$\sqrt{b^2}=\sqrt{225}$

$b=15$

30. In the diagram below, a right circular cone has a diameter of 8 and a slant height of 7.
Find the volume of the cone rounded to the *nearest tenth.*



$V=\frac{1}{3}\pi r^2h$

$V=\frac{1}{3}\pi(4)^2(5..)$

$V=96.3$

$a^2+b^2=c^2$

$4^2+b^2=7^2$

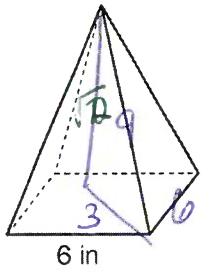
$16+b^2=49$

$-16 \quad -16$

$\sqrt{b^2}=\sqrt{33}$

$b=5..$

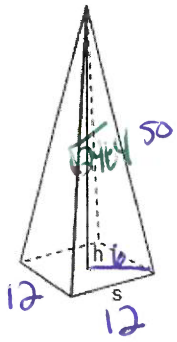
31. As shown in the diagram below, a regular pyramid has a square base whose side measures 6 inches. If the slant height measures 9 inches, find its volume to the nearest cubic inch.



$a^2+b^2=c^2$
$3^2+b^2=9^2$
$9+b^2=81$
$-9 \quad -9$
$\sqrt{b^2}=\sqrt{72}$
$b=\sqrt{72}$

$V=\frac{1}{3}lwh$
$V=\frac{1}{3}(6)(6)(\sqrt{72})$
$V=$ 102 in^3

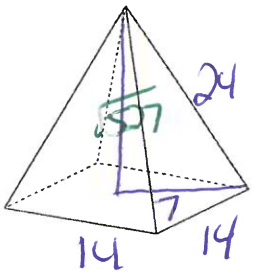
32. A side, *s*, of the base of the pyramid is 12 meters, and the slant height 50 meters. What is the volume of the pyramid to the nearest cubic meter?



$a^2+b^2=c^2$
$6^2+b^2=50^2$
$36+b^2=2500$
$-36 \quad -36$
$\sqrt{b^2}=\sqrt{2464}$
$b=\sqrt{2464}$

$V=\frac{1}{3}lwh$
$V=\frac{1}{3}(12)(12)(\sqrt{2464})$
$V=$ 2383 m^3

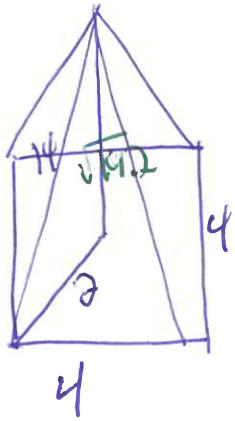
33. A regular pyramid has a square base with an edge length of 14 cm and a slant height of 24 cm. Find its volume to the nearest cubic centimeter.



$a^2+b^2=c^2$
$7^2+b^2=24^2$
$49+b^2=576$
$-49 \quad -49$
$\sqrt{b^2}=\sqrt{527}$
$b=\sqrt{527}$

$V=\frac{1}{3}lwh$
$V=\frac{1}{3}(14)(14)\sqrt{527}$
$V=$ 1500 cm^3

34. Find the volume of a square pyramid with a base with edge length 4 inches and a slant height of 14 inches.



$a^2+b^2=c^2$
$2^2+b^2=14^2$
$4+b^2=196$
$-4 \quad -4$
$\sqrt{b^2}=\sqrt{192}$
$b=\sqrt{192}$

$V=\frac{1}{3}lwh$
$V=\frac{1}{3}(4)(4)(\sqrt{192})$
$V=$ 74 in^3

density = mass / volume

Name Schlansky
Mr. Schlansky

Date _____
Geometry

Density

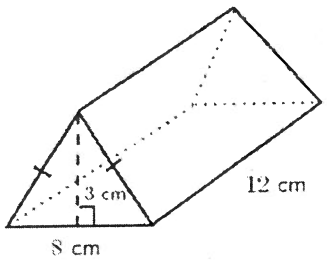
1. A brick that weighs 1824 grams has dimensions that measure 4 cm by 3 cm by 8 cm. To the nearest tenth, what is the density of the brick?

$d = \frac{1824 \text{ grams}}{96 \text{ cm}^3}$

$d = 19 \text{ g/cm}^3$

$V = lwh$
$V = 4(3)(8)$
$V = 96 \text{ cm}^3$

2. Clay in the shape of a triangular prism shown below has a mass of 1260 grams. What is its density?



$d = \frac{1260}{144 \text{ cm}^3}$

$d = 8.75 \text{ g/cm}^3$

$V = \frac{1}{2}lwh$
$V = \frac{1}{2}(8)(3)(12)$
$V = 144 \text{ cm}^3$

3. A cylindrical candleholder has a diameter of 4.5 cm and a height of 20 cm. If the candleholder has a mass of 2900 g, rounded to the nearest whole number, what is its density?

$d = \frac{2900 \text{ g}}{318\ldots \text{cm}^3}$

$d = 9 \text{ g/cm}^3$

$V = \pi r^2 h$
$V = \pi(2.25)^2(20)$
$V = 318\ldots \text{cm}^3$

4. A square pyramid with a base with an edge of 6 inches and a height of 12 inches has a mass of 684 grams. Find the density to the nearest tenth.

$d = \frac{684\ g}{432\ in^3}$

$d = 1.6\ g/in^3$

$V = \frac{1}{3}lwh$

$V = 6(6)(12)$

$V = 432\ in^3$

5. What is the density of a solid sphere of clay that has a diameter of 3.2 inches and has a mass of 552 grams? Round your answer to the nearest tenth.

$d = \frac{552\ g}{17\ldots\ in^3}$

$d = 32.2\ g/in^3$

$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(1.6)^3$

$V = 17\ldots\ in^3$

6. A wooden cube has an edge length of 6 centimeters and a mass of 137.8 grams. Determine the density of the cube, to the *nearest thousandth*. State which type of wood the cube is made of, using the density table below.

Type of Wood	Density (g/cm^3)
Pine	0.373
Hemlock	0.431
Elm	0.554
Birch	0.601
Ash	0.638
Maple	0.676
Oak	0.711

$d = \frac{137.8\ g}{216\ cm^3}$

$d = .638\ g/cm^3$

Ash

$V = lwh$

$V = 6(6)(6)$

$V = 216\ cm^3$

Compound Volume: If a shape is made up of multiple shapes on top of each other, find the volume of each and add them together.

Find the volume of each of the following shapes and round to the *nearest tenth* if necessary.

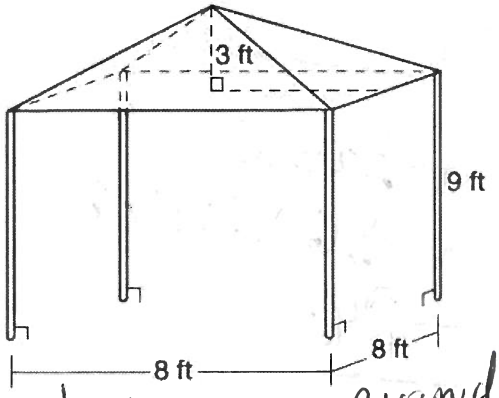
1.

rectangular prism
$V = lwh$
$V = 12(20)(45)$
$V = 10800$

half-cylinder
$V = \frac{1}{2}\pi r^2 h$
$V = \frac{1}{2}\pi(10)^2(45)$
$V = 7068..$

$10800 + 7068.. = 17868.6$

2.

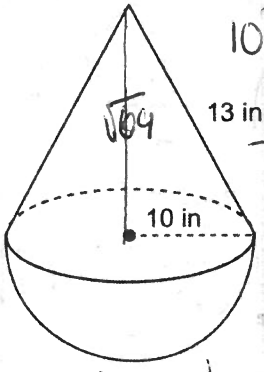


rectangular prism
$V = lwh$
$V = 8(8)(9)$
$V = 576$

pyramid
$V = \frac{1}{3}lwh$
$V = \frac{1}{3}(8)(8)(3)$
$V = 64$

$576 + 64 = 640$

3.



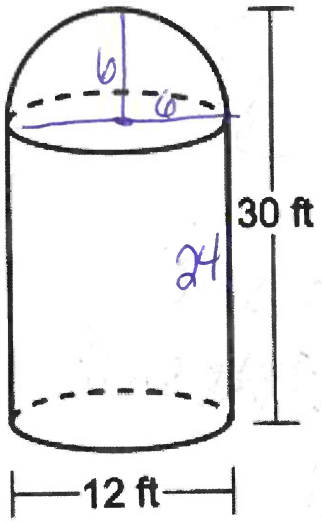
$a^2 + b^2 = c^2$
$10^2 + b^2 = 13^2$
$100 + b^2 = 169$
$-100 \quad -100$
$\sqrt{b^2} = \sqrt{69}$
$b = \sqrt{69}$

cone
$V = \frac{1}{3}\pi r^2 h$
$V = \frac{1}{3}\pi(10)^2(\sqrt{69})$
$V = 869..$

hemisphere
$V = \frac{1}{2}(\frac{4}{3}\pi r^3)$
$V = \frac{1}{2}(\frac{4}{3}\pi(10)^3)$
$V = 2094.$

$869.. + 2094 = 2964.3$

4.



cylinder
$V = \pi r^2 h$
$V = \pi(6)^2(24)$
$V = 2714..$

hemisphere
$V = \frac{1}{2}(\frac{4}{3}\pi(6)^3)$
$V = 452..$

$2714.. + 452. = 3166.7$

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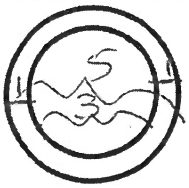
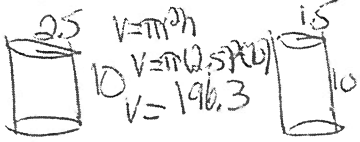
Hollow Volume

1. A hollow cylinder has a height of 10 inches, an outer diameter of 5 inches, and a thickness of 1 inch. Find the volume to the hollow cylinder to the nearest cubic inch.



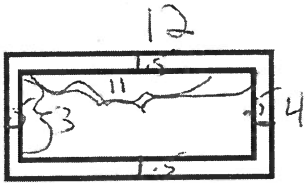


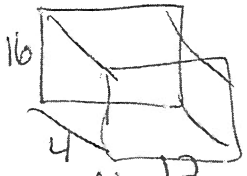
$V=\pi r^2 h$
$V=\pi(2.5)^2(10)$
$V=196.3$

$V=\pi r^2 h$
$V=\pi(1.5)^2(10)$
$V=70.6$

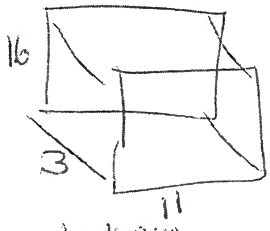
196.3
− 70.6
126 in³

2. A hollow rectangular prism has a length of 12 cm, a width of 4 cm, and a height of 16 cm. Find the volume of the hollow rectangular prism to the nearest cubic centimeter.





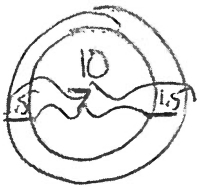
$V=lwh$
$V=16(12)(4)$
$V=768$



$V=16(3)(11)$
$V=528$

768
− 528
240 cm³

3. A hollow sphere has an outer diameter of 10 feet, and a thickness of 1.5 feet. Find the volume of the hollow sphere to the nearest tenth of a cubic foot.

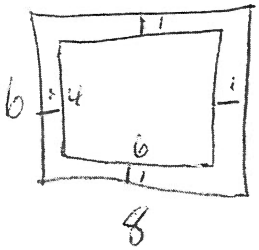


$V=\frac{4}{3}\pi r^3$ $V=\frac{4}{3}\pi(5)^3$ $V=523..$

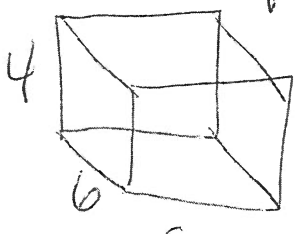
$V=\frac{4}{3}\pi(3.5)^3$
$V=179..$

523..
− 179..
344.0 ft³

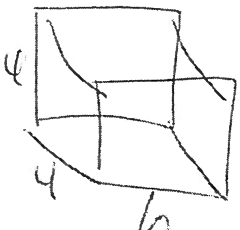
4. A hollow rectangular prism has a length of 8 meters, a width of 6 meters, and a height of 4 meters. Find the volume of the hollow rectangular prism to the nearest cubic meter.



$V=lwh$



$V=8(6)(4)$
$V=192$



$V=6(4)(4)$
$V=96$

192
− 96
96 m³

inches → ft: ÷12

ft → inches: ·12

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Volume with Conversions

Convert the following units and round to the nearest tenth if necessary

1. 24 inches to feet

$\frac{24}{12}=2$ ft

2. 54 inches to feet

$\frac{54}{12}=4.5$ ft

2. 3 inches to feet

$\frac{3}{12}=.25$ ft

4. 4 inches to feet

$\frac{4}{12}=\frac{1}{3}$ ft

5. 2 feet to inches

$2\cdot12=24$ in

6. 5 feet to inches

$5\cdot12=60$ in

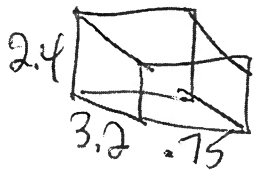
7. 3.5 feet to inches

$3.5\cdot12=42$ in

8. 9.25 feet to inches

$9.25\cdot12=111$ in

9. What is the volume, to the *nearest cubic foot*, of a rectangular prism that is 2.4 feet high, 3.2 feet ~~wide~~, and 9 inches ~~high~~ long?



$\frac{9\text{ in}}{12}=.75$ ft

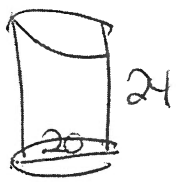
$V=lwh$

$V=.75(3.2)(2.4)$

$V=5.76\text{ ft}^3$

40 109

10. A cylinder has a diameter of 20 inches and a height of 2 feet. Find the volume rounded to the *nearest cubic inch.*



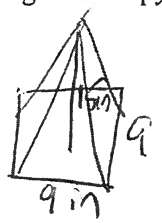
$2ft \cdot 12 = 24in$

$V = \pi r^2 h$

$V = \pi(10)^2(24)$

$V = 7540 in^3$

11. A regular pyramid has a square base. A side of the base measures 0.75 feet and the height of the pyramid is 15 inches. What is the volume of the pyramid in cubic inches?



$.75ft \cdot 12 = 9in$

$V = \frac{1}{3}lwh$

$V = \frac{1}{3}(9)(9)(15)$

$V = 405 in^3$

12. Find the volume of a cone whose diameter is 15 inches and height of 2 feet rounded to the *nearest cubic foot.*

$\frac{15in}{12} = \frac{5}{4}ft$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{5}{8}\right)^2(2)$

$V = 0.8ft^3$

13. A child's tent can be modeled as a pyramid with a square base whose sides measure 60 inches and whose height measures 84 inches. What is the volume of the tent, to the *nearest cubic foot*?

$\frac{60in}{12} = 5ft$

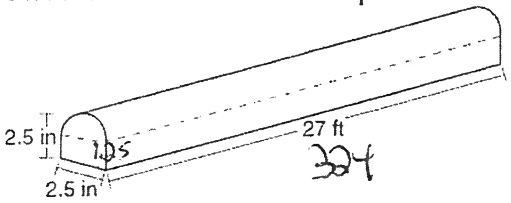
$\frac{84in}{12} = 7ft$

$V = \frac{1}{3}lwh$

$V = \frac{1}{3}(5)(5)(7)$

$V = 58ft^3$

14. Find the volume of the shape below to the *nearest cubic inch.*



$27ft \cdot 12 = 324in^3$

rectangular prism

$V = lwh$

$V = 2.5(1.25)(324)$

$V = 1012.5$

half cylinder

$V = \frac{1}{2}\pi r^2 h$

$V = \frac{1}{2}\pi(1.25)^2(3.24)$

$V = 795..$

$1012.5 + 795..$

$1808 in^3$

41 (10

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Unit Analysis

1. A block of wood has a volume of 200 cm^3. The cost of the wood is \$.10 per gram and the density of the wood is 2.1 g/cm^3. What would be the cost of producing 15 of these blocks of wood.

$$200\ cm^3 \times \frac{2.1\ g}{1\ cm^3} \times \frac{.10\ \$}{1\ g} \times 15 = \$630.00$$

2. A cylindrical test tube has a volume of 45 in^3. The liquid inside has weighs 4 ounces per cubic inch and the cost of the liquid is \$.12 per ounce. How much will it cost to fill the test tube to 80% of its capacity?

$$45\ in^3 \times \frac{4\ oz}{1\ in^3} \times \frac{.12\ \$}{1\ oz} \times .8 = \$17.28$$

3. The volume of a pool is 25,000 gallons. The cost of the water to fill the pool is \$120 per 8000 gallons. How much will it cost to fill the pool up 90%?

$$25{,}000\ gal \times \frac{120\ \$}{8000\ gal} \times .9 = \frac{25{,}000(120)(.9)}{8000} = \$337.50$$

4. An object made of steel has a volume of 24.1 cm^3. The steel costs \$1.25 for 500 grams and has a density of 3.1 g/cm^3. How much will it cost to make 25 of these objects?

$$24.1\ cm^3 \times \frac{3.1\ g}{1\ cm^3} \times \frac{1.25\ \$}{500\ g} \times 25 = \frac{24.1(3.1)(1.25)(25)}{500}$$

\$4.67

5. A stone brick has a volume of 150 in^3. The stone weighs 5 grams per cubic inch and it costs \$4.52 for 500 grams of stone. How much will it cost to purchase enough stone to make 12 bricks?

$$150\ in^3 \times \frac{5\ g}{1\ in^3} \times \frac{4.52\ \$}{500\ g} \times 12 = \frac{150(5)(4.52)(12)}{500}$$

\$81.36

5. A stone brick has a volume of 150 in^3. The stone weighs 5 grams per cubic inch and it costs \$4.52 for 500 grams of stone. How much will it cost to purchase enough stone to make 12 bricks?

$150\ in^3 \cdot \frac{5\ g}{1\ in^3} \cdot \frac{4.52\ \$}{500\ g} \times 12$ $\quad \frac{150(5)(4.52)(12)}{500} = \$81.36$

6. A machinist creates a solid steel part for a wind turbine engine. The part has a volume of 1015 cubic centimeters. Steel can be purchased for \$0.29 per ~~kilogram~~ 1000g, and has a density of 7.95 g/cm^3. If the machinist makes 500 of these parts, what is the cost of the steel, to the *nearest dollar*?

$1015\ cm^3 \cdot \frac{7.95\ g}{1\ cm^3} \cdot \frac{.29\ \$}{1000\ g} \cdot 500$ $\quad \frac{1015(7.95)(.29)(500)}{1000}$

\$1170

7. A water tower has a volume of 1000 liters and the cost of the water is \$250 per ~~cubic~~ ~~kiloliter.~~ 1000 l How much will it cost to fill the water tower up to 60% of its capacity?

$1000\ l \cdot \frac{\$250}{1000\ l} \times .6$ $\quad \frac{1000(250)(.6)}{1000} = \$150$

8. A wax candle has a volume of 885 cubic centimeters. The wax costs \$1.24 per ~~kilogram~~ 1000g and has a density of 1.9 g/cm^3. How much will it cost to make 80 candles?

$885\ cm^3 \cdot \frac{1.9\ g}{1\ cm^3} \cdot \frac{1.24\ \$}{1000\ g} \times 80$ $\quad \frac{885(1.9)(1.24)(80)}{1000} = \$166.80$

9. An object has a volume of 12 cubic inches and the material it is made from has a density of 7.6 g/in^3. If the cost of the material is \$1.25 per ~~kilogram~~ 1000g, how much will it cost to make 50 of these objects?

$12\ in^3 \cdot \frac{7.6\ g}{1\ in^3} \cdot \frac{1.25\ \$}{1000\ g} \times 50$ $\quad \frac{12(7.6)(1.25)(50)}{1000} = \$5.70$

Modeling Volume

1) Check units. Convert if necessary. To convert units: Multiply to get units to cancel out. Example: $3\ in \bullet \frac{2.54\ cm}{1\ in}$

2) FIND VOLUME (Likely to be compound volume (add) or displaced volume (subtract)

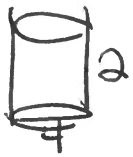
3) Begin unit analysis. Start with volume!

Example, a volume of 12 cubic inches has a density of $7.6\ g/in^3$, which costs \$1.25 per kilogram, and 50 are needed that are each filled up to 85%:

$$12\ in^3 \bullet \frac{7.6\ g}{1\ in^3} \bullet \frac{1kg}{1000\,g} \bullet \frac{\$1.25}{1\ kg} \bullet 50 \bullet .85$$

*If given volume, substitute for V and do Algebra!

1. Cylindrical bricks are needed to fill a hole in a homeowner's backyard. Each brick is to have a diameter of 4 cm and a height of 2 cm. The weight of the concrete that the brick is going to be made from is 2.1 ounces per cubic centimeter. If the concrete costs \$.14 per ounce, how much would it cost to purchase four bricks? Round your answer to the *nearest cent*.



$V = \pi r^2 h$

$V = \pi(2)^2(2)$

$V = 25.\ldots cm^3$

$25.\ldots cm^3 \cdot \frac{2.1\ oz}{1\ cm^3} \cdot \frac{.14\ \$}{1\ oz} \times 4 = \$29.56$

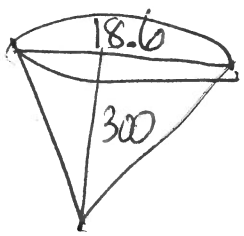
2. A town in upstate New York keeps sand in a silo that is in the shape of a cone. They use this sand to help de-ice the roads after a snowstorm. The silo has a diameter of 18.6 meters and a height of ~~3 kilometers~~ 300 meters. The weight of the sand is 1.2 ounces per cubic meter. If the sand costs \$.12 per ounce, how much will it cost the town to fill 80% of the silo?

$3\ km \cdot \frac{1000\ m}{1\ km} = 300\ m$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(9.3)^2(300)$

$V = 2717.\ldots m^3$

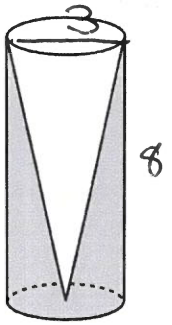


$2717 m^3 \cdot \frac{1.2\ oz}{1\ m^3} \cdot \frac{.12\ \$}{1\ oz} \times .8$

$\$3130.17$

3. Walter wants to make 100 candles in the shape of a cone for his new candle business. The mold shown below will be used to make the candles. Each mold will have a height of 8 inches and a diameter of 3 inches.

Walter goes to a hobby store to buy the wax for his candles. The wax costs \$0.10 per ounce. If the weight of the wax is 0.52 ounce per cubic inch, how much will it cost Walter to buy the wax for 100 candles?



$V = \frac{1}{3}\pi r^2 h$

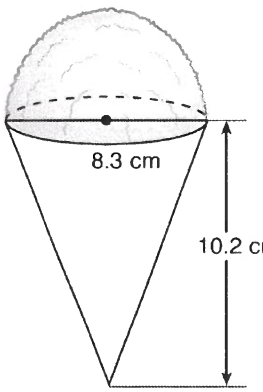
$V = \frac{1}{3}\pi(1.5)^2(8)$

$V = 18\ldots in^3$

$18\ldots in^3 \cdot \frac{.52\ oz}{1\ in^3} \cdot \frac{.10\$}{1\ oz} \times 100$

$18\ldots(.52)(.10)(100) = \$98.02$

4. A snow cone consists of a paper cone completely filled with shaved ice and topped with a hemisphere of shaved ice, as shown in the diagram below. The inside diameter of both the cone and the hemisphere is 8.3 centimeters. The height of the cone is 10.2 centimeters. The desired density of the shaved ice is 0.697 g/cm^3, and the cost, per ~~kilogram~~ 1000 g, of ice is \$3.83. Determine and state the cost of the ice needed to make 50 snow cones.



Hemisphere: $V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$V = \frac{1}{2}\left(\frac{4}{3}\pi(4.15)^3\right)$

$V = 149\ldots$

Cone: $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(4.15)^2(10.2)$

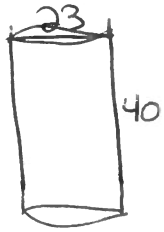
$V = 183\ldots$

$149\ldots + 183\ldots = 333\ldots\ cm^3$

$333\ldots\ cm^3 \cdot \frac{.697\ g}{1\ cm^3} \cdot \frac{3.83\$}{1000\ g} \times 50$

$\frac{333\ldots(.697)(3.83)(50)}{1000} = \$44.53$

5. A cylindrical casing is to be put around a garbage can in a busy street in Manhattan. The diameter is 25 inches. The height of the case will be 40 inches and the casing will be 1 inch thick. The density of the metal is .841 grams per cubic inch. What will be the mass of the casing?



$V=\pi r^2 h$ $V=\pi r^2 h$

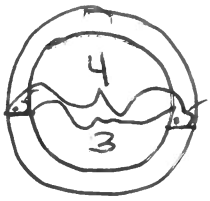
$V=\pi(12.5)^2(40)$ $V=\pi(11.5)^2(40)$

$V=19634..$ $V=16619...$

$19634.. - 16619.. = 3015...in^3$

$3015...in^3 \cdot \frac{.841\ g}{1\ in^3} = 2536\ g$

6. A bakery sells hollow chocolate spheres. The larger diameter of each sphere is 4 cm. The thickness of the chocolate of each sphere is 0.5 cm. Determine and state, to the *nearest tenth of a cubic centimeter*, the amount of chocolate in each hollow sphere. The bakery packages 8 of them into a box. If the density of the chocolate is 1.308 g/cm^3, determine and state, to the *nearest gram*, the total mass of the chocolate in the box.



outer

$V=\frac{4}{3}\pi r^3$

$V=\frac{4}{3}\pi(2)^3$

$V=33...$

inner

$V=\frac{4}{3}\pi r^3$

$V=\frac{4}{3}\pi(1.5)^3$

$V=14...$

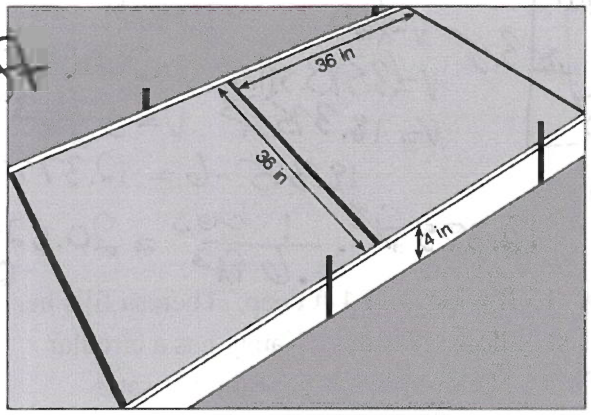
$33... - 14.. = 19...cm^3$

$19...cm^3 \cdot \frac{1.308\ g}{1\ cm^3} \times 8 = 203\ grams$

7. Ian needs to replace two concrete sections in his sidewalk, as modeled below. Each section is 36 inches by 36 inches and 4 inches deep. He can mix his own concrete for $3.25 per cubic foot. How much money will it cost Ian to replace the two concrete sections?

$36 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = 3 \text{ ft}$ ↓ convert first

$4 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = \frac{1}{3} \text{ ft}$



$V = lwh$

$V = 3(3)\left(\frac{1}{3}\right)$

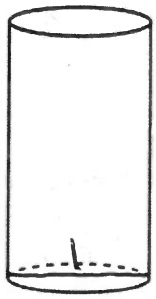
$V = 3 \text{ ft}^3$

$3 \text{ ft}^3 \cdot \frac{3.25 \$}{1 \text{ ft}^3} \times 2 = \$19.50$

8. A concrete footing is a cylinder that is placed in the ground to support a building structure. The cylinder is 4 feet tall and 12 inches in diameter. A contractor is installing 10 footings. If a bag of concrete mix makes $\frac{2}{3}$ of a cubic foot of concrete, determine and state the minimum number of bags of concrete mix needed to make all 10 footings.

convert first

$12 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = 1 \text{ ft}$



$V = \pi r^2 h$

$V = \pi(.5)^2(4)$

$V = 3.14 \text{ ft}^3$

$\frac{3.14(10)}{\frac{2}{3}} = 47.12 \text{ bags}$

48 bags

$3.14 \text{ ft}^3 \cdot \frac{1 \text{ bag}}{\frac{2}{3} \text{ ft}^3} \times 10$

9. A gardener wants to buy enough mulch to cover a rectangular garden that is 3 feet by 10 feet. One bag contains 2 cubic feet of mulch and costs $3.66. How much will the minimum number of bags cost to cover the garden with mulch 3 inches deep?

it's not asking about cost

$3 \text{ in} \cdot \frac{1 \text{ ft}}{12 \text{ in}} = \frac{1}{4} \text{ ft}$

$V = lwh$

$V = 3(10)\left(\frac{1}{4}\right)$

$V = 7.5 \text{ ft}^3$

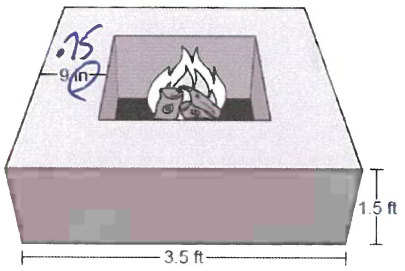
convert first

$7.5 \text{ ft}^3 \cdot \frac{1 \text{ bag}}{2 \text{ ft}^3} = \frac{7.5(1 \text{ bag})}{2} = 3.75 \text{ bags}$

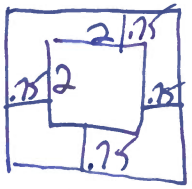
$\frac{7.5}{2} = 3.75 \text{ bags}$

4 bags

10. Josh is making a square-based fire pit out of concrete for his backyard, as modeled by the right prism below. He plans to make the outside walls of the fire pit 3.5 feet on each side with a height of 1.5 feet. The concrete walls of the fire pit are going to be 9 inches thick. If a bag of concrete mix will fill 0.6 ft^3, determine and state the minimum number of bags needed to build the fire pit.



9 in · 1 ft / 12 in = .75 ft



Outside: $V = lwh$; $V = (3.5)(3.5)(1.5)$; $V = 18.375\ ft^3$

Inside: $V = lwh$; $V = 2(2)(1.5)$; $V = 6\ ft^3$

$18.375 - 6 = 12.375\ ft^3$

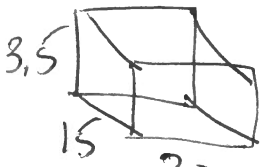
$12.375\ ft^3 \cdot \frac{1\ bag}{.6\ ft^3} = 20.625\ bags$ (21 bags)

11. Theresa has a rectangular pool 30 ft long, 15 ft wide, and 4 ft deep. Theresa fills her pool using city water at a rate of $3.95 per 100 gallons of water. Nancy has a circular pool with a diameter of 24 ft and a depth of 4 ft. Nancy fills her pool with a water delivery service at a rate of $200 per 6000 gallons. If Theresa and Nancy both fill their pools 6 inches from the top of the pool, determine and state who paid more to fill her pool. [$1\ ft^3$ water = 7.48 gallons]

6 in · 1 ft / 12 in = .5 ft

4 − .5 ft = 3.5 ft high

Theresa



$V = lwh$

$V = 3.5(15)(30)$

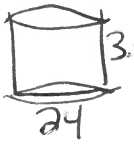
$V = 1575\ ft^3$

$1575\ ft^3 \cdot \frac{7.48\ gal}{1\ ft^3} \cdot \frac{\$3.95}{100\ gal}$

$\frac{1575(7.48)(3.95)}{100} = \$465.$

Theresa paid more!

Nancy



$V = \pi r^2 h$

$V = \pi(12)^2(3.5)$

$V = 1583.\ ft^3$

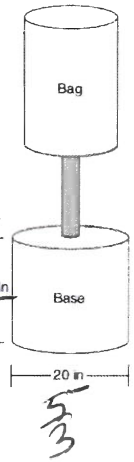
$1583\ ft^3 \cdot \frac{7.48\ gal}{1\ ft^3} \cdot \frac{\$200}{6000\ gal}$

$\frac{1583(7.48)(200)}{6000} = \$394.$

11. Shae has recently begun kickboxing and purchased training equipment as modeled in the diagram below. The total weight of the bag, pole, and unfilled base is 270 pounds. The cylindrical base is 18 inches tall with a diameter of 20 inches. The dry sand used to fill the base weighs 95.46 lbs per cubic foot.

To the *nearest pound*, determine and state the total weight of the training equipment if the base is filled to 85% of its capacity.

18 in · 1 ft / 12 in = 1.5



20 in · 1 ft / 12 in = 1.5 ft

$\frac{5}{3}$ ft

$\frac{5}{3}$

total weight = weight of equipment + weight of sand

$= 270 + 265.. = 536$ pounds

$V = \pi r^2 h$

$V = \pi(\frac{5}{6})^2(1.5)$

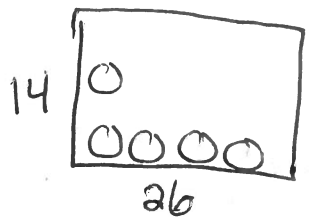
$V = 3.27..\ ft^3$

$3.27..\ ft^3 \cdot \frac{95.46\ lb}{1\ ft^3} \cdot .85 = 265..\ lb$

Shelf/Box Questions

-Draw a two dimensional diagram of the shelf/bottom of the box
-Find how many of each object with fit in each dimension by dividing the dimension by the diameter/width of the object and sketch that into the diagram
*For boxes, add in the third dimension
-Multiply the amount in each dimension by each other to come up with the total number.

1. Boxes of baseball cards are being put on a display shelf. Each box is a cube with edge length of 6 inches. The display shelf is 26 inches by 14 inches. The boxes must completely fit on the shelf and cannot be stacked on top of each other. What is the maximum number of boxes that can fit on the shelf?

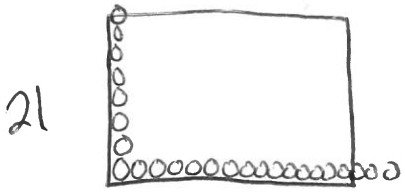


4x2=8

$\frac{26}{6} = 4.\overline{3} = 4$

$\frac{14}{6} = 2.\overline{3} = 2$

2. Cylindrical soup cans with a base diameter of 2.5 inches and a height of 4 inches are to be put on a display shelf. The display shelf measures 21 inches by 45 inches. The cans must completely fit on the shelf and cannot be stacked on top of each other. What is the maximum number of cans that can fit on the shelf?

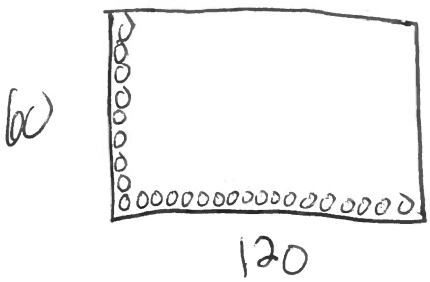


18x8=144

$\frac{21}{2.5} = 8.4 = 8$

$\frac{45}{2.5} = 18$

3. Lacrosse balls have a diameter of 6.47 centimeters and are to be put on a shelf that measures 120 centimeters by 60 centimeters. The balls must completely fit on the shelf and cannot be stacked on top of each other. What is the maximum number of balls that can fit on the shelf?



18x9=162

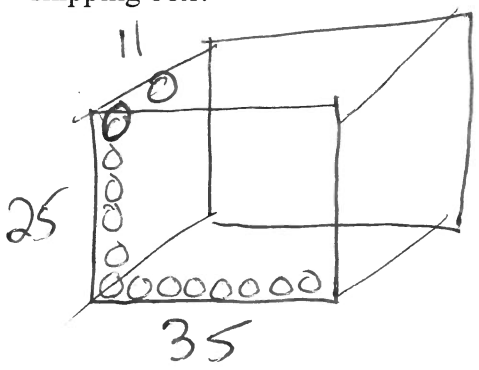
$\frac{60}{6.47}$ $\frac{120}{6.47}$

9.27... 18.5...

9 18

89 ~~43~~ 18

4. Funko Pops come in cubic packages with edge length of 4 inches. They are to be packed into a shipping box that is a rectangular prism that measures 35 inches by 25 inches by 11 inches. What are the maximum number of Funko Pops that can fit into the shipping box?



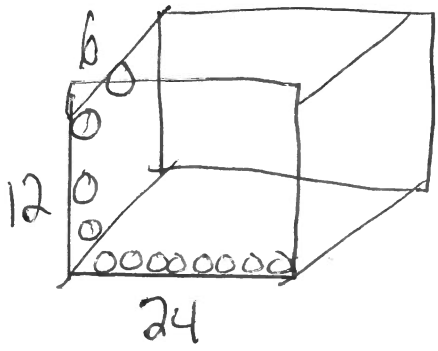
$8 \times 6 \times 2 = 96$

$\frac{35}{4}$ $\frac{25}{4}$ $\frac{11}{4}$

8.75 6.25 2.75

8 6 2

5. Baseballs that have a diameter of 2.8 inches are to be packed into a rectangular shipping box that has dimensions 24 inches by 12 inches by 6 inches. What is the maximum number of baseballs that can fit into the shipping box?



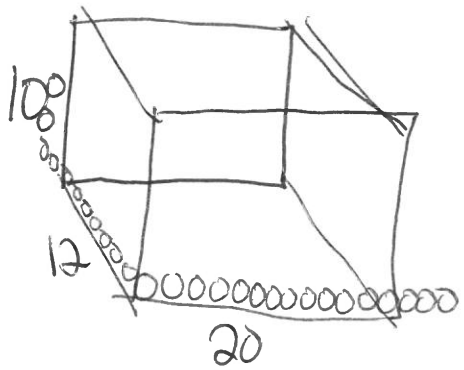
$\frac{24}{2.8}$ $\frac{12}{2.8}$ $\frac{6}{2.8}$

8.57... 4.28... 2.14...

8 4 2

$8 \times 4 \times 2 = 64$

6. Ice cream cones are to be packed into a shipping box that has a base that measures 20 inches by 12 inches and has a height of 10 inches. The cones have a diameter of 1.2 inches and a height of 3.2 inches. How many cones can be packed into the box?



$\frac{20}{1.2}$ $\frac{12}{1.2}$ $\frac{10}{3.2}$

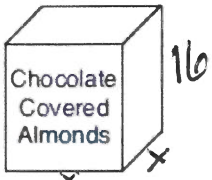
16.6... 10 3.125

16 10 3

$16 \times 10 \times 3 = 480$

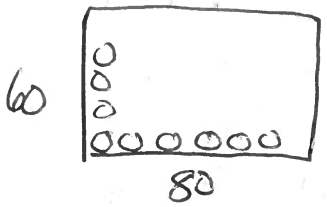
7. A manufacturer is designing a new container for their chocolate-covered almonds. Their original container was a cylinder with a height of 18 cm and a diameter of 14 cm. The new container can be modeled by a rectangular prism with a square base and will contain the same amount of chocolate-covered almonds. Same Volume
If the new container's height is 16 cm, determine and state, to the *nearest tenth of a centimeter*, the side length of the new container if both containers contain the same amount of almonds. A store owner who sells the chocolate-covered almonds displays them on a shelf whose dimensions are 80 cm long and 60 cm wide. The shelf can only hold one layer of new containers when each new container sits on its square base. Determine and state the maximum number of new containers the store owner can fit on the shelf.

$V = \pi r^2 h$
$V = \pi(7)^2(18)$
$V = 2770.\ldots\ cm^3$



$V = lwh$
$2770.\ldots = x(x)(16)$
$\frac{2770.\ldots}{16} = \frac{16x^2}{16}$
$\sqrt{173.\ldots} = \sqrt{x^2}$
$13.2 = x$

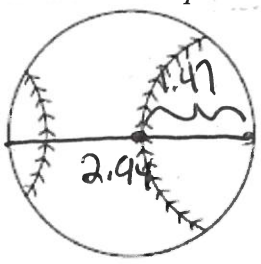
$\frac{80}{13.2} = 6.06\ldots$ 6
$\frac{60}{13.2} = 4.54\ldots$ 4



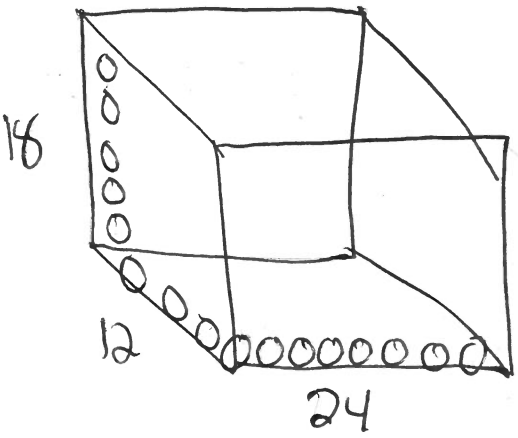
$6 \times 4 = 24$

8. A packing box for baseballs is the shape of a rectangular prism with dimensions of 2 ft × 1 ft × 18 in. Each baseball has a diameter of 2.94 inches.
Determine and state the maximum number of baseballs that can be packed in the box if they are stacked in layers and each layer contains an equal number of baseballs. The weight of a baseball is approximately 0.025 pound per cubic inch. Determine and state, to the *nearest pound*, the total weight of all the baseballs in the fully packed box.

24 in
2 ft × 12 in / 1 ft
1 ft × 12 in / 1 ft
12 in



$\frac{24}{2.94} = 8.16\ldots$ 8
$\frac{12}{2.94} = 4.08\ldots$ 4
$\frac{18}{2.94} = 6.12\ldots$ 6



$8 \times 4 \times 6 =$ 192 baseballs

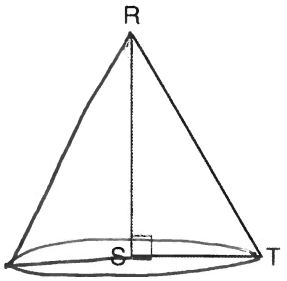
$V = \frac{4}{3}\pi r^3$
$V = \frac{4}{3}\pi(1.47)^3$
$V = 13.3\ldots\ in^3$

$13.3\ldots\ in^3 \times \frac{.025\ lb}{1\ in^3} \times 192$

64 pounds

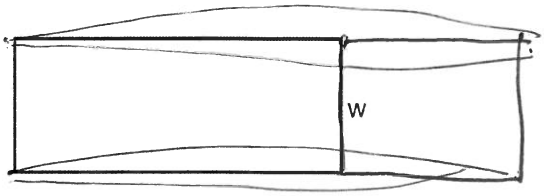
3 dimensional rotations ALMOST ALWAYS form a cylinder or cone
Reflect the shape in 2 dimensions and connect the images with curves

1. Which object is formed when right triangle *RST* shown below is rotated around leg $\overline{RS}$?



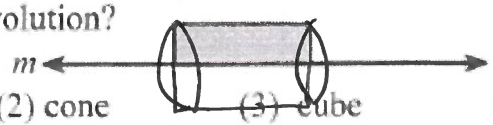
1) a pyramid with a square base
2) an isosceles triangle
3) a right triangle
4) a cone

2. If the rectangle below is continuously rotated about side *w*, which solid figure is formed?



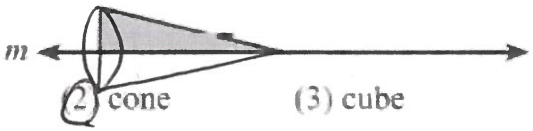
1) pyramid
2) rectangular prism
3) cone
4) cylinder

3. If you rotated the shaded figure below about line *m*, which solid would result from the revolution?



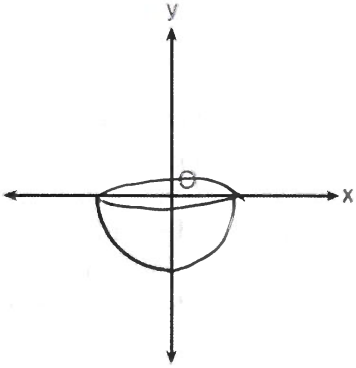
(1) cylinder (2) cone (3) cube (4) sphere

4. If you rotated the triangular region of the figure below about line *m*, what solid would result from the revolution?



(1) cylinder (2) cone (3) cube (4) sphere

5. Circle *O* is centered at the origin. In the diagram below, a quarter of circle *O* is graphed.
Which three-dimensional figure is generated when the quarter circle is continuously rotated about the *y*-axis?



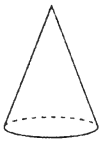
1) cone
2) sphere
3) cylinder
4) hemisphere

6. If an equilateral triangle is continuously rotated around one of its medians, which 3-dimensional object is generated?

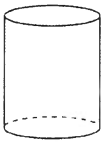
1) cone
2) pyramid
3) prism
4) sphere

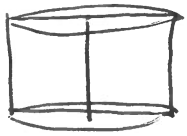
7. A student has a rectangular postcard that he folds in half lengthwise. Next, he rotates it continuously about the folded edge. Which three-dimensional object below is generated by this rotation?

1)

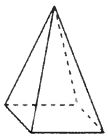


3)

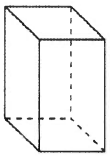




2)



4)

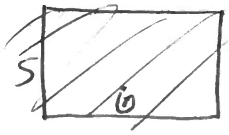


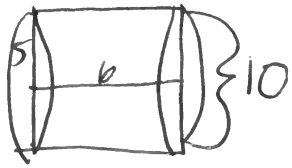
8. An isosceles right triangle whose legs measure 6 is continuously rotated about one of its legs to form a three-dimensional object. The three-dimensional object is a

1) cylinder with a diameter of 6
2) cylinder with a diameter of 12
3) cone with a diameter of 6
4) cone with a diameter of 12

9. Which three-dimensional figure will result when a rectangle 6 inches long and 5 inches wide is continuously rotated about the longer side?

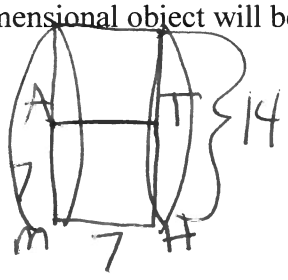
1) a rectangular prism with a length of 6 inches, width of 6 inches, and height of 5 inches
2) a rectangular prism with a length of 6 inches, width of 5 inches, and height of 5 inches
3) a cylinder with a radius of 5 inches and a height of 6 inches
4) a cylinder with a radius of 6 inches and a height of 5 inches



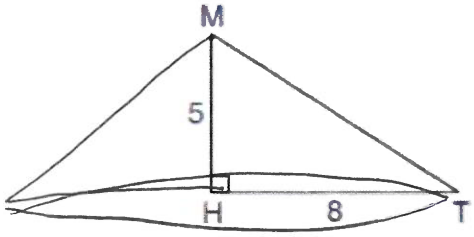


10. Square *MATH* has a side length of 7 inches. Which three-dimensional object will be formed by continuously rotating square *MATH* around side $\overline{AT}$?

1) a right cone with a base diameter of 7 inches
2) a right cylinder with a diameter of 7 inches
3) a right cone with a base radius of 7 inches
4) a right cylinder with a radius of 7 inches



11. In right triangle MTH shown below, $m\angle H = 90°$, $HT = 8$, and $HM = 5$. Determine and state, to the *nearest tenth*, the volume of the three-dimensional solid formed by rotating $\triangle MTH$ continuously around $\overline{MH}$.

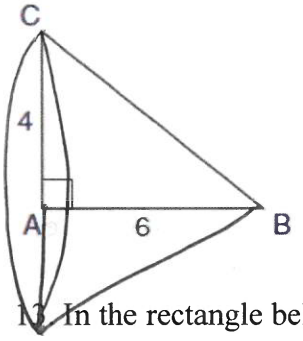


$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(8)^2(5)$

$V = 335.1$

12. In the diagram below, right triangle ABC has legs whose lengths are 4 and 6. What is the volume, in terms of π, of the three-dimensional object formed by continuously rotating the right triangle around $\overline{AB}$?

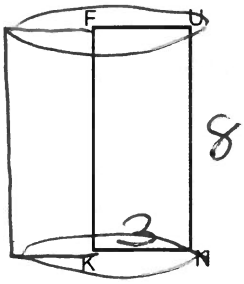


$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(4)^2(6)$

$V = 32\pi$

13. In the rectangle below, $\overline{UN} = 8\,in$ and $\overline{KN} = 3\,in$. Find the volume of the three dimensional object created by rotating rectangle FUNK continuously about side $\overline{FK}$ in terms of π.

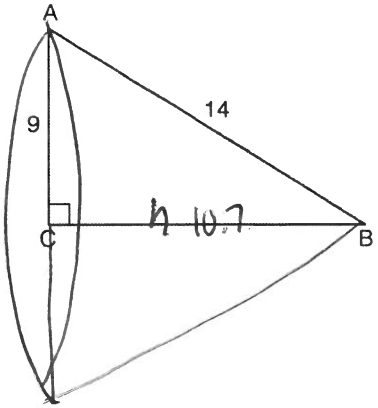


$V = \pi r^2 h$

$V = \pi(3)^2(8)$

$V = 72\pi$

14. In the diagram of right triangle ABC shown below, $AB = 14$ and $AC = 9$. What is the volume of the three dimensional object formed when the triangle is continuously rotated about side $\overline{BC}$ to the nearest tenth.



$a^2 + b^2 = c^2$

$9^2 + h^2 = 14^2$

$81 + h^2 = 196$

$-81 \quad -81$

$\sqrt{h^2} = \sqrt{115}$

$h = 10.7$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi(9)^2(10.7)$

$V = 909.6$

Shortcut
slope is always the same

origin
multiply the scale factor and b

not origin
same line
same equation

Name Schlansky
Mr. Schlansky

Date _____
Geometry

Line Dilations Practice

1. The line $y=3x-2$ is dilated by a scale factor of 2 and centered at the origin. Write an equation that represents the image of the line after the dilation.

multiply scale factor and b

1) $y=3x-2$
2) $y=3x-4$
3) $y=6x-2$
4) $y=6x-4$

$m=3$
$b=2(-2)=-4$

2. The line $y=3x-2$ is dilated by a scale factor of 2 and centered at (-1,-5). Write an equation that represents the image of the line after the dilation.

not origin
same equation

1) $y=3x-2$
2) $y=3x-4$
3) $y=6x-2$
4) $y=6x-4$

3. The line $y=-\frac{1}{2}x+6$ is dilated by a scale factor of 4 and centered at (2,5). Write an equation that represents the image of the line after the dilation.

not origin
same equation

1) $y=-\frac{1}{2}x+6$
2) $y=-\frac{1}{2}x+24$
3) $y=-2x+6$
4) $y=-2x+24$

4. The line $y=-\frac{1}{2}x+6$ is dilated by a scale factor of 4 and centered at the origin. Write an equation that represents the image of the line after the dilation.

multiply scale factor and b

1) $y=-\frac{1}{2}x+6$
2) $y=-\frac{1}{2}x+24$
3) $y=-2x+6$
4) $y=-2x+24$

$m=-\frac{1}{2}$
$b=4(6)=24$

5. Line $y = 3x - 1$ is transformed by a dilation with a scale factor of 2 and centered at $(3, 8)$. The line's image is

1) $y = 3x - 8$
2) $y = 3x - 4$
3) $y = 3x - 2$
4) $y = 3x - 1$

not origin
same equation!
multiply scale factor and b

6. The line $y = 2x - 4$ is dilated by a scale factor of $\frac{3}{2}$ and centered at the origin. Which equation represents the image of the line after the dilation?

1) $y = 2x - 4$
2) $y = 2x - 6$
3) $y = 3x - 4$
4) $y = 3x - 6$

$m = 2$
$b = \frac{3}{2}(-4) = -6$

origin
multiply scale factor and b

7. Line MN is dilated by a scale factor of 2 centered at the origin. If $\overleftrightarrow{MN}$ is represented by $y = -3x + 6$, which equation can represent $\overleftrightarrow{M'N'}$, the image of $\overleftrightarrow{MN}$?

1) $y = -3x + 12$
2) $y = -3x + 6$
3) $y = -6x + 12$
4) $y = -6x + 6$

multiply scale factor and b

$m = -3$
$b = 2(6) = 12$

8. Line MN is dilated by a scale factor of 2 centered at the point $(0, 6)$. If $\overleftrightarrow{MN}$ is represented by $y = -3x + 6$, which equation can represent $\overleftrightarrow{M'N'}$, the image of $\overleftrightarrow{MN}$?

1) $y = -3x + 12$
2) $y = -3x + 6$
3) $y = -6x + 12$
4) $y = -6x + 6$

not origin
same equation

9. The line $y = 4x - 2$ is dilated by a scale factor of 3 and centered at the point (-1,-6). Which equation represents the image of the line after the dilation?

1) $y = 4x - 2$
2) $y = 4x - 6$
3) $y = 12x - 2$
4) $y = 12x - 6$

not origin
same equation

10. The line $y = \frac{1}{2}x + 5$ is dilated by a scale factor of 4 and centered at the point (4,7). Which equation represents the image of the line after the dilation?

1) $y = \frac{1}{2}x + 20$
2) $y = \frac{1}{2}x + 5$
3) $y = 2x + 20$
4) $y = 2x + 5$

not origin
same equation

11. The equation of line h is $2x+y=1$. Line m is the image of line h after a dilation of scale factor 4 with respect to the origin. What is the equation of the line m?

1) $y=-2x+1$
2) $y=-2x+4$
3) $y=2x+4$
4) $y=2x+1$

$2x+y=1$ → multiply scale factor and b
$-2x \quad -2x$
$y=-2x+1$
$m=-2$
$b=4(1)=4$

12. The line $2x+3y=8$ is dilated by a scale factor of 3 and centered at the point (1,2). Which equation represents the image of the line after the dilation?

not origin
same equation

1) $y=-\frac{2}{3}x+\frac{8}{3}$
2) $y=-\frac{2}{3}x+8$
3) $y=-2x+\frac{8}{3}$
4) $y=-2x+8$

$2x+3y=8$ → must put into $y=mx+b$ form
$-2x \quad -2x$
$\frac{3y}{3}=\frac{-2x+8}{3}$ $y=-\frac{2}{3}x+\frac{8}{3}$

13. Line $y-2x=4$ is transformed by a dilation with a scale factor of 2 centered at the origin. What is the equation of the line's image?

$y-2x=4$
$+2x \quad +2x$
$y=2x+4$
$m=2$
$b=2(4)=8$
$y=2x+8$

multiply scale factor and b

14. The equation of a line is given by the equation $2x+2y=6$. Write an equation for the image of the line after a dilation of 2 centered at (3,0).

not origin
same equation
$2x+2y=6$
*you don't have to get y by itself.

15. The equation of line l is $y+2x=1$. Line m is the image of line l after a dilation of 3 centered at the origin. What is the equation of line m.

multiply scale factor and b
$y+2x=1$
$-2x \quad -2x$
$y=-2x+1$
$m=-2$
$b=3(1)=3$
$y=-2x+3$

If I don't have the scale factor and center, all I know is the image is parallel and the slope must be the same.

16. The line $y=2x-1$ is dilated centered at (4,1). Which linear equation could be its image?

1) $y=-2x+3$ (same slope)
2) $y=2x+7$
3) $y=-\frac{1}{2}x-4$
4) $y=\frac{1}{2}x$

17. The line $y=\frac{2}{3}x+3$ is dilated centered at the origin. Which linear equation could be its image? (same slope)

1) $2x+3y=7$
2) $2x-3y=7$
3) $3x-2y=7$
4) $3x+2y=7$

$3x-2y=7$, $-3x$ $-3x$, $-2y=-3$

$-2x$ $-2x$, $\frac{-3y}{-3}=\frac{-2x+7}{-3}$, $y=\frac{2}{3}x-\frac{7}{3}$

18. The line $3y=-2x+8$ is transformed by a dilation centered at the origin. Which linear equation could be its image? (same slope)

1) $2x+3y=5$
2) $2x-3y=5$
3) $3x+2y=5$
4) $3x-2y=5$

$2x+3y=5$, $-2x$ $-2x$, $\frac{3y}{3}=\frac{-2x+5}{3}$, $y=-\frac{2}{3}x+\frac{5}{3}$

$\frac{3y}{3}=\frac{-2x+8}{3}$, $y=-\frac{2}{3}x+\frac{8}{3}$

19. The line represented by the equation $4y=3x+7$ is transformed by a dilation centered at the origin. Which linear equation could represent its image? (same slope)

1) $3x-4y=9$
2) $3x+4y=9$
3) $4x-3y=9$
4) $4x+3y=9$

$3x-4y=9$, $-3x$ $-3x$, $\frac{-4y}{-4}=\frac{-3x+9}{-4}$, $y=\frac{3}{4}x-\frac{9}{4}$

$\frac{4y}{4}=\frac{3x+7}{4}$, $y=\frac{3}{4}x+\frac{7}{4}$

20. The line $-3x+4y=8$ is transformed by a dilation centered at the origin. Which linear equation could represent its image? (same slope)

1) $y=\frac{4}{3}x+8$
2) $y=\frac{3}{4}x+8$
3) $y=-\frac{3}{4}x-8$
4) $y=-\frac{4}{3}x-8$

$-3x+4y=8$, $+3x$ $+3x$, $\frac{4y}{4}=\frac{3x+8}{4}$, $y=\frac{3}{4}x+2$

21. Line l is represented by the equation $y = 4x - 1$. Emely says that the equation of line l after a dilation with a scale factor of 3 centered at (2,7) is $y = 4x - 3$. Is Emely correct? Explain your answer.

no, the center of dilation is on the line which means the equation is the same.

$7 = 4(2) - 1$
$7 = 7$ ✓

22. Josue believes that the image of $y = \frac{2}{3}x + 2$ after a dilation of scale factor 2 centered at the origin is $3y = 2x + 6$. Is Josue correct? Explain your answer.

(0,0) multiply scale factor and b

$\frac{3y}{3} = \frac{2x}{3} + \frac{6}{3}$

$y = \frac{2}{3}x + 2$

$m = \frac{2}{3}$

$b = 2(2) = 4$

$y = \frac{2}{3}x + 4$

$0 = \frac{2}{3}(0) + 2$
$0 = 2$ ✗

No, the center is not on the line. If origin is center, multiply scale factor and b.

23. Line n is represented by the equation $3x + 4y = 20$. Determine and state the equation of line p, the image of line n, after a dilation of scale factor $\frac{1}{3}$ centered at the point (4, 2). [The use of the set of axes below is optional.] Explain your answer.

(x, y)

$3x + 4y = 20$

The center of dilation is on the line so the image is the same line.

$3(4) + 4(2) = 20$
$12 + 8 = 20$
$20 = 20$ ✓

24. Aliyah says that when the line $4x + 3y = 24$ is dilated by a scale factor of 2 centered at the point (3,4), the equation of the dilated line is $y = -\frac{4}{3}x + 16$. Is Aliyah correct? Explain why. [The use of the set of axes below is optional.]

(x, y)

$4(3) + 3(4) = 24$

$12 + 12 = 24$

$24 = 24$ ✓

$4x + 3y = 24$
$-4x \quad -4x$

$\frac{3y}{3} = \frac{-4x + 24}{3}$

$y = -\frac{4}{3}x + 8$

No, the equation should be $y = -\frac{4}{3}x + 8$ because the center of dilation is on the line so the equation stays the same.

Equation of a line through a point flip and negate

1) Find m using parallel (same slope) or perpendicular (negative reciprocal slopes).
2) Substitute into $y-y_1=m(x-x_1)$. Don't forget to negate x_1 and y_1.
3) If it's multiple choice, you may have to distribute and isolate y.

x_1 y_1

1. What is the equation of a line that passes through the point $(-3,-11)$ and is parallel to the line whose equation is $2x-y=4$?

1) $y=2x+5$
2) $y=2x-5$
3) $y=\frac{1}{2}x+\frac{25}{2}$
4) $y=-\frac{1}{2}x-\frac{25}{2}$

$-2x \quad -2x$

$\frac{-y}{-1}=\frac{-2x+4}{-1}$

$y=2x-4$

$m=2$

same slope

$y-y_1=m(x-x_1)$

$y+11=2(x+3)$

$y+11=2x+6$

$-11 \quad -11$

$y=2x-5$

$m\parallel=2$, $x_1=-3$, $y_1=-11$

x_1 y_1

2. What is an equation of the line that passes through the point $(-2,5)$ and is perpendicular to the line whose equation is $y=\frac{1}{2}x+5$?

negative reciprocal slopes

$m=\frac{1}{2}$

1) $y-5=\frac{1}{2}(x+2)$
2) $y-5=-2(x+2)$
3) $y+5=\frac{1}{2}(x-2)$
4) $y+5=-2(x-2)$

$y-y_1=m(x-x_1)$

$y-5=-2(x+2)$

$m\perp=-2$, $x_1=-2$, $y_1=5$

x_1 y_1

3. What is an equation of the line that contains the point $(3,-1)$ and is perpendicular to the line whose equation is $y=-3x+2$?

1) $y=-3x+8$
2) $y=-3x$
3) $y=\frac{1}{3}x$
4) $y=\frac{1}{3}x-2$

$m=-3$

negative reciprocal slopes

$y-y_1=m(x-x_1)$

$y+1=\frac{1}{3}(x-3)$

$y+1=\frac{1}{3}x-1$

$-1 \quad -1$

$y=\frac{1}{3}x-2$

$m\perp=\frac{1}{3}$, $x_1=3$, $y_1=-1$

x_1 y_1

4. An equation of the line that passes through $(2,-1)$ and is parallel to the line $2y+3x=8$ is

1) $y+1=-\frac{3}{2}(x-2)$
2) $y+1=\frac{2}{3}(x-2)$
3) $y-1=-\frac{3}{2}(x+2)$
4) $y-1=\frac{2}{3}(x+2)$

same slope

$-3x \quad -3x$

$\frac{2y}{2}=\frac{-3x+8}{2}$

$y=-\frac{3}{2}x+4$

$m=-\frac{3}{2}$

$m\parallel=-\frac{3}{2}$, $x_1=2$, $y_1=-1$

$y-y_1=m(x-x_1)$

$y+1=-\frac{3}{2}(x-2)$

= 3/5

negative reciprocal slopes

5. What is an equation of the line that is perpendicular to the line whose equation is $y = \frac{3}{5}x - 2$ and that passes through the point $(3, -6)$?

1) $y = \frac{5}{3}x - 11$

2) $y = -\frac{5}{3}x + 11$

3) $y = -\frac{5}{3}x - 1$

4) $y = \frac{5}{3}x + 1$

$x_1\ y_1$

$y - y_1 = m(x - x_1)$

$y + 6 = -\frac{5}{3}(x - 3)$

$y + 6 = -\frac{5}{3}x + 5$

$-6 \quad -6$

$y = -\frac{5}{3}x - 1$

$m\perp = -\frac{5}{3}$

$x_1 = 3$

$y_1 = -6$

6. The equation of a line is $y = \frac{2}{3}x + 5$. What is an equation of the line that is perpendicular to the given line and that passes through the point $(4, 2)$?

negative reciprocal slopes

1) $y = \frac{2}{3}x - \frac{2}{3}$

2) $y = \frac{3}{2}x - 4$

3) $y = -\frac{3}{2}x + 7$

4) $y = -\frac{3}{2}x + 8$

$m = \frac{2}{3}$

$x_1\ y_1$

$y - y_1 = m(x - x_1)$

$y - 2 = -\frac{3}{2}(x - 4)$

$y - 2 = -\frac{3}{2}x + 6$

$+2 \quad +2$

$y = -\frac{3}{2}x + 8$

$m\perp = -\frac{3}{2}$

$x_1 = 4$

$y_1 = 2$

7. What is an equation of the line that passes through the point $(6, 8)$ and is perpendicular to a line with equation $y = \frac{3}{2}x + 5$?

1) $y - 8 = \frac{3}{2}(x - 6)$

2) $y - 8 = -\frac{2}{3}(x - 6)$

3) $y + 8 = \frac{3}{2}(x + 6)$

4) $y + 8 = -\frac{2}{3}(x + 6)$

$m = \frac{3}{2}$

$y - y_1 = m(x - x_1)$

$y - 8 = -\frac{2}{3}(x - 6)$

negative reciprocal slopes

$m\perp = -\frac{2}{3}$

$x_1 = 6$

$y_1 = 8$

8. What is an equation of a line which passes through $(6, 9)$ and is perpendicular to the line whose equation is $4x - 6y = 15$?

1) $y - 9 = -\frac{3}{2}(x - 6)$

2) $y - 9 = \frac{2}{3}(x - 6)$

3) $y + 9 = -\frac{3}{2}(x + 6)$

4) $y + 9 = \frac{2}{3}(x + 6)$

$x_1\ y_1$

$-4x \quad -4x$

$-6y = -4x + 15$

$\frac{-6y}{-6} = \frac{-4x}{-6} + \frac{15}{-6}$

$y = \frac{2}{3}x - \frac{5}{2}$

negative reciprocal slopes

$m\perp = -\frac{3}{2}$

$x_1 = 6$

$y_1 = 9$

$y - y_1 = m(x - x_1)$

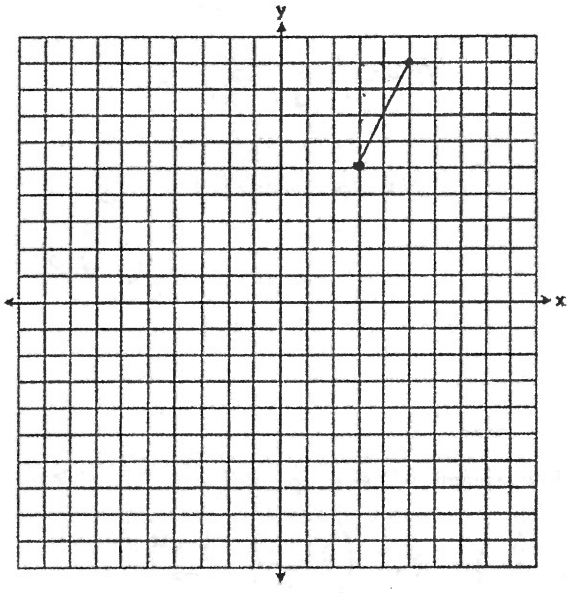
$y - 9 = -\frac{3}{2}(x - 6)$

Name Schlansky
Mr. Schlansky

Date _____
Geometry

Perpendicular Bisector

1. Write an equation of the perpendicular bisector of the line segment whose endpoints are (3,5) and (5,9).



$m = \frac{\Delta y}{\Delta x}$

$m = \frac{4}{2}$

$m = 2$

$mp = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$mp = \left(\frac{3+5}{2}, \frac{5+9}{2}\right)$

$mp = \left(\frac{8}{2}, \frac{14}{2}\right)$

$mp = (4,7)$

$m\perp = -\frac{1}{2}$

$x_1 = 4$

$y_1 = 7$

$y - y_1 = m(x - x_1)$

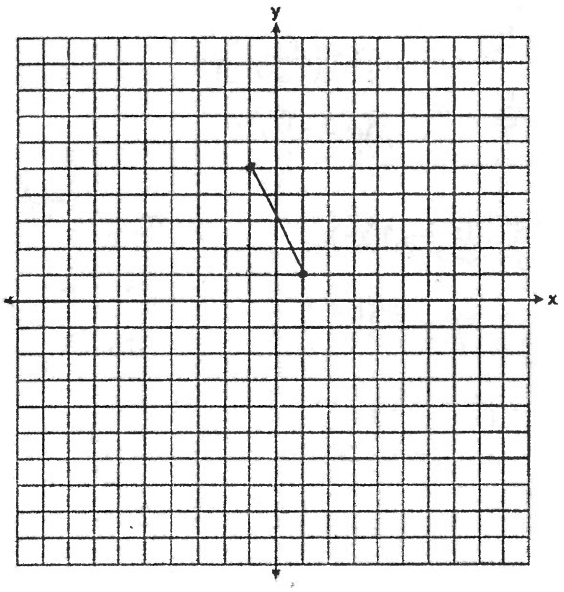
$y - 7 = -\frac{1}{2}(x-4)$

$y - 7 = -\frac{1}{2}x + 2$

$+7 \quad +7$

$y = -\frac{1}{2}x + 9$

2. Write an equation of the perpendicular bisector of the line segment whose endpoints are (-1,5) and (1,1).



$m = \frac{\Delta y}{\Delta x}$

$m = \frac{-4}{2}$

$m = -2$

$mp = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$mp = \left(\frac{-1+1}{2}, \frac{5+1}{2}\right)$

$mp = \left(\frac{0}{2}, \frac{6}{2}\right)$

$mp = (0,3)$

$m\perp = \frac{1}{2}$

$x_1 = 0$

$y_1 = 3$

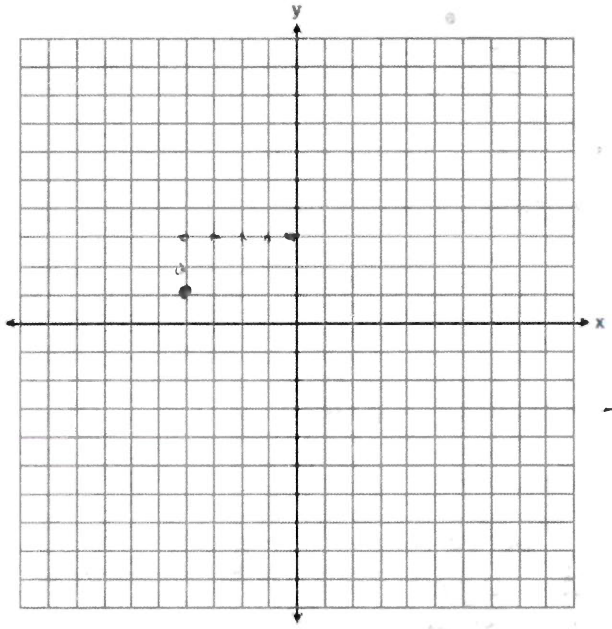
$y - y_1 = m(x - x_1)$

$y - 3 = \frac{1}{2}x$

$+3 \quad +3$

$y = \frac{1}{2}x + 3$

3. Write an equation of the perpendicular bisector of the line segment whose endpoints are (-4,1) and (0,3) in both point slope and slope intercept form.



$m = \frac{\Delta y}{\Delta x}$ $mp = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{-4+0}{2}, \frac{1+3}{2}\right)$

$m = \frac{2}{4}$ $(-2,2)$

$m = \frac{1}{2}$

$m\perp = 2$ $y - y_1 = m(x - x_1)$

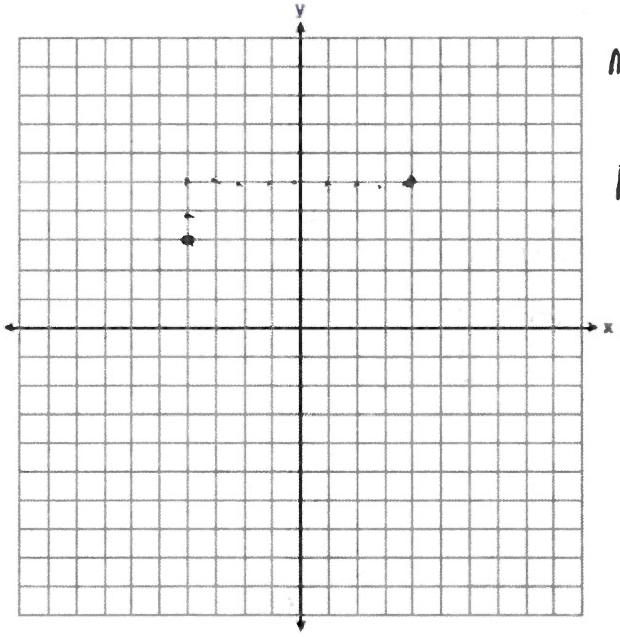
$x_1 = -2$ $y - 2 = 2(x+2)$

$y_1 = 2$ $y - 2 = 2x + 4$

$+2 \quad +2$

$y = 2x + 6$

4. Write an equation of the perpendicular bisector of the line segment whose endpoints are (-4,3) and (4,5) in both point slope and slope intercept form.



$m = \frac{\Delta y}{\Delta x}$ $mp = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$m = \frac{2}{8}$ $= \left(\frac{-4+4}{2}, \frac{3+5}{2}\right)$

$m = \frac{1}{4}$ $(0,4)$

$m\perp = -4$ $y - y_1 = m(x - x_1)$

$x_1 = 0$ $y - 4 = -4(x - 0)$

$y_1 = 4$ $y - 4 = -4x + 0$

$+4 \quad +4$

$y = -4x + 4$

5. Line segment NY has endpoints $N(-11, 5)$ and $Y(5, -7)$. What is the equation of the perpendicular bisector of $\overline{NY}$?

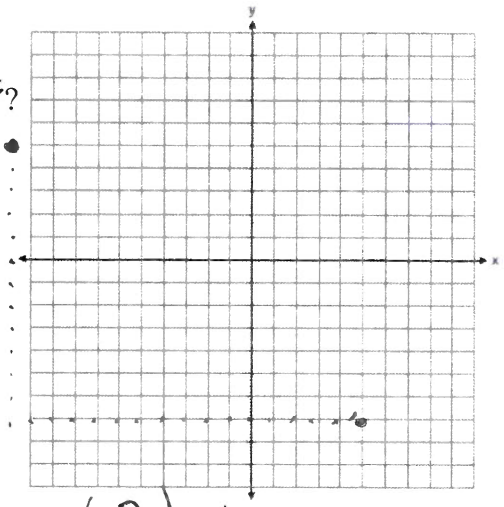
1) $y + 1 = \frac{4}{3}(x + 3)$
2) $y + 1 = -\frac{3}{4}(x + 3)$
3) $y - 6 = \frac{4}{3}(x - 8)$
4) $y - 6 = -\frac{3}{4}(x - 8)$

$m = \frac{\Delta y}{\Delta x}$ $\quad m = \frac{-12}{16}$ $\quad m = -\frac{3}{4}$

$mp = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ $\quad = \left(\frac{-11+5}{2}, \frac{5+-7}{2}\right)$ $\quad (-3, -1)$

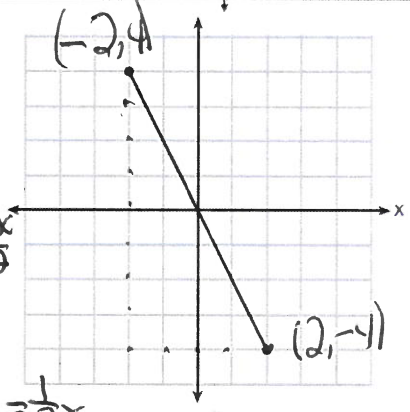
$m_\perp = \frac{4}{3}$ $\quad x_1 = -3$ $\quad y_1 = -1$

$y - y_1 = m(x - x_1)$ $\quad y + 1 = \frac{4}{3}(x + 3)$



6. What is an equation of the perpendicular bisector of the line segment shown in the diagram below?

1) $y + 2x = 0$
2) $y - 2x = 0$
3) $2y + x = 0$
4) $2y - x = 0$



$m = \frac{\Delta y}{\Delta x}$ $\quad m = \frac{-8}{4}$ $\quad m = -2$

$mp = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ $\quad = \left(\frac{-2+2}{2}, \frac{4+-4}{2}\right)$ $\quad (0, 0)$

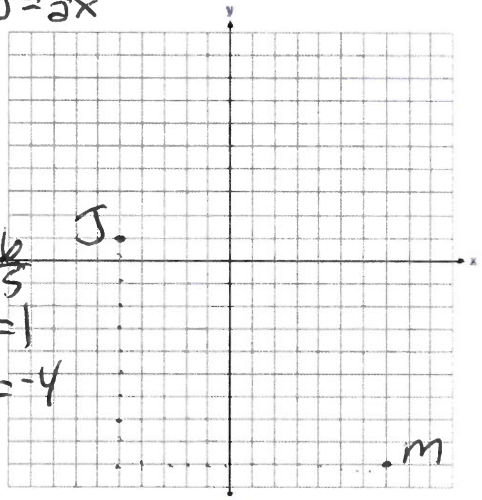
$m_\perp = \frac{1}{2}$ $\quad x = 0$ $\quad y_1 = 0$

$y - y_1 = m(x - x_1)$ $\quad y - 0 = \frac{1}{2}(x - 0)$ $\quad y = \frac{1}{2}x$

$\frac{2y}{2} = \frac{2x}{2}$ $\quad +x \; +x$

7. Segment JM has endpoints $J(-5, 1)$ and $M(7, -9)$. An equation of the perpendicular bisector of $\overline{JM}$ is

1) $y - 4 = \frac{5}{6}(x + 1)$
2) $y + 4 = \frac{5}{6}(x - 1)$
3) $y - 4 = \frac{6}{5}(x + 1)$
4) $y + 4 = \frac{6}{5}(x - 1)$



$m = \frac{\Delta y}{\Delta x}$ $\quad m = \frac{-10}{12}$ $\quad m = \frac{-5}{6}$

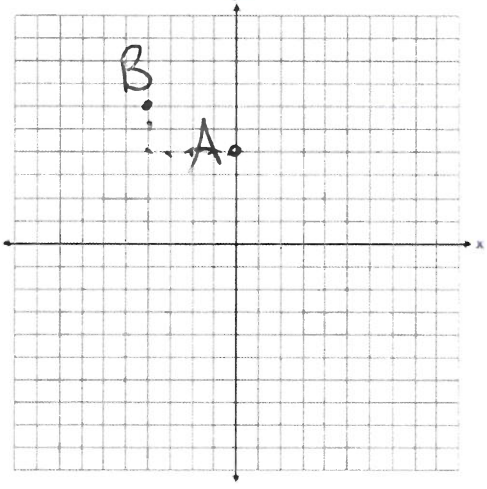
$mp = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ $\quad = \left(\frac{-5+7}{2}, \frac{1+-9}{2}\right)$ $\quad (1, -4)$

$y - y_1 = m(x - x_1)$ $\quad y + 4 = \frac{6}{5}(x - 1)$

$m_\perp = \frac{6}{5}$ $\quad x_1 = 1$ $\quad y_1 = -4$

8. The endpoints of $\overline{AB}$ are $A(0, 4)$ and $B(-4, 6)$. Which equation of a line represents the perpendicular bisector of $\overline{AB}$?

1) $y = -\frac{1}{2}x + 4$
2) $y = -2x + 1$
3) $y = 2x + 8$
4) $y = 2x + 9$



$m = \frac{-2}{4} = -\frac{1}{2}$

$mp = \left(\frac{0+-4}{2}, \frac{4+6}{2}\right)$ $\quad (-2, 5)$

$m_\perp = 2$ $\quad x_1 = -2$ $\quad y_1 = 5$

$y - y_1 = m(x - x_1)$ $\quad y - 5 = 2(x + 2)$ $\quad y - 5 = 2x + 4$ $\quad +5 \quad +5$ $\quad y = 2x + 9$

Center: negate what's in the parenthesis
*If no parenthesis, the coordinate is 0
radius: take the square root of the right hand side

Name Schlansky
Mr. Schlansky

Date _____
Geometry

Graphing Circles

1. What are the center and the radius of the circle whose equation is $(x+5)^2+(y-1)^2=4$

1) center = $(5,-1)$; radius = 4
2) center = $(-5,1)$; radius = 4
3) center = $(5,-1)$; radius = 2
4) center = $(-5,1)$; radius = 2

Center: (-5,1)
radius: 2

2. What are the center and the radius of the circle whose equation is $(x-3)^2+(y+4)^2=36$

1) center = $(3,-4)$; radius = 6
2) center = $(-3,4)$; radius = 6
3) center = $(3,-4)$; radius = 36
4) center = $(-3,4)$; radius = 36

Center: (3,-4)
radius: 6

3. The equation of a circle is $x^2+(y-7)^2=\frac{25}{16}$. What are the center and radius of the circle?

1) center = $(0,7)$; radius = $\frac{5}{4}$
2) center = $(0,7)$; radius = $\frac{25}{16}$
3) center = $(0,-7)$; radius = $\frac{5}{4}$
4) center = $(0,-7)$; radius = $\frac{25}{16}$

Center: (0,7)
radius $\frac{\sqrt{25}}{\sqrt{16}}=\frac{5}{4}$

4. What are the center and the radius of the circle whose equation is $(x-3)^2+(y+3)^2=36$

1) center = $(3,-3)$; radius = 6
2) center = $(-3,3)$; radius = 6
3) center = $(3,-3)$; radius = 36
4) center = $(-3,3)$; radius = 36

Center: (3,-3)
radius: 6

5. What are the center and the radius of the circle whose equation is $(x-5)^2+(y+3)^2=16$?

1) $(-5,3)$ and 16
2) $(5,-3)$ and 16
3) $(-5,3)$ and 4
4) $(5,-3)$ and 4

Center: (5,-3)
radius: 4

6. The equation of a circle is $(x-4)^2+(y-5)^2=\frac{49}{4}$. What are the center and radius of the circle?

1) center = $(-4,-5)$; radius = $\frac{49}{4}$

2) center = $(-4,-5)$; radius = $\frac{7}{2}$

3) center = $(4,5)$; radius = $\frac{49}{4}$

4) center = $(4,5)$; radius = $\frac{7}{2}$

Center: $(4,5)$

radius: $\frac{\sqrt{49}}{\sqrt{4}}=\frac{7}{2}$

7. A circle is represented by the equation $x^2+(y+3)^2=13$. What are the coordinates of the center of the circle and the length of the radius?

1) $(0,3)$ and 13

2) $(0,3)$ and $\sqrt{13}$

3) $(0,-3)$ and 13

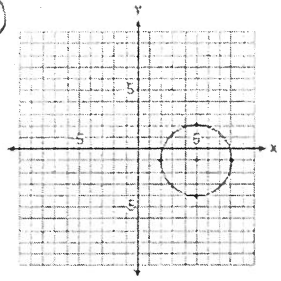
4) $(0,-3)$ and $\sqrt{13}$

Center: $(0,-3)$

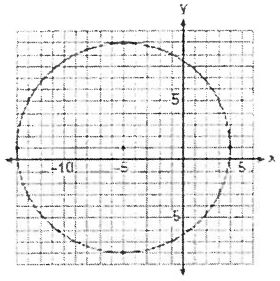
radius: $\sqrt{13}$

8. Which graph represents a circle with the equation $(x-5)^2+(y+1)^2=9$?

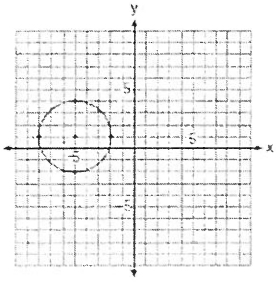
1)



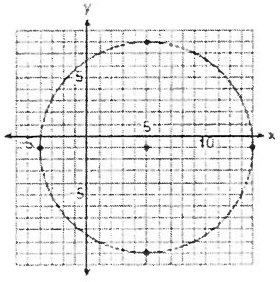
3)



2)



4)

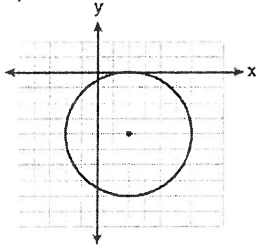


Center: $(5,-1)$

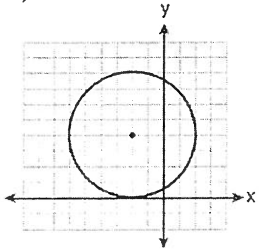
radius: 3

9. The equation of a circle is $(x-2)^2+(y+4)^2=4$. Which diagram is the graph of the circle?

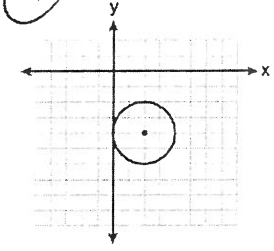
1)



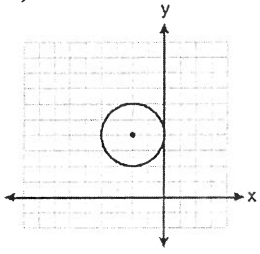
2)



3)



4)



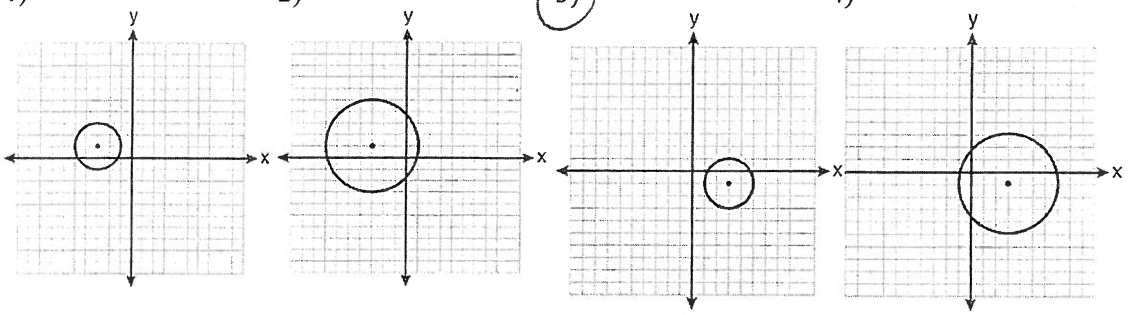
Center: $(2,-4)$

radius: 2

Center: (3,-1)
radius: 2

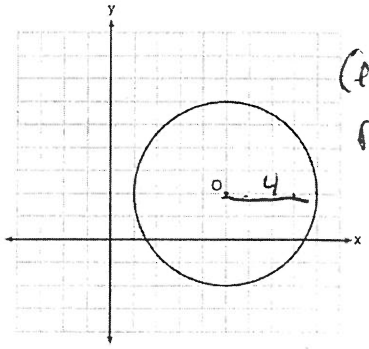
10. Which graph represents a circle with the equation $(x-3)^2 + (y+1)^2 = 4$?

1) 2) 3) 4)



11. Which of the following is the equation of the given circle?

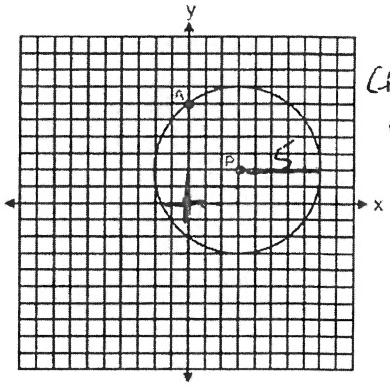
1) $(x-5)^2+(y-2)^2=16$ (5,2) r=4
2) $(x+5)^2+(y+2)^2=16$
3) $(x-5)^2+(y-2)^2=4$
4) $(x+5)^2+(y+2)^2=4$



Center: (5,2)
radius=4

12. Which of the following is the equation of the given circle?

1) $(x-3)^2+(y-2)^2=25$ (3,2) r=5
2) $(x+3)^2+(y+2)^2=25$
3) $(x-3)^2+(y-2)^2=5$
4) $(x+3)^2+(y+2)^2=5$

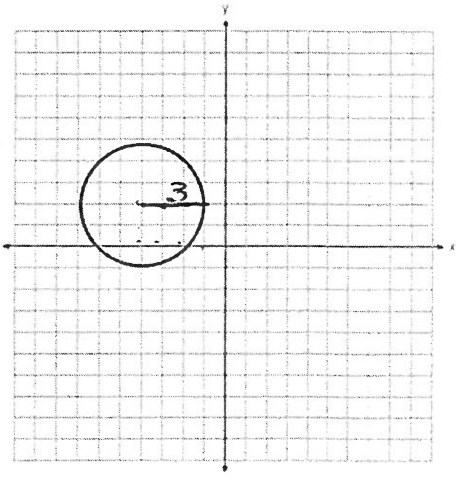


Center: (3,2)
radius: 5

13. Which of the following is the equation of the given circle?

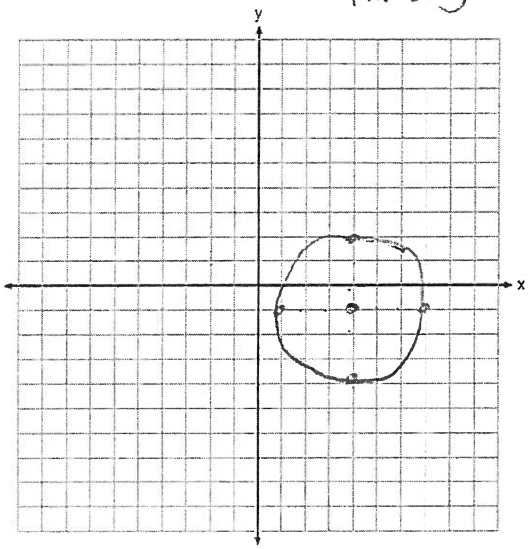
1) $(x-4)^2+(y+2)^2=9$
2) $(x-4)^2+(y+2)^2=3$
3) $(x+4)^2+(y-2)^2=9$ (-4,2), r=3
4) $(x+4)^2+(y-2)^2=3$

Center: (-4,2)
radius: 3

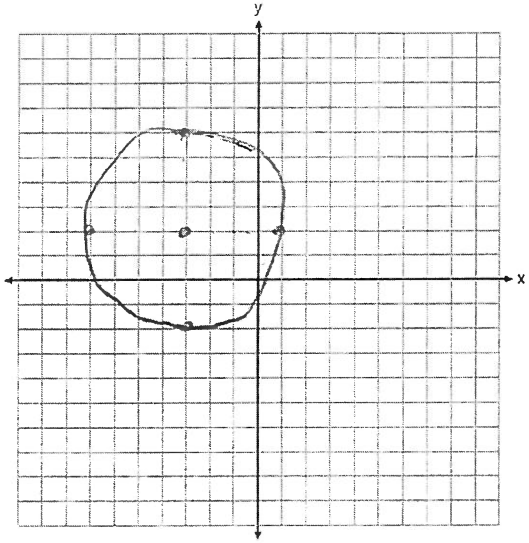


Graph the following circles on the provided graphs

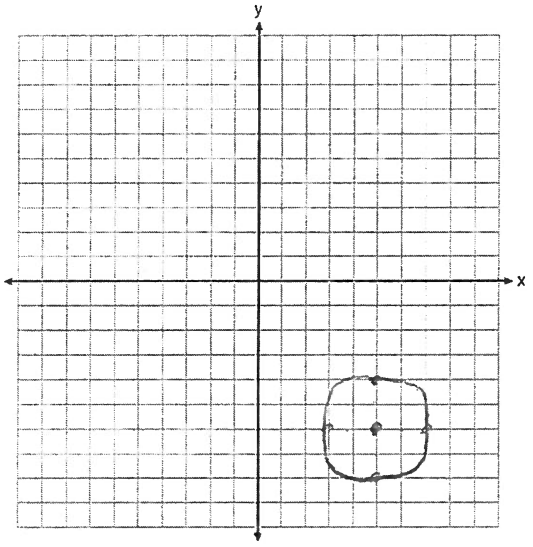
14. $(x-4)^2+(y+1)^2=9$ Center: (4,-1) radius: 3



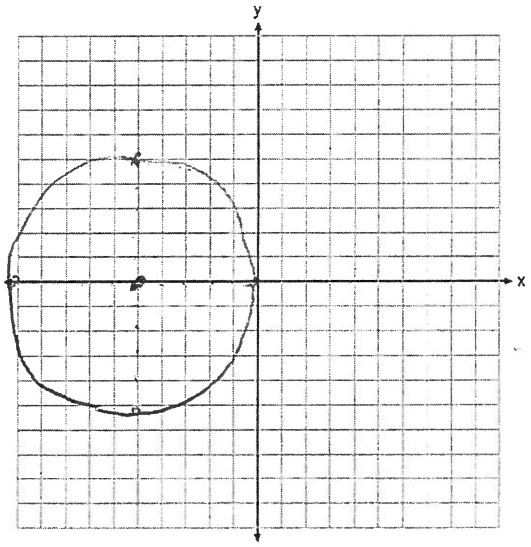
15. $(x+3)^2+(y-2)^2=16$ Center: (-3,2) radius: 4



16. $(x-5)^2+(y+6)^2=4$ Center: (5,-6) radius: 2



17. $(x+5)^2+y^2=25$ Center: (-5,0) radius: 5



Finding Center and Radius of a Circle Given Standard Form
Completing the Square

1) Write the x's together, y's together, and move constant to the other side

$$x^2+bx+y^2+by=c$$

2) Add $\left(\frac{b}{2}\right)^2$ to both sides for each variable

3) Factor each trinomial (Both factors must be the same)

4) Rewrite the factors as a binomial squared

Conics App

1) Apps, Conics, Circles, 2

2) Make sure the equation is equal to 0! If not, move D to the other side

3) ALPHA SOLVE for center and radius

*If radius is a decimal, guess and check for the appropriate radical

1. $x^2+y^2+16x+6y+1=0$

$(\frac{16}{2})^2=64$ $(\frac{6}{2})^2=9$

$-1\ -1$

$x^2+16x+y^2+6y=-1$

$x^2+16x+\boxed{64}+y^2+6y+\boxed{9}=-1+\boxed{64}+\boxed{9}$

$(x+8)^2+(y+3)^2=72$

center: $(-8,-3)$ radius: $\sqrt{72}$

or Apps, Conics, A=1, B=16, C=6, D=1, Alpha Enter, center $(-8,-3)$, radius: 8.485, $\sqrt{64}-\sqrt{81}$, $\sqrt{72}$

2. $x^2+y^2-12x-14y=15$

$(\frac{-12}{2})^2=36$ $(\frac{-14}{2})^2=49$

$x^2-12x+y^2-14y=15$

$x^2-12x+\boxed{36}+y^2-14y+\boxed{49}=15+\boxed{36}+\boxed{49}$

$(x-6)^2+(y-7)^2=100$

center: $(6,7)$ radius: 10

or Apps, Conics, A=1, B=-12, C=-14, D=-15, Alpha Enter, center $(6,7)$, radius: 10

3. $x^2+y^2-4x+8y+\frac{31}{4}=0$ $-\frac{31}{4}$

$-\frac{31}{4}$

$(\frac{8}{2})^2=16$

$(\frac{-4}{2})^2=4$

$x^2-4x+y^2+8y=-\frac{31}{4}$

$x^2-4x+\boxed{4}+y^2+8y+\boxed{16}=-\frac{31}{4}+\boxed{4}+\boxed{16}$

$(x-2)^2+(y+4)^2=\frac{49}{4}$

Center: $(2,-4)$

radius: $\frac{\sqrt{49}}{\sqrt{4}}=\frac{7}{2}$

or Apps, conics, A=1, B=-4, C=8, D=$\frac{31}{4}$, Alpha Enter, Center: $(2,-4)$, radius: 3.5

*use Conics app if multiple choice

4. What are the coordinates of the center of a circle whose equation is $x^2 + y^2 - 16x + 6y + 53 = 0$?

1) (−8,−3)
2) (−8,3)
3) (8,−3)
4) (8,3)

$\left(\frac{-16}{2}\right)^2$ $\left(\frac{6}{2}\right)^2$ 64 9

$x^2-16x+y^2+6y=-53$

$x^2-16x+\boxed{64}+y^2+6y+\boxed{9}=-53+\boxed{64}+\boxed{9}$

$(x-8)^2+(y+3)^2=20$

Center $(8,-3)$ $r=\sqrt{20}$

5. The equation of a circle is $x^2 + y^2 + 6y = 7$. What are the coordinates of the center and the length of the radius of the circle?

1) center (0,3) and radius 4
2) center (0,−3) and radius 4
3) center (0,3) and radius 16
4) center (0,−3) and radius 16

$x^2+y^2+6y+\boxed{9}=7+\boxed{9}$ $\left(\frac{6}{2}\right)^2=9$

$x^2+(y+3)^2=16$

$(0,-3)$ $r=4$

6. What are the coordinates of the center and length of the radius of the circle whose equation is $x^2 + 6x + y^2 - 4y = 23$?

1) (3,−2) and 36
2) (3,−2) and 6
3) (−3,2) and 36
4) (−3,2) and 6

$x^2+6x+\boxed{9}+y^2-4y+\boxed{4}=23+\boxed{9}+\boxed{4}$

$\left(\frac{6}{2}\right)^2=9$ $\left(\frac{-4}{2}\right)^2=4$

$(x+3)^2+(y-2)^2=36$

$(-3,2)$ $r=6$

7. The equation of a circle is $x^2 + y^2 + 12x = -27$. What are the coordinates of the center and the length of the radius of the circle?

1) center (6, 0) and radius 3
2) center (6, 0) and radius 9
3) center (−6, 0) and radius 3
4) center (−6, 0) and radius 9

$\left(\frac{12}{2}\right)^2=36$

$x^2+12x+\boxed{36}+y^2=-27+\boxed{36}$

$(x+6)^2+y^2=9$

$(-6,0)$ $r=6$

8. An equation of circle M is $x^2 + y^2 + 6x - 2y + 1 = 0$. What are the coordinates of the center and the length of the radius of circle M?

1) center (3,−1) and radius 9
2) center (3,−1) and radius 3
3) center (−3, 1) and radius 9
4) center (−3, 1) and radius 3

-1 -1

$\left(\frac{6}{2}\right)^2$ $\left(\frac{-2}{2}\right)^2$ 9 1

$x^2+6x+y^2-2y=-1$

$x^2+6x+\boxed{9}+y^2-2y+\boxed{1}=-1+\boxed{9}+\boxed{1}$

$(x+3)^2+(y-1)^2=9$

$(-3,1)$ $r=3$

$(\frac{2}{2})^2$ $(\frac{-16}{2})^2$
1 64

9. What are the coordinates of the center and length of the radius of the circle whose equation is $x^2 + y^2 + 2x - 16y + 49 = 0$?

1) center $(1, -8)$ and radius 4
2) center $(-1, 8)$ and radius 4
3) center $(1, -8)$ and radius 16
4) center $(-1, 8)$ and radius 16

$x^2+2x+y^2-16y = -49$
$-49 \quad -49$
$x^2+2x+\boxed{1}+y^2-16y+\boxed{64} = -49+\boxed{1}+\boxed{64}$
$(x+1)^2+(y-8)^2 = 16$
$(-1, 8) \quad r=4$

10. What are the coordinates of the center and the length of the radius of the circle whose equation is $x^2 + y^2 - 12y - 20.25 = 0$?

1) center $(0, 6)$ and radius 7.5
2) center $(0, -6)$ and radius 7.5
3) center $(0, 12)$ and radius 4.5
4) center $(0, -12)$ and radius 4.5

$+20.25 \quad +20.25$
$x^2+y^2-12y = 20.25$
$(\frac{-12}{2})^2 = 36$
$x^2+y^2-12y+\boxed{36} = 20.25+\boxed{36}$
$x^2+(y-6)^2 = 56.25$
center: $(0, 6)$
$r = \sqrt{56.25} = 7.5$

11. What is an equation of a circle whose center is $(1, 4)$ and diameter is 10? $r = 5$

$(1, 4) \; r=5$
$a=1$
$b=-2$
$c=-8$
$d=-8$

1) $x^2 - 2x + y^2 - 8y = 8$
2) $x^2 + 2x + y^2 + 8y = 8$
3) $x^2 - 2x + y^2 - 8y = 83$
4) $x^2 + 2x + y^2 + 8y = 83$

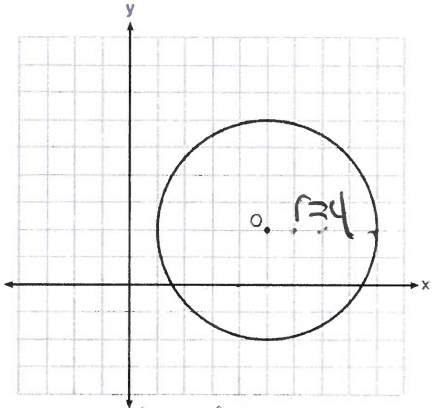
use conics app
or
go through all 4 circles

12. What is an equation of circle O shown in the graph below?

1) $x^2 + 10x + y^2 + 4y = -13$
2) $x^2 - 10x + y^2 - 4y = -13$
3) $x^2 + 10x + y^2 + 4y = -25$
4) $x^2 - 10x + y^2 - 4y = -25$

$a=1$
$b=-10$
$c=-4$
$d=13$

Center: (5, 2) radius: 4



Center: (5, 2)

Partitions

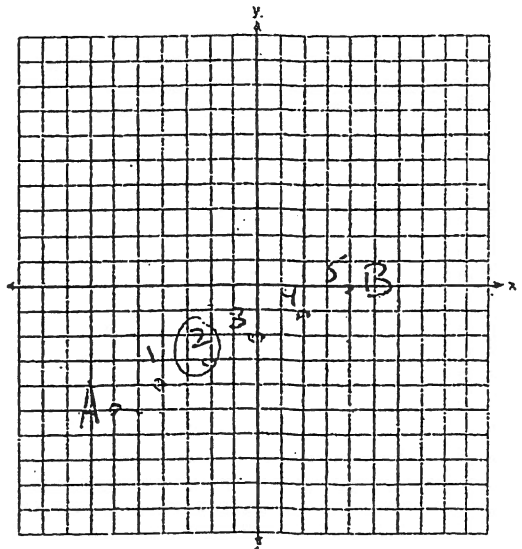
1) **Find** $\frac{\Delta x}{p}$ *and* $\frac{\Delta y}{p}$ where p is the number of partitions.
2) **Count those values out on the graph between the two endpoints**
3) **Circle and state the point that matches the given ratio.**
   BE CAREFUL WHICH POINT YOU START FROM!

1. The coordinates of the endpoints of $\overline{AB}$ are $A(-6,-5)$ and $B(4,0)$. Point P is on $\overline{AB}$. Determine and state the coordinates of point P, such that $AP:PB$ is 2:3. $p=5$

$\frac{\Delta x}{p}$ $\frac{\Delta y}{p}$ $(-2,-3)$

$\frac{10}{5}$ $\frac{5}{5}$

2 1

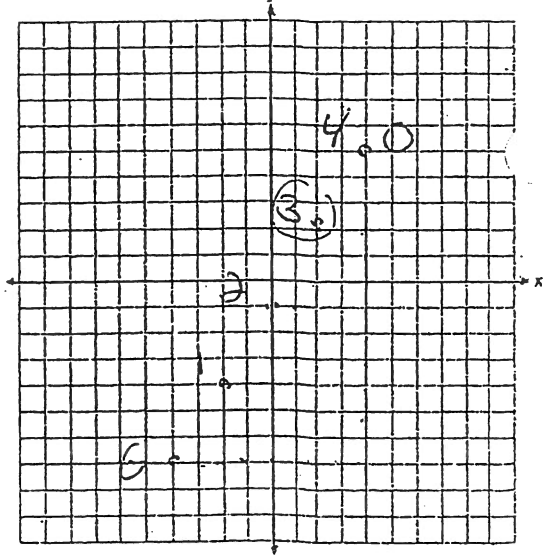


2. What are the coordinates of the point on the directed line segment from $G(-4,-7)$ to $O(4,5)$ that partitions the segment into a ratio of 3 to 1? $p=4$

$\frac{\Delta x}{p}$ $\frac{\Delta y}{p}$ $(2,2)$

$\frac{8}{4}$ $\frac{12}{4}$

2 3

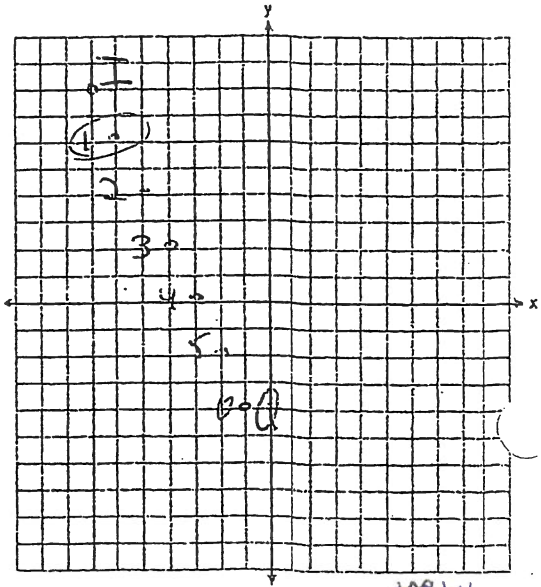


3. Directed line segment IQ has endpoints whose coordinates are $I(-7,8)$ and $Q(-1,-4)$. Determine the coordinates of point J that divides the segment in the ratio 1 to 5. $p=6$

$\frac{\Delta x}{p}$ $\frac{\Delta y}{p}$ $(-6,6)$

$\frac{6}{6}$ $\frac{12}{6}$

1 2



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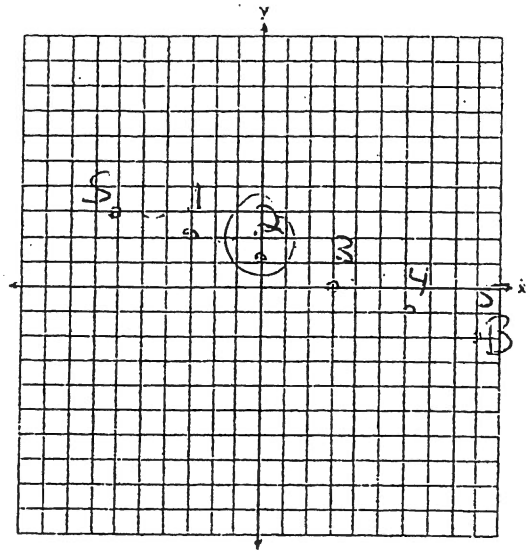
4. Directed line segment SB has endpoints whose coordinates are $S(-6,3)$ and $B(9,-2)$. Determine the coordinates of point J that divides the segment in the ratio 2 to 3. $p=5$

$\frac{\Delta x}{p}$ $\frac{\Delta y}{p}$ $(0,1)$

$\frac{15}{5}$ $\frac{5}{5}$

3 1

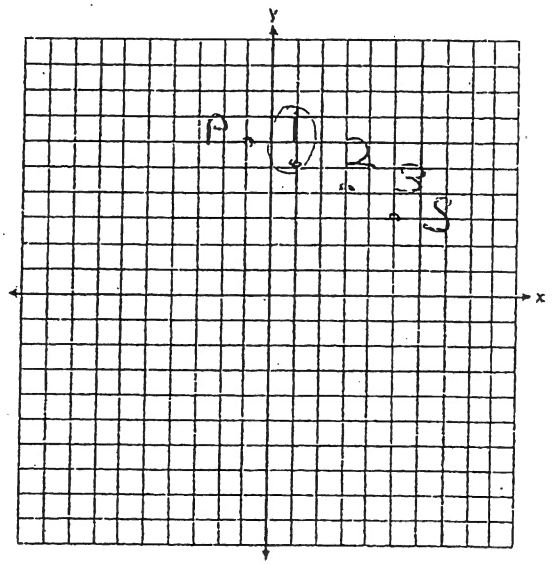


5. What are the coordinates of the point on the directed line segment from $P(-1,6)$ to $S(5,3)$ that partitions the segment into a ratio of 1 to 2? $p=3$

$\frac{\Delta x}{p}$ $\frac{\Delta y}{p}$ $(1,5)$

$\frac{6}{3}$ $\frac{3}{3}$

2 1

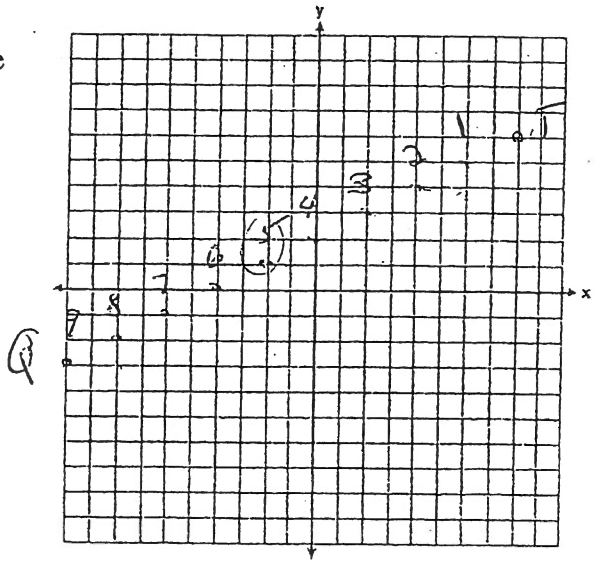


6. Directed line segment JQ has endpoints whose coordinates are $J(8,6)$ and $Q(-10,-3)$. Determine the coordinates of point O that divides the segment in the ratio 5 to 4. $p=9$

$\frac{\Delta x}{p}$ $\frac{\Delta y}{p}$ $(-2,1)$

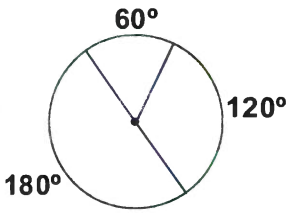
$\frac{18}{9}$ $\frac{9}{9}$

2 1

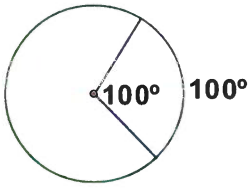


Circle Angle and Segment Rules:

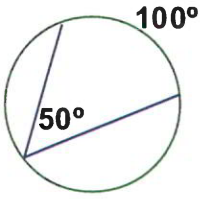
The arcs of a circle add to 360°
A diameter cuts a circle into 2 halves of 180° each



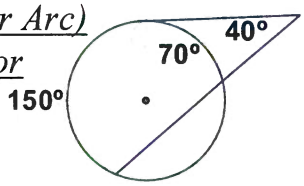
Central Angle: Has its vertex at the center of the circle
Central angle is equal to the measure of the intercepted arc

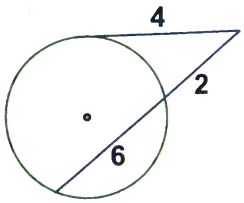


Inscribed Angle: Has its vertex on the circle
Inscribed angle is half of the measure of the intercepted arc

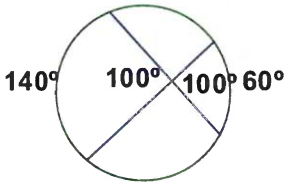


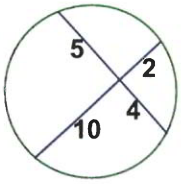
Exterior Segments/Angles:
Angles: *2(Exterior Angle) = (Major Arc – Minor Arc)*
Segments: *Whole • Exterior = Whole • Exterior*



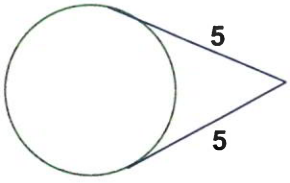


Intersecting Segments/Angles:
Angles: *2(Vertical Angle) = Arc + Arc*
Segments: *Part • Part = Part • Part*

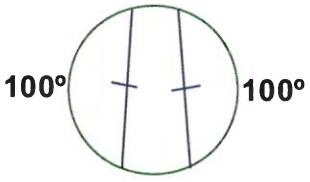




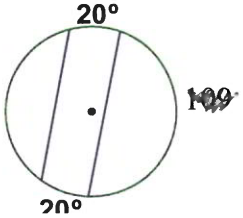
Two tangents drawn from the same point are congruent



Congruent chords intercept congruent arcs



Parallel chords intercept congruent arcs



143
143

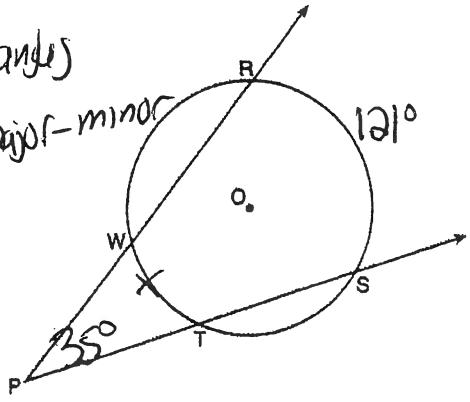
2(exterior angle) = major - minor

1. As shown in the diagram below, secants $\overrightarrow{PWR}$ and $\overrightarrow{PTS}$ are drawn to circle O from external point P.

If $m\angle RPS = 35°$ and $m\overset{\frown}{RS} = 121°$, determine and state $m\overset{\frown}{WT}$.

- exterior
- arcs and angles

2(EA) = major - minor

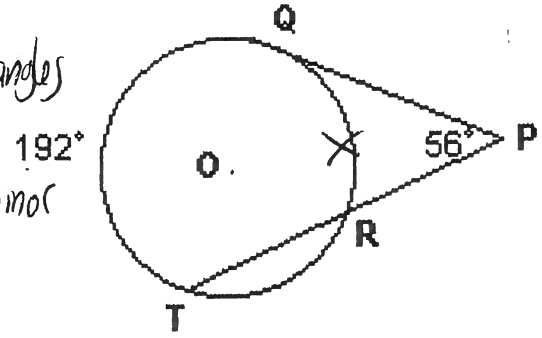


2(EA) = major - minor
2(35) = 121 - x
70 = 121 - x
-121 -121
$\frac{-51}{-1} = \frac{-x}{-1}$
51 = x

2. In the diagram of circle O, $\overline{PQ}$ is tangent to O at Q and $\overline{PRT}$ is a secant. If $m\angle P = 56$ and $m\overset{\frown}{QT} = 192$, find $m\overset{\frown}{QR}$

exterior
arcs and angles

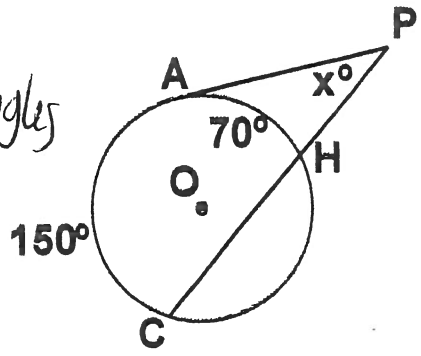
EA) = major - minor



2(EA) = major - minor
2(56) = 192 - x
112 = 192 - x
-192 -192
$\frac{-80}{-1} = \frac{-x}{-1}$
80 = x

3. $\overset{\frown}{AC} = 150°$, $\overset{\frown}{AH} = 70°$, find $m\angle APH$

- exterior
arcs and angles



2(EA) = major - minor
2x = 150 - 70
$\frac{2x}{2} = \frac{80}{2}$
x = 40

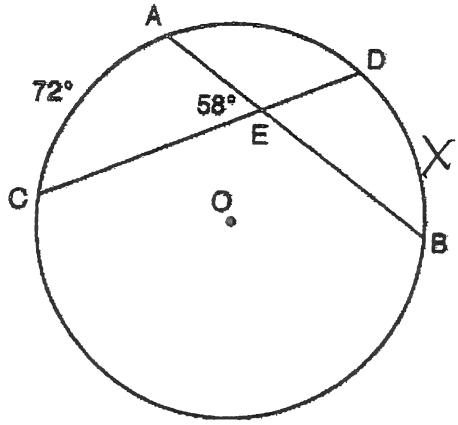
2(EA) = major - minor

2(vertical angle) = arc + arc

4. In the diagram below of circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E. If $m\overset{\frown}{AC} = 72°$ and $m\angle AEC = 58°$, how many degrees are in $m\overset{\frown}{DB}$?

- interior
- arcs and angles

2(VA) = arc + arc

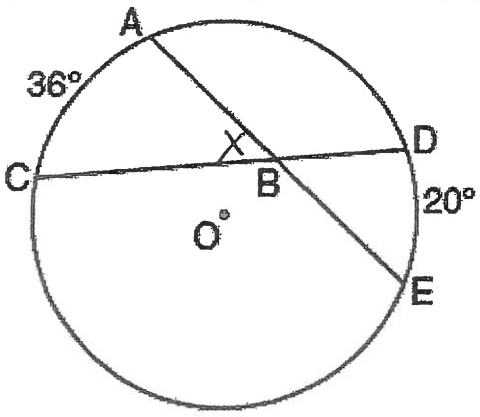


2(VA) = arc + arc
2(58) = x + 72
116 = x + 72
−72 −72
44 = x

5. In the diagram below of circle O, chords $\overline{AE}$ and $\overline{DC}$ intersect at point B, such that $m\overset{\frown}{AC} = 36$ and $m\overset{\frown}{DE} = 20$. What is $m\angle ABC$?

- interior
- arcs and angles

2(VA) = arc + arc

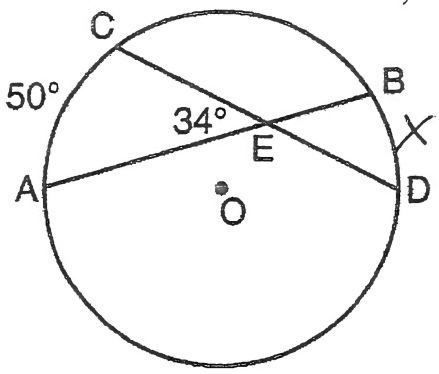


2(VA) = arc + arc
2x = 36 + 20
$\frac{2x}{2} = \frac{56}{2}$
x = 28

6. In the diagram below of circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E. If $m\angle AEC = 34$ and $m\overset{\frown}{AC} = 50$, what is $m\overset{\frown}{DB}$?

- interior
- arcs and angles

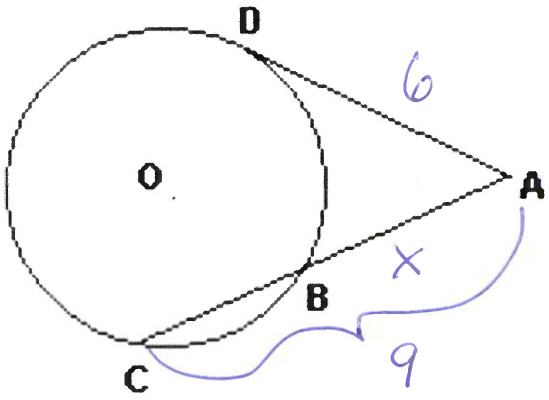
2(VA) = arc + arc



2(VA) = arc + arc
2(34) = x + 50
68 = x + 50
−50 −50
18 = x

Whole · exterior = whole · exterior

7. In the diagram, $\overline{AD}$ is tangent to circle O at D, and $\overline{CBA}$ is a secant. If $AD = 6$ and $AC = 9$, what is AB?



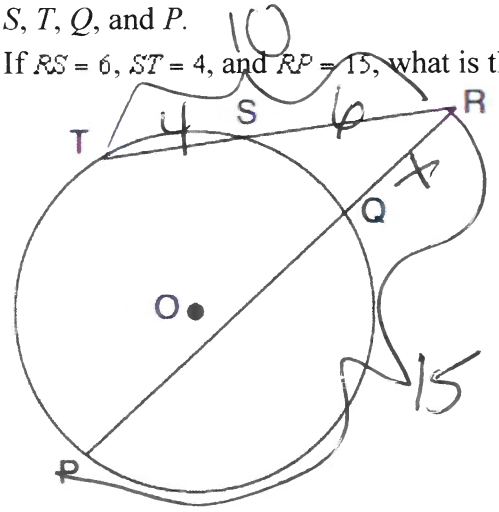
$W \cdot e = w \cdot e$

$6 \cdot 6 = 9 \cdot x$

$\frac{36}{9} = \frac{9x}{9}$

$4 = x$

8. In the diagram below, secants $\overline{RST}$ and $\overline{RQP}$, drawn from point R, intersect circle O at S, T, Q, and P.
If $RS = 6$, $ST = 4$, and $RP = 15$, what is the length of $\overline{RQ}$?



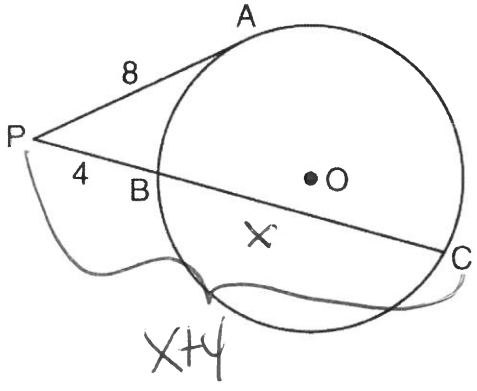
$W \cdot e = w \cdot e$

$10 \cdot 6 = 15 \cdot x$

$\frac{60}{15} = \frac{15x}{15}$

$4 = x$

9. In the diagram below of circle O, $\overline{PA}$ is tangent to circle O at A, and $\overline{PBC}$ is a secant with points B and C on the circle.
If $PA = 8$ and $PB = 4$, what is the length of $\overline{BC}$?



$W \cdot e = w \cdot e$

$8 \cdot 8 = (x+4)4$

$64 = 4x + 16$

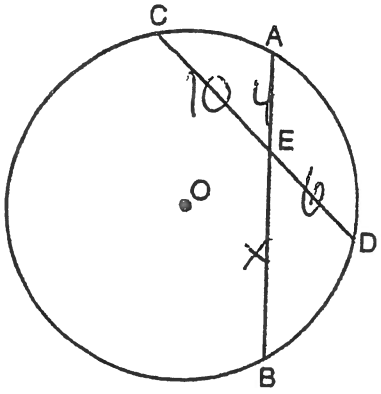
$-16 \quad -16$

$\frac{48}{4} = \frac{4x}{4}$

$12 = x$

Part · Part = Part · Part

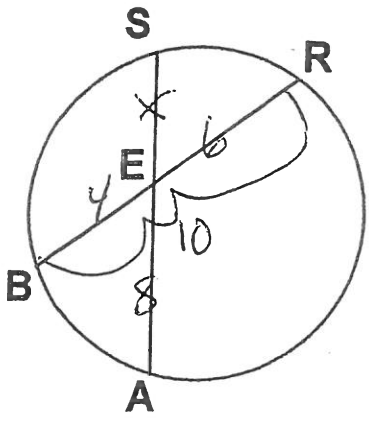
10. In the diagram below of circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E.
If $CE = 10$, $ED = 6$, and $AE = 4$, what is the length of $\overline{EB}$?



P·P=P·P
10·6 = 4·x
60 = 4x
4 4
15 = x

- Segments
- interior
P·P=P·P

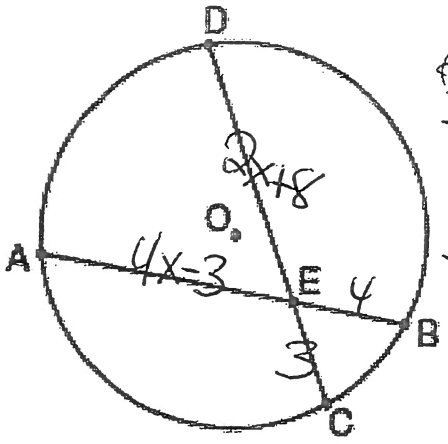
11. If $\overline{BR} = 10$, $\overline{BE} = 4$, $\overline{AE} = 8$, find $\overline{ES}$



P·P=P·P
6·4 = 8·x
24 = 8x
8 8
3 = x

- Segments
- interior
P·P=P·P
*10 is not a part

12. In the diagram of circle O below, chord $\overline{AB}$ intersects chord $\overline{CD}$ at E, $DE = 2x + 8$, $EC = 3$, $AE = 4x - 3$, and $EB = 4$.
What is the value of x?

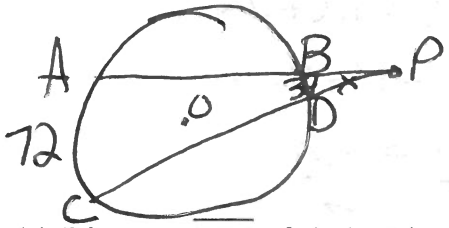


P·P=P·P
3(2x+8) = 4(4x-3)
6x+24 = 16x-12
-6x -6x
24 = 10x-12
+12 +12
36 = 10x
10 10
3.6 = x

- Segments
- interior
P·P=P·P

147

13. In circle O two secants, $\overline{ABP}$ and $\overline{CDP}$, are drawn to external point P. If $m\overset{\frown}{AC} = 72°$, and $m\overset{\frown}{BD} = 34°$, what is the measure of $\angle P$?



$2(EA) = major - minor$

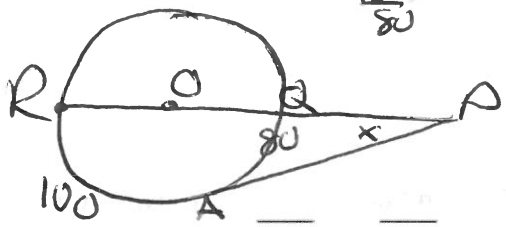
$2x = 72 - 34$

$\frac{2x}{2} = \frac{38}{2}$

$x = 19$

14. Diameter $\overline{ROQ}$ of circle O is extended through Q to point P, and tangent $\overline{PA}$ is drawn. If $m\overset{\frown}{RA} = 100°$, what is $m\angle P$?

$180 - 100 = 80$



$2(EA) = major - minor$

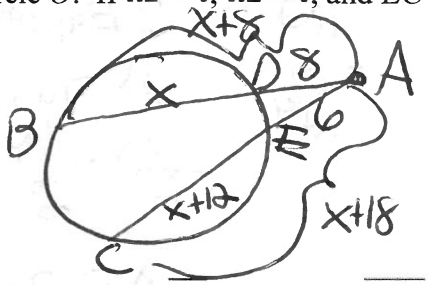
$2x = 100 - 80$

$\frac{2x}{2} = \frac{20}{2}$

$x = 10$

15. In circle O, secants $\overline{ADB}$ and $\overline{AEC}$ are drawn from external point A such that points D, B, E, and C are on circle O. If $AD = 8$, $AE = 6$, and EC is 12 more than BD, the length of $\overline{BD}$ is

1) 6
2) 22
3) 36
4) 48



$w \cdot e = w \cdot e$

$(x+8)8 = (x+18)(6)$

$8x + 64 = 6x + 108$

$-6x \quad -6x$

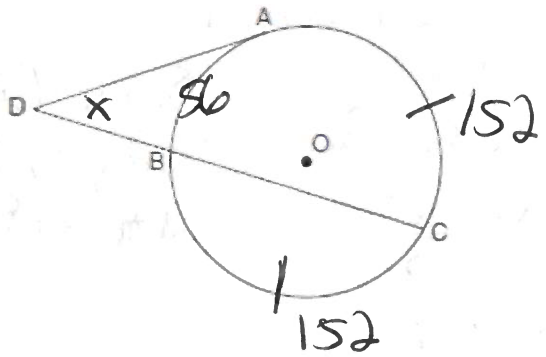
$2x + 64 = 108$

$-64 \quad -64$

$\frac{2x}{2} = \frac{44}{2}$

$x = 22$

16. In the diagram below, tangent $\overline{DA}$ and secant $\overline{DBC}$ are drawn to circle O from external point D, such that $\overset{\frown}{AC} \cong \overset{\frown}{BC}$. If $m\overset{\frown}{BC} = 152°$, determine and state $m\angle D$.



$2(EA) = major - minor$

$2x = 152 - 56$

$\frac{2x}{2} = \frac{96}{2}$

$x = 48$

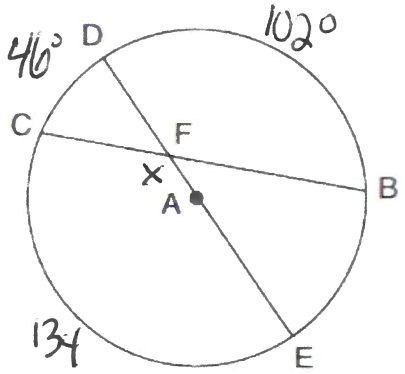
$152 + 152 + x = 360$

$304 + x = 360$

$-304 \quad -304$

$x = 56$

17. In circle A below, chord $\overline{BC}$ and diameter $\overline{DAE}$ intersect at F. If $m\overset{\frown}{CD} = 46°$ and $m\overset{\frown}{DB} = 102°$, what is $m\angle CFE$?



180
−46
134

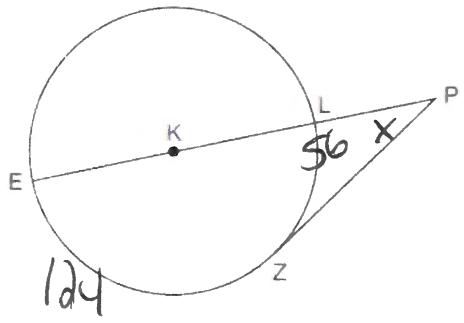
$2(VA) = arc + arc$

$2x = 134 + 102$

$\frac{2x}{2} = \frac{236}{2}$

$x = 118$

18. In the diagram below of circle K, secant $\overline{PLKE}$ and tangent $\overline{PZ}$ are drawn from external point P. If $m\overset{\frown}{LZ} = 56°$, determine and state the degree measure of angle P.



180
−56
124

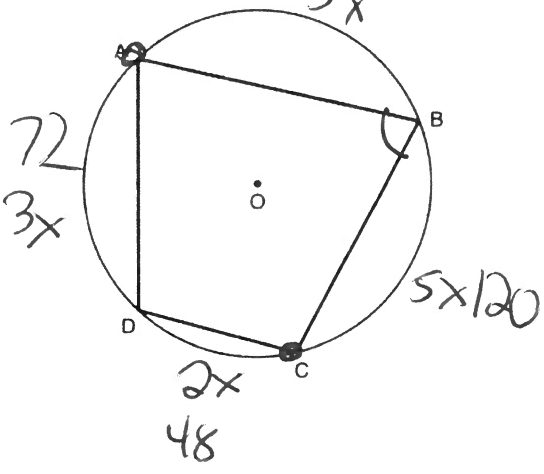
$2(EA) = major - minor$

$2x = 124 - 56$

$\frac{2x}{2} = \frac{68}{2}$

$x = 34$

19. In the diagram below, quadrilateral $ABCD$ is inscribed in circle O, and $m\overset{\frown}{CD} : m\overset{\frown}{DA} : m\overset{\frown}{AB} : m\overset{\frown}{BC} = 2 : 3 : 5 : 5$. Determine and state $m\angle B$.



$5x + 5x + 2x + 3x = 360$

$\frac{15x}{15} = \frac{360}{15}$

$x = 24$

$5(24) = 120$

$3(24) = 72$

$2(24) = 48$

$\angle B = \frac{1}{2}(AC)$

$\angle B = \frac{1}{2}(120)$

$\angle B = 60°$

14a

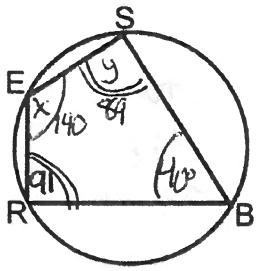
The opposite angles add to 180°

Name Schlansky
Mr. Schlansky

Date _____
Geometry

Quadrilaterals Inscribed In a Circle

1. In the diagram below, quadrilateral *SBRE* is inscribed in the circle. If $m\angle BRE = 91°$ and $m\angle SBR = 40°$, find $m\angle BSE$ and $m\angle SER$



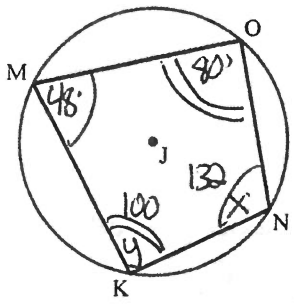
89

$x + 40 = 180$
$-40 \quad -40$
$x = 140$

140

$91 + y = 180$
$-91 \quad -91$
$y = 89$

2. In the diagram below, quadrilateral MONK is inscribed in circle J, $m\angle KMO = 48°$ and $m\angle MON = 80°$. Find the measures of $m\angle KNO$ and $m\angle MKN$.



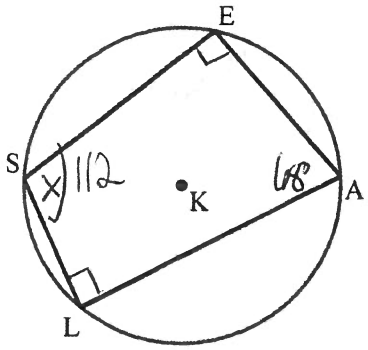
$x + 48 = 180$
$-48 \quad -48$
$x = 132$

$80 + y = 180$
$-80 \quad -80$
$y = 100$

90°

3. In the diagram below, quadrilateral SEAL is inscribed in circle K, $\overline{SE} \perp \overline{EA}$ and $m\angle EAL = 68°$. Find the measures of $m\angle SLA$ and $m\angle ESL$.

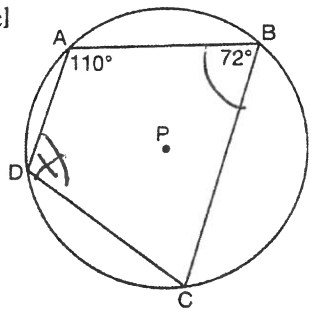
90 112



$x + 68 = 180$
$-68 \quad -68$
$x = 112$

150
~~120~~

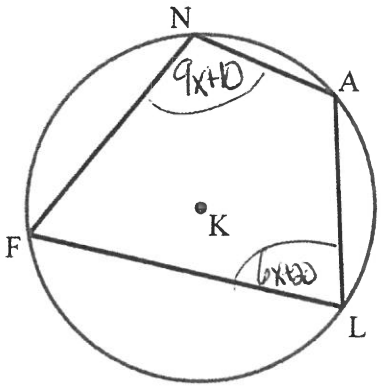
4. In the diagram below, quadrilateral *ABCD* is inscribed in circl



What is m∠*ADC*?
1) 70°
2) 72°
3) 108°
4) 110°

$x+72=180$
$-72\ -72$
$x=108$

5. In the diagram below, quadrilateral FLAN is inscribed in circle K, $m\angle FNA = 9x+10$ and $m\angle FLA = 6x+20$. Find the measures of $m\angle FLA$.

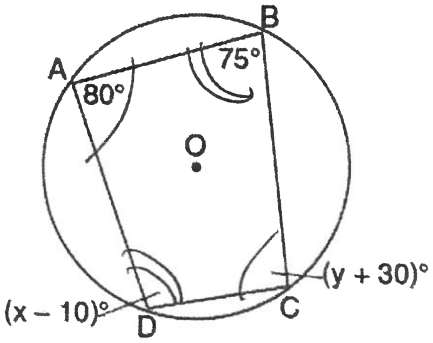


$9x+10+6x+20=180$
$15x+30=180$
$-30\ -30$
$\frac{15x}{15}=\frac{150}{15}$
$x=10$

$6x+20$
$6(10)+20$
$80°$

6. Quadrilateral *ABCD* is inscribed in circle *O*, as shown below.

If m∠*A* = 80°, m∠*B* = 75°, m∠*C* = (*y* + 30)°, and m∠*D* = (*x* − 10)°, which statement is true?
1) $x = 85$ and $y = 50$
2) $x = 90$ and $y = 45$
3) $x = 110$ and $y = 75$
4) $x = 115$ and $y = 70$



$80+y+30=180$
$y+110=180$
$-110\ -110$
$y=70$

$75+x-10=180$
$x+65=180$
$-65\ -65$
$x=115°$

151
~~121~~

Area of a Sector = $\frac{\theta \pi r^2}{360}$

θ = *central angle*, r = *radius*

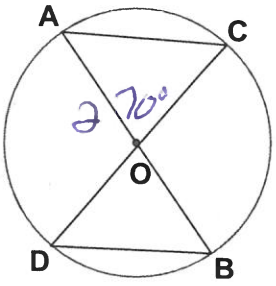
*You may need to use a linear pair (180), central angle (equal to intercepted arc), inscribed angle (half of the intercepted arc), or any other geometry concept to find the central angle and/or radius.

1. In circle O, $m\angle AOC = 70$ and $\overline{AO} = 2$ *in*. Find the area of sector COA to the *nearest tenth of a square inch*.

$A = \frac{\theta \pi r^2}{360}$

$A = \frac{70\pi(2)^2}{360}$

$A = 2.4$

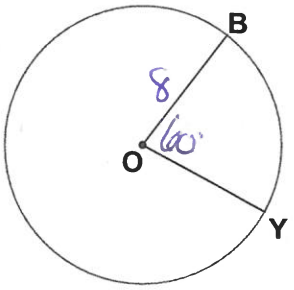


2. In circle O, if $\angle$ BOY = 60° and $\overline{BO} = 8$ *cm*, find the area of sector BOY in terms of π.

$A = \frac{\theta \pi r^2}{360}$

$A = \frac{60\pi(8)^2}{360}$

$A = \frac{32}{3}\pi$

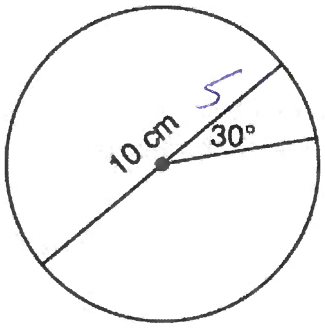


3. A circle with a diameter of 10 cm and a central angle of 30° is drawn below. What is the area, to the *nearest tenth of a square centimeter*, of the sector formed by the 30° angle?

$A = \frac{\theta \pi r^2}{360}$

$A = \frac{30\pi(5)^2}{360}$

$A = 6.5$



4. Determine and state, in terms of π, the area of a sector that intercepts a 40° arc of a circle with a radius of 4.5.

$A = \frac{\theta \pi r^2}{360}$

$A = \frac{40\pi(4.5)^2}{360}$

$A = 2.25\pi$

5. In the diagram below of circle O, $GO = 8$ and $m\angle GOJ = 60°$. What is the area, in terms of π, of the shaded region?

1) $\frac{4\pi}{3}$

2) $\frac{20\pi}{3}$

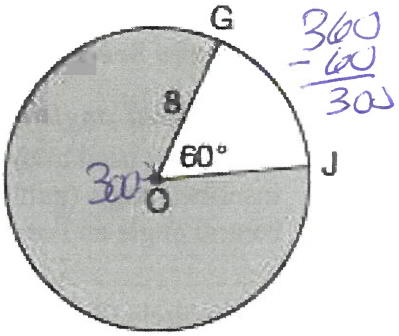
3) $\frac{32\pi}{3}$

4) $\frac{160\pi}{3}$

$A = \frac{\theta\pi r^2}{360}$

$A = \frac{300\pi(8)^2}{360}$

$A = \frac{160}{3}\pi$



360 − 60 = 300

6. In circle O below, $OA = 6$, and $m\angle COA = 100°$. What is the area of the shaded sector?

1) 10π

2) 26π

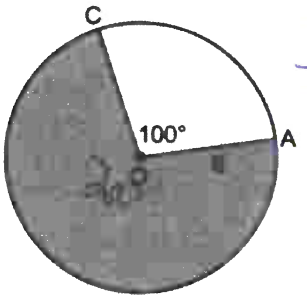
3) $\frac{10\pi}{3}$

4) $\frac{26\pi}{3}$

$A = \frac{\theta\pi r^2}{360}$

$A = \frac{260\pi(6)^2}{360}$

$A = 26\pi$



360 − 100 = 260

7. Circle O with a radius of 9 is drawn below. The measure of central angle AOC is 120°. What is the area of the shaded sector of circle O?

1) 6π

2) 12π

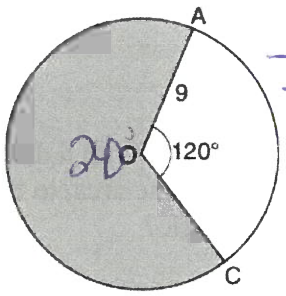
3) 27π

4) 54π

$A = \frac{\theta\pi r^2}{360}$

$A = \frac{240\pi(9)^2}{360}$

$A = 54\pi$



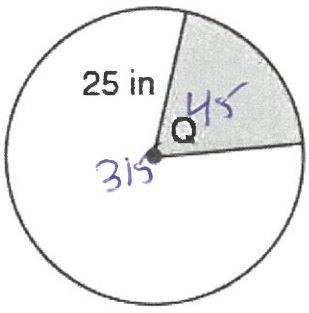
360 − 120 = 240

8. In the diagram below, the circle has a radius of 25 inches. The measure of angle Q is 45. Find the area of the *unshaded* sector in terms of π.

$A = \frac{\theta\pi r^2}{360}$

$A = \frac{315\pi(25)^2}{360}$

$A = \frac{4375}{8}\pi$



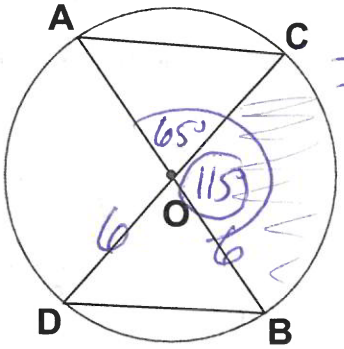
360 − 45 = 315

9. In circle O, $m\angle AOC = 65$ and $\overline{DO} = 6$ *in*. Find the area of sector COB in terms of π.

$A = \frac{\theta \pi r^2}{360}$

$A = \frac{115\pi(6)^2}{360}$

$A = \frac{23}{2}\pi$



$180 - 65 = 115$

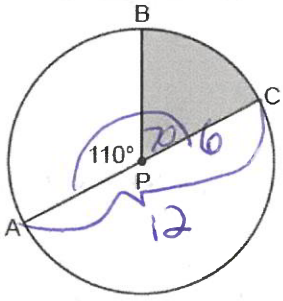
10. In circle *P* below, diameter $\overline{AC}$ and radius $\overline{BP}$ are drawn such that $m\angle APB = 110°$. If $AC = 12$, what is the area of shaded sector *BPC*?

1) $\frac{7}{6}\pi$
2) 7π
3) 11π
4) 28π

$A = \frac{\theta \pi r^2}{360}$

$A = \frac{70\pi(6)^2}{360}$

$A = 7\pi$



$180 - 110 = 70$

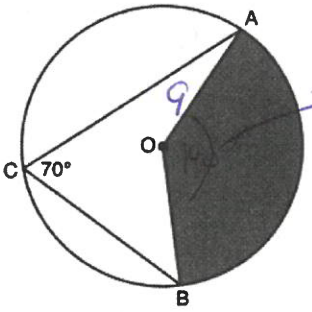
11. In the diagram below of circle *O*, $\overline{AC}$ and $\overline{BC}$ are chords, and $m\angle ACB = 70°$. If $OA = 9$, the area of the shaded sector *AOB* is

1) 3.5π
2) 7π
3) 15.75π
4) 31.5π

$A = \frac{\theta \pi r^2}{360}$

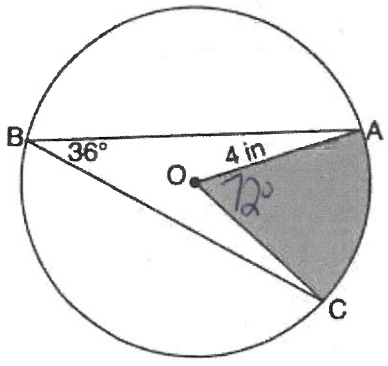
$A = \frac{140\pi(9)^2}{360}$

$A = \frac{63}{2}\pi$



$360 - 140 = 220$

12. In the diagram below of circle *O*, the measure of inscribed angle *ABC* is 36° and the length of $\overline{OA}$ is 4 inches. Determine and state, to the *nearest tenth of a square inch*, the area of the shaded sector.



$A = \frac{\theta \pi r^2}{360}$

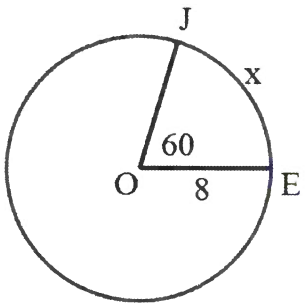
$A = \frac{72\pi(4)^2}{360}$

$A = 10.1$

Arc Length: $L=\frac{\theta \pi d}{360}$, where s = arc length, θ = central angle (in radians), r = radius

Find the arc length of the following arcs rounded to the nearest tenth

1.

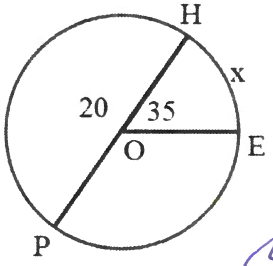


$L=\frac{\theta \pi d}{360}$

$L=\frac{60\pi(16)}{360}$

$L=8.4$

2.

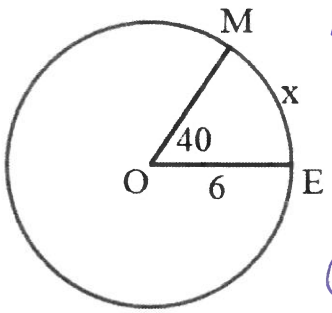


$L=\frac{35\pi(20)}{360}$

$L=6.1$

Find the arc length of the following arcs in terms of π

3.

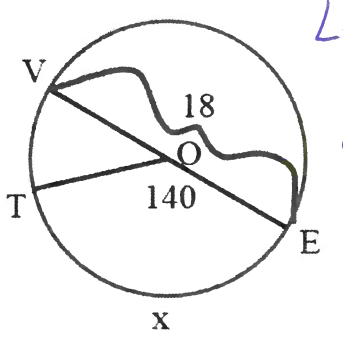


$L=\frac{\theta \pi d}{360}$

$L=\frac{40\pi(12)}{360}$

$L=\frac{4}{3}\pi$

4.

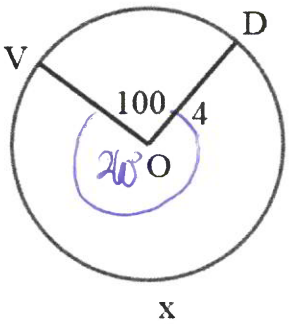


$L=\frac{140\pi(18}{360}$

$L=7\pi$

Find the arc length of the following arcs rounded to the nearest hundredth

5.

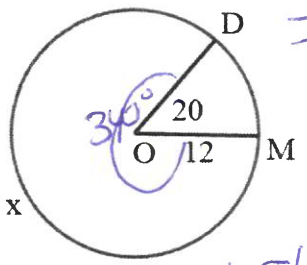


$L=\frac{260\pi(8)}{360}$

$L=18.15$

$360-100=260$

6.

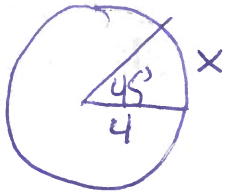


$360-20=340$

$L=\frac{340\pi(24)}{360}$

$L=71.21$

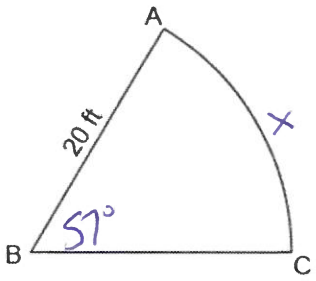
7. Find the arc length of a sector that has a radius of 4 inches and has a central angle of 45° to the nearest tenth of an inch.



$L = \frac{45\pi(8)}{360}$

$L = 3.1$

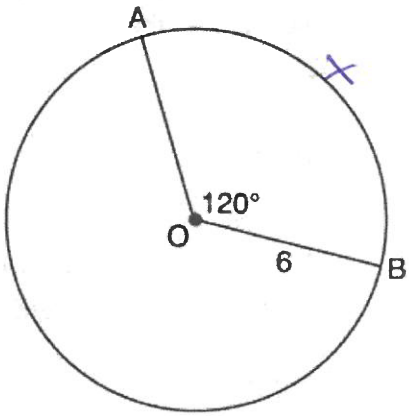
8. A sprinkler system is set up to water the sector shown in the accompanying diagram, with angle *ABC* measuring 57 degrees and radius *AB*=20 feet. What is the length of arc *AC*, to the nearest tenth of a foot?



$L = \frac{57\pi(40)}{360}$

$L = 19.9$

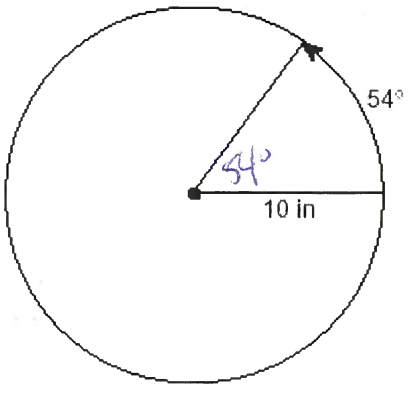
9. The diagram below shows circle *O* with radii $\overline{OA}$ and $\overline{OB}$. The measure of angle *AOB* is 120°, and the length of a radius is 6 inches.
Which expression represents the length of arc *AB*, in inches?



$L = \frac{120\pi(12)}{360}$

$L = 12.6$

10. A ball is rolling in a circular path that has a radius of 10 inches, as shown in the accompanying diagram. What distance has the ball rolled when the subtended arc is 54°? Express your answer to the *nearest hundredth of an inch.*



$L = \frac{54\pi(20)}{360}$

$L = 9.42$

Geometry with Algebra
Substitute into appropriate formula
Solve the equation
OR
USE EQUATION SOLVER (Math, Up)
E1: Left Hand Side, E2: Right Hand Side
Graph, 10, Graph

Volume	Triangle Area	Circle Sectors
Rectangular prism: $V = lwh$ Triangular prism: $V = \frac{1}{2}lwh$ Pyramid: $V = \frac{1}{3}lwh$ Cylinder: $V = \pi r^2 h$ Cone: $V = \frac{1}{3}\pi r^2 h$ Sphere: $V = \frac{4}{3}\pi r^3$	$A = \frac{1}{2}ab\sin C$	$A = \frac{\theta \pi r^2}{360}$ $L = \frac{\theta \pi d}{360}$

1. A right circular cylinder has a volume of 1,000 cubic inches and a height of 8 inches. What is the radius of the cylinder to the *nearest tenth of an inch*?

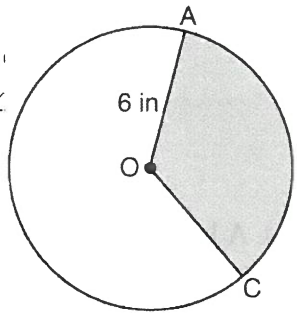
1) 6.3
2) 11.2
3) 19.8
4) 39.8

$V = \pi r^2 h$
$1000 = \pi r^2 (8)$

2. In the diagram below of circle O, the area of the shaded sector A is 12π in^2 and the length of $\overline{OA}$ is 6 inches. Determine and state m∠

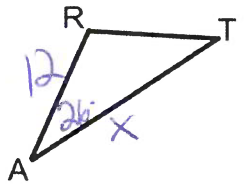
1) 60
2) 120
3) 45
4) 145

$A = \frac{\theta \pi r^2}{360}$ $12\pi = \frac{x \pi (6)^2}{360}$



3. The area of ΔART is 48 square inches. If $\overline{AR} = 12$ and $\angle TAR = 26$, find $\overline{AT}$ to the nearest tenth of an inch.

1) 17.4
2) 18.2
3) 21.7
4) 24.9



$A = \frac{1}{2}ab\sin C$
$48 = \frac{1}{2}(12)(x)\sin 26$

4. The arc length of a sector of a circle is 12.8 and the central angle is 40. Find the radius to the nearest tenth.

1) 36.7
2) 73.4
3) 46.1
4) 83.2

$L = \frac{\theta \pi d}{360}$ $\quad 12.8 = \frac{40 \pi d}{360}$

5. The volume of a cylinder is 12,566.4 cm^3. The height of the cylinder is 8 cm. Find the radius of the cylinder to the *nearest tenth of a centimeter*.

1) 12.3
2) 22.4
3) 7.9
4) 501.8

$V = \pi r^2 h$

$12566.4 = \pi r^2 (8)$

6. The area of a sector of a circle with a radius measuring 15 cm is 75π cm^2. What is the measure of the central angle that forms the sector?

1) 72°
2) 120°
3) 144°
4) 180°

$A = \frac{\theta \pi r^2}{360}$

$75\pi = \frac{x \pi (15)^2}{360}$

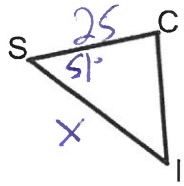
7. The Parkside Packing Company needs a rectangular shipping box. The box must have a length of 11 inches and a width of 8 inches. Find, to the *nearest tenth of an inch*, the height of the box such that the volume is 800 cubic inches.

1) 9.1
2) 14.7
3) 42.1
4) 7.9

$V = lwh$

$800 = 11(8)(h)$

8. The area of ΔSCI is 124 square centimeters. If $\overline{SC} = 25$ and $\angle CSI = 51$, find $\overline{SI}$ to the nearest tenth of a centimeter.



$A = \frac{1}{2}ab \sin C$

$124 = \frac{1}{2}(25)(x) \sin 51$

1) 22.5
2) 20.1
3) 13.6
4) 12.8

9. The volume of a sphere is approximately 44.6022 cubic centimeters. What is the radius of the sphere, to the *nearest tenth of a centimeter*?

1) 2.2
2) 3.3
3) 4.4
4) 4.7

$V = \frac{4}{3}\pi r^3$

$44.6022 = \frac{4}{3}\pi r^3$

10. What is the measure of a central angle whose arc length is 6 meters and whose radius measures 8 meters?

1) 43.0
2) 21.5
3) 47.2
4) 37.5

$L = \frac{\theta \pi d}{360}$

$6 = \frac{x\pi(16)}{360}$

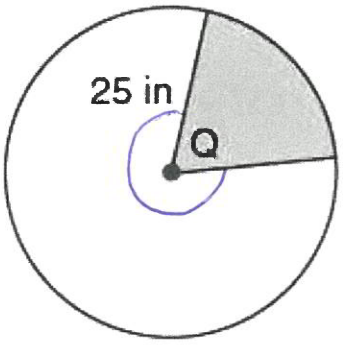
11. Find the radius of a sphere with a volume of 576π cubic inches. Find the answer to the nearest tenth of an inch.

1) 4.9
2) 15.1
3) 9.2
4) 7.6

$V = \frac{4}{3}\pi r^3$

$576\pi = \frac{4}{3}\pi r^3$

12. In the diagram below, the circle has a radius of 25 inches. The area of the *unshaded* sector is 500π in^2.
Determine and state the degree measure of angle Q, the central angle of the shaded sector.



$A = \frac{\theta \pi r^2}{360}$

$500\pi = \frac{x\pi(25)^2}{360}$

$288 = x$

$360 - 288 = 72^\circ$

Name Schlansky
Mr. Schlansky

Date _____
Geometry

Factoring Trinomials and Solving Quadratic Equations

1. $x^2 + 4x - 12$ m: 1,12 / 2,10 / 3,4 — $(x+6)(x-2)$

2. $y^2 + 3y + 2$ m: +2 — $(y+2)(y+1)$

3. $m^2 - 8m + 15$ m: 1,15 / 3,5 — $(m-5)(m-3)$

4. $x^2 - 8x - 20$ m: 1,20 / 2,10 / 4,5 — $(x-10)(x+2)$

5. $y^2 + 5y - 14$ m: 1,14 / 2,7 — $(y+7)(y-2)$

6. $x^2 + 1x - 12$ m: 1,12 / 2,6 / 3,4 — $(x+4)(x-3)$

7. $x^2 - 3x - 10$ m: 1,10 / 2,5 — $(x-5)(x+2)$

8. $x^2 - 7x + 12$ m: 1,12 / 2,6 / 3,4 — $(x-4)(x-3)$

9. $x^2 - 9x + 20$ m: 1,20 / 2,10 / 4,5 — $(x-5)(x-4)$

10. $x^2 - 9x - 36$ m: 1,36 / 2,18 / 3,12 / 4,9 / 6,6 — $(x-12)(x+3)$

Solve the following equations for x:

11. $x^2 - 5x = 6$

$-6 \quad -6$

$x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0$

$x-6=0 \quad x+1=0$

$+6 \quad +6 \quad -1 \quad -1$

$x=6 \quad x=-1$

1,6 2,3

12. $x^2 + 4x = 45$

$-45 \quad -45$

$x^2 + 4x - 45 = 0$

$(x+9)(x-5) = 0$

$x+9=0 \quad x-5=0$

$-9 \quad -9 \quad +5 \quad +5$

$x=-9 \quad x=5$

1,45 3,15 5,9

13. $x^2 = 3x + 18$

$-3x - 18 \quad -3x - 18$

$x^2 - 3x - 18 = 0$

$(x-6)(x+3) = 0$

$x-6=0 \quad x+3=0$

$+6 \quad +6 \quad -3 \quad -3$

$x=6 \quad x=-3$

1,18 2,9 3,6

14. $x^2 = 8x + 33$

$-8x - 33 \quad -8x - 33$

$x^2 - 8x - 33 = 0$

$(x-11)(x+3) = 0$

$x-11=0 \quad x+3=0$

$+11 \quad +11 \quad -3 \quad -3$

$x=11 \quad x=-3$

1,33 3,11

15. $x^2 - 7x = 3x - 16$

$-3x + 16 \quad -3x + 16$

$x^2 - 10x + 16 = 0$

$(x-8)(x-2) = 0$

$x-8=0 \quad x-2=0$

$+8 \quad +8 \quad +2 \quad +2$

$x=8 \quad x=2$

1,16 2,8 4,4

16. $x^2 + 5x = 8x + 10$

$-8x - 10 \quad -8x - 10$

$x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x-5=0 \quad x+2=0$

$+5 \quad +5 \quad -2 \quad -2$

$x=5 \quad x=-2$

1,10 2,5

17. $x(x-2) = 3(x+8)$

$x^2 - 2x = 3x + 24$

$-3x - 24 \quad -3x - 24$

$x^2 - 5x - 24 = 0$

$(x-8)(x+3) = 0$

$x-8=0 \quad x+3=0$

$+8 \quad +8 \quad -3 \quad -3$

$x=8 \quad x=-3$

1,24 2,12 3,8 4,6

18. $x(x+7) = 3(x+7)$

$x^2 + 7x = 3x + 21$

$-3x - 21 \quad -3x - 21$

$x^2 + 4x - 21 = 0$

$(x+7)(x-3) = 0$

$x+7=0 \quad x-3=0$

$-7 \quad -7 \quad +3 \quad +3$

$x=-7 \quad x=3$

1,21 3,7

19. $(x-2)(x+3) = 3x+2$

×	x	-2
$+3$	$+3x$	-6

$x^2 + x - 6$

$x^2 + x - 6 = 3x + 2$

$-3x - 2 \quad -3x - 2$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x-4=0 \quad x+2=0$

$+4 \quad +4 \quad -2 \quad -2$

$x=4 \quad x=-2$

1,8 2,4

20. $(x+3)(x+3) = 36$

×	x	$+3$
$+3$	$+3x$	$+9$

$x^2 + 6x + 9$

$x^2 + 6x + 9 = 36$

$-36 \quad -36$

$x^2 + 6x - 27 = 0$

$(x+9)(x-3) = 0$

$x+9=0 \quad x-3=0$

$-9 \quad -9 \quad +3 \quad +3$

$x=-9 \quad x=3$

1,27 3,9

Name Schlansky
Mr. Schlansky

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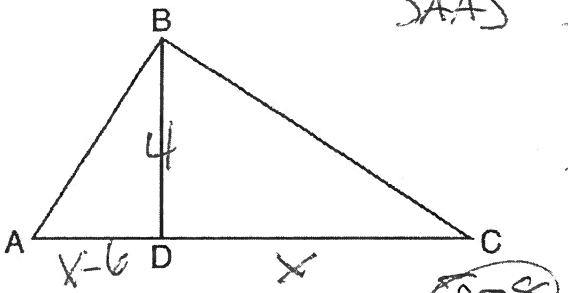
Geometry

Similar Triangles with Quadratics

1. In the diagram below of right triangle ABC, altitude $\overline{BD}$ is drawn to hypotenuse $\overline{AC}$. If $BD = 4$, $AD = x - 6$, and $CD = x$, what is the length of $\overline{CD}$?



SAAS $\frac{S}{A} = \frac{A}{S}$

$\frac{x-6}{4} = \frac{4}{x}$

$x(x-6) = 16$

$x^2 - 6x = 16$

$-16 \quad -16$

$x^2 - 6x - 16 = 0$

$(x-8)(x+2) = 0$

$x-8=0 \quad | \quad x+2=0$

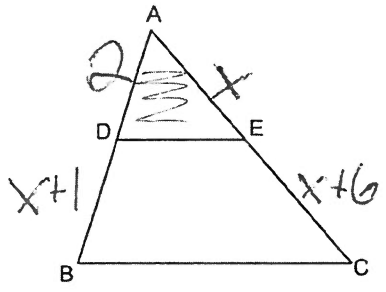
$+8 \quad +8 \quad -2 \quad -2$

$x = 8 \quad x = -2$ reject

1,16
2,8
4,4

$CD = 8$

2. In triangle ABC, $\overline{DE} \parallel \overline{BC}$. If $\overline{AD} = 2$, $\overline{DB} = x + 1$, $\overline{AE} = x$, and $\overline{EC} = x + 6$, find $\overline{AE}$



$\frac{\text{top}}{\text{top}} = \frac{\text{bottom}}{\text{bottom}}$

$\frac{2}{x} = \frac{x+1}{x+6}$

$x(x+1) = 2(x+6)$

$x^2 + 1x = 2x + 12$

$-2x - 12 \quad -2x - 12$

$x^2 - 1x - 12 = 0$

$(x-4)(x+3) = 0$

$x-4=0 \quad | \quad x+3=0$

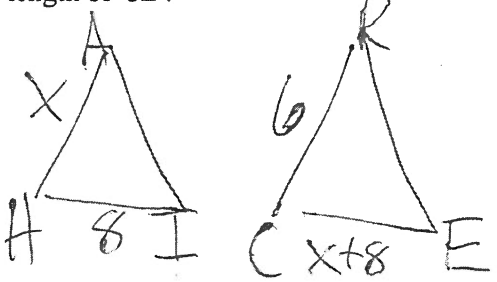
$+4 \quad +4 \quad -3 \quad -3$

$x = 4 \quad x = -3$ reject

1,12
2,6
3,4

$AE = 4$

3. $\Delta HAI \sim \Delta CRE$. If $\overline{HA} = x$, $\overline{CR} = 6$, $\overline{HI} = 8$, and $\overline{CE} = x + 8$, determine and state the length of $\overline{CE}$.



$\frac{x}{6} = \frac{8}{x+8}$

$x(x+8) = 48$

$x^2 + 8x = 48$

$-48 \quad -48$

$x^2 + 8x - 48 = 0$

$(x+12)(x-4) = 0$

$x+12=0 \quad | \quad x-4=0$

$-12 \quad -12 \quad +4 \quad +4$

$x = -12$ reject $\quad x = 4$

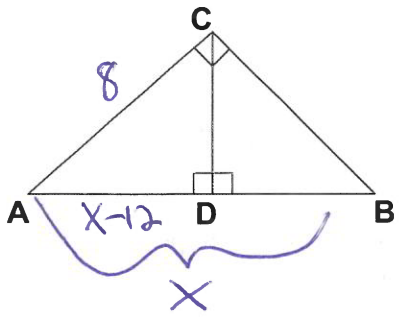
1,48
2,24
3,16
4,12
6,8

$\overline{CE} = x + 8$

$\overline{CE} = 4 + 8$

$\overline{CE} = 12$

4. Altitude $\overline{CD}$ is drawn to right triangle ABC. If $\overline{AC} = 8$, $\overline{AB} = x$, and $\overline{AD} = x - 12$. Find the measure of $\overline{AD}$.



$\frac{H}{L} = \frac{L}{S}$

$\frac{x}{8} = \frac{8}{x-12}$

Math up, Graph, 10, Graph $x = 16$

$x^2 - 12x = 64$

$-64 \quad -64$

$x^2 - 12x - 64 = 0$

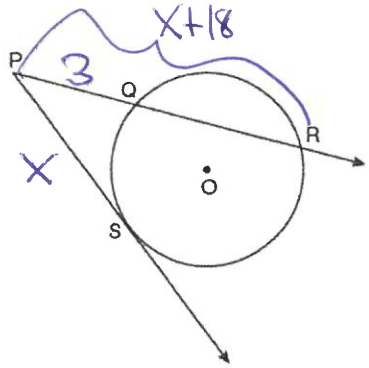
$(x-16)(x+4) = 0$

$x = 16 \quad x = -4$

$AD = 16 - 12$

$AD = 4$

5. In the diagram below, $\overline{PS}$ is a tangent to circle O at point S, $\overline{PR}$ is a secant, $PS = x$, $PQ = 3$, and $PR = x + 18$. What is the length of $\overline{PS}$?



(Not drawn to scale)

whole · exterior = whole · exterior

$(x+18)(3) = (x)(x)$ Math, up Graph, 10, Graph

$3x + 54 = x^2$

$-3x - 54 \quad -3x - 54$

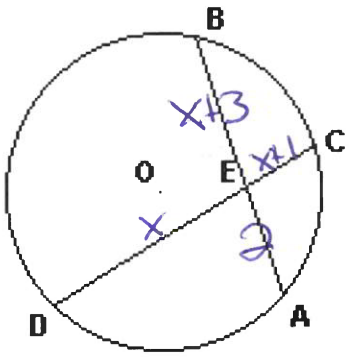
$0 = x^2 - 3x - 54$

$0 = (x-9)(x+6)$

$x = 9 \quad x = -6$

$PS = 9$

6. In the diagram of circle O, chords $\overline{BA}$ and $\overline{CD}$ intersect at E. $AE = 2$, $EB = x + 3$ $CE = x + 1$, and $ED = x$. Find ED.



Part · part = part · part

$x(x+1) = 2(x+3)$ Math, up Graph, 10, Graph

$x^2 + 1x = 2x + 6$

$-2x - 6 - 2x - 6$

$x^2 - 1x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3 \quad x = -2$

$ED = 3$

Name Schlansky
Mr. Schlansky

Date _____
Geometry

Reducing Radicals

Reducing Radicals

-Separate into two radicals (perfect squares and non perfect squares). Find the largest perfect square that divides in

-Take the square root of the perfect square. Bring the non-perfect square down

1. $\sqrt{45}$
$\sqrt{9}\sqrt{5}$
$3\sqrt{5}$

2. $\sqrt{50}$
$\sqrt{25}\sqrt{2}$
$5\sqrt{2}$

3. $\sqrt{162}$
$\sqrt{81}\sqrt{2}$
$9\sqrt{2}$

4. $\sqrt{32}$
$\sqrt{16}\sqrt{2}$
$4\sqrt{2}$

5. $\sqrt{48}$
$\sqrt{16}\sqrt{3}$
$4\sqrt{3}$

6. $\sqrt{75}$
$\sqrt{25}\sqrt{3}$
$5\sqrt{3}$

7. $\sqrt{48}$
$\sqrt{16}\sqrt{3}$
$4\sqrt{3}$

8. $\sqrt{200}$
$\sqrt{100}\sqrt{2}$
$10\sqrt{2}$

9. $\sqrt{98}$
$\sqrt{49}\sqrt{2}$
$7\sqrt{2}$

10. $\sqrt{125}$
$\sqrt{25}\sqrt{5}$
$5\sqrt{5}$

11. $\sqrt{147}$
$\sqrt{49}\sqrt{3}$
$7\sqrt{3}$

12. $\sqrt{192}$
$\sqrt{64}\sqrt{3}$
$8\sqrt{3}$

B5
1
4
9
16
25
36
49
64
81
100

Perfect Squares

1

4

9

16

25

36

49

64

81

100

Geometry with Reducing Radicals

Apply the appropriate rule

Algebraically	Calculator Trick (MC Only)
1) Separate into two radicals and find the largest perfect square that goes into it (Y = #/X, 2nd Table) 2) Take the square root of the perfect square. Keep the non perfect square in the radical.	-Type the radical into the calculator -Type each answer into the calculator -See what matches up

1. Triangle ABC shown below is a right triangle with altitude $\overline{AD}$ drawn to the hypotenuse $\overline{BC}$.

If $BD = 2$ and $DC = 10$, what is the length of $\overline{AB}$?

1) $2\sqrt{2}$
2) $2\sqrt{5}$
3) $2\sqrt{6}$ 4.89..
4) $2\sqrt{30}$

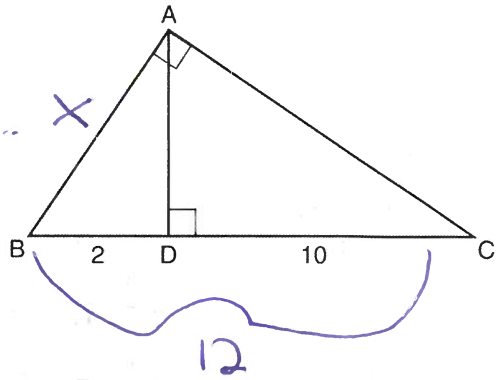
$\frac{H}{L} = \frac{L}{S}$

$\frac{12}{x} = \frac{x}{2}$

$\sqrt{x^2} = \sqrt{24}$ (or) 4.89..

$\sqrt{4}\sqrt{6}$

$2\sqrt{6}$



2. In the diagram below of circle O, chord $\overline{AB}$ bisects chord $\overline{CD}$ at E. If $AE = 8$ and $BE = 9$, find the length of $\overline{CE}$ in simplest radical form.

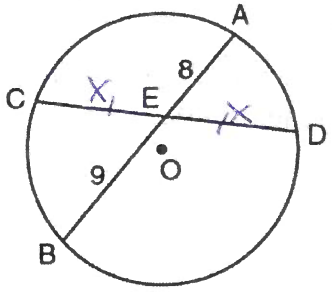
Part·part = part·part

$x(x) = 9(8)$

$\sqrt{x^2} = \sqrt{72}$

$\sqrt{36}\sqrt{2}$

$6\sqrt{2}$



3. In the diagram below of right triangle ABC, altitude $\overline{BD}$ is drawn to hypotenuse $\overline{AC}$, $AC = 16$, and $CD = 7$.

What is the length of $\overline{BD}$?

1) $3\sqrt{7}$ 7.93..
2) $4\sqrt{7}$
3) $7\sqrt{3}$
4) 12

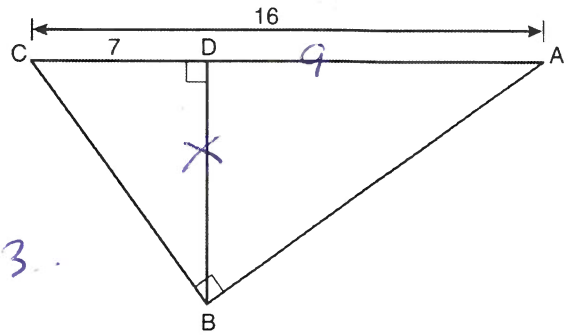
$\frac{S}{A} = \frac{A}{S}$

$\frac{7}{x} = \frac{x}{9}$

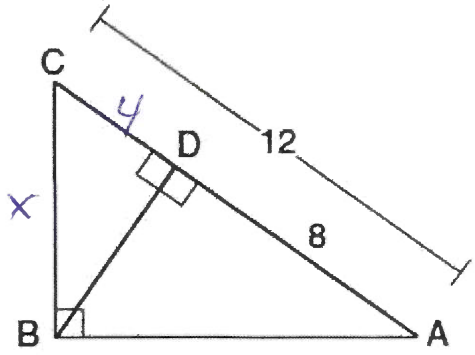
$\sqrt{x^2} = \sqrt{63}$ (or) 7.93..

$x = \sqrt{9}\sqrt{7}$

$3\sqrt{7}$



4. In the diagram below of $\triangle ABC$, $\angle ABC$ is a right angle, $AC = 12$, $AD = 8$, and altitude $\overline{BD}$ is drawn.



What is the length of $\overline{BC}$?

$\frac{\#}{L} = \frac{L}{S}$

$\frac{12}{x} = \frac{x}{4}$

$\sqrt{x^2} = \sqrt{48}$

$\sqrt{16}\sqrt{3}$

$x = 4\sqrt{3}$

5. A cone has a base with a diameter of 4 and a slant height of 4. Its height can be represented by

1) $2\sqrt{3}$ 3.46
2) $4\sqrt{3}$
3) $6\sqrt{2}$
4) $4\sqrt{2}$

$a^2 + b^2 = c^2$

$2^2 + b^2 = 4^2$

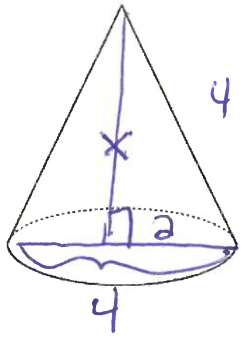
$4 + b^2 = 16$

$-4 \quad -4$

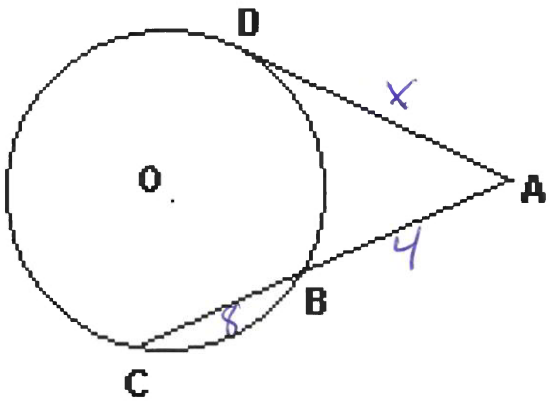
$\sqrt{b^2} = \sqrt{12}$ or 3.46

$\sqrt{4}\sqrt{3}$

$b = 2\sqrt{3}$



6. In the diagram, $\overline{AD}$ is tangent to circle O at D, and $\overline{CBA}$ is a secant. If $\overline{AB} = 4$ and $\overline{BC} = 8$, find AD in simplest radical form.



whole · exterior = whole · exterior

$x \cdot x = 12 \cdot 4$

$\sqrt{x^2} = \sqrt{48}$

$\sqrt{16}\sqrt{3}$

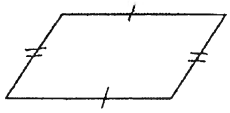
$x = 4\sqrt{3}$

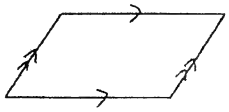
Name Schlansky
Mr. Schlansky

Date _____
Geometry

Quadrilateral Properties

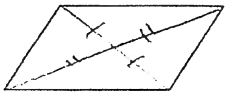
1. A quadrilateral whose diagonals bisect each other and are perpendicular is a rhombus
1) rhombus
2) rectangle
3) trapezoid
4) parallelogram

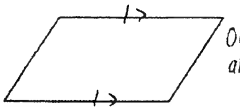

Two pairs of opposite sides are congruent


Two pairs of opposite sides are parallel

Two pairs of opposite angles are congruent

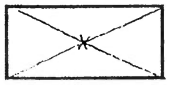
2. If the diagonals of a quadrilateral do *not* bisect each other, then the quadrilateral could be a not a P-gram
1) rectangle
2) rhombus
3) square
4) trapezoid

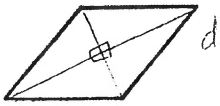

Diagonals bisect each other


One pair of opposite sides are congruent and parallel

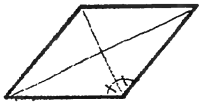
3. A quadrilateral whose diagonals are always congruent and perpendicular to each other must be a rectangle rhombus
1) rectangle
2) rhombus
3) square
4) trapezoid

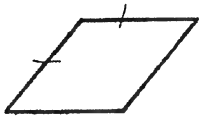
A right angle (consecutive sides perpendicular)


Congruent diagonals


diagonals are perpendicular to each other

4. Which quadrilateral has diagonals that always bisect its angles and also bisect each other? (rhombus)
1) rhombus
2) rectangle
3) parallelogram
4) isosceles trapezoid


diagonals bisect the angles


Consecutive sides are congruent

5. Which quadrilateral has diagonals that always are congruent and also bisect each other? rectangle
1) isosceles trapezoid
2) rectangle
3) rhombus
4) parallelogram

6. The diagonals of a quadrilateral are congruent but do not bisect each other. This quadrilateral is

not p-gram

1) an isosceles trapezoid
2) a parallelogram
3) a rectangle
4) a rhombus

7. Given three distinct quadrilaterals, a square, a rectangle, and a rhombus, which quadrilaterals must have perpendicular diagonals? rhombus

1) the rhombus, only
2) the rectangle and the square
3) the rhombus and the square
4) the rectangle, the rhombus, and the square

8. A parallelogram must be a rhombus when its one of the 3 ways to prove rhombus

1) Diagonals are congruent.
2) Opposite sides are parallel.
3) Diagonals are perpendicular.
4) Opposite angles are congruent.

9. A parallelogram must be a rectangle when its one of the 2 ways to prove rectangle

1) diagonals are perpendicular
2) diagonals are congruent
3) opposite sides are parallel
4) opposite sides are congruent

has rectangle, needs rhombus

10. A rectangle must be a square when its one of the 3 ways to prove rhombus

1) Angles are right angles
2) diagonals are congruent
3) diagonals are perpendicular to each other
4) opposite sides are parallel

has rhombus, needs rectangle

11. A rhombus must be a square when its one of the 2 ways to prove rectangle

1) consecutive sides are congruent
2) diagonals are congruent
3) opposite angles are congruent
4) diagonals are perpendicular to each other

ways to prove a

12. A parallelogram must be a rectangle when its one of the 2 rectangle
1) consecutive sides are congruent
2) opposite angles are congruent
3) angles are right angles
4) opposite sides are parallel

13. Which of the following properties does not make a parallelogram a rhombus?
1) diagonals bisect the angles
2) diagonals are perpendicular to each other not one of the 3 ways to prove rhombus
3) opposite angles are congruent
4) consecutive sides are congruent

needs rectangle

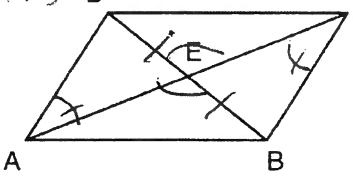
14. Which of the following properties does not make a rhombus a square?
1) Diagonals are congruent
2) Diagonals are perpendicular to each other ✗ not one of the two rectangle properties
3) Angles are right angles
4) Consecutive angles are congruent

in order for consecutive angles to be congruent, they must be right angles

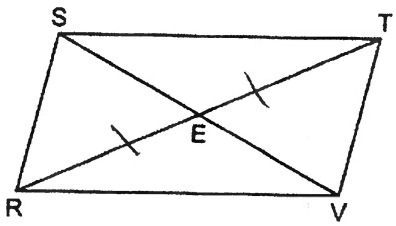
15. In the diagram below, parallelogram $ABCD$ has diagonals $\overline{AC}$ and $\overline{BD}$ that intersect at point E.

not one of the 5 p-gram properties or alternate interior angles

Which expression is *not* always true?
1) $\angle DAE \cong \angle BCE$ alt int ∠s
2) $\angle DEC \cong \angle BEA$ vertical angles
3) $\overline{AC} \cong \overline{DB}$ congruent diagonals ✗
4) $\overline{DE} \cong \overline{EB}$ diagonals bisect each other



16. In the diagram below of parallelogram $RSTV$, diagonals $\overline{SV}$ and $\overline{RT}$ intersect at E.



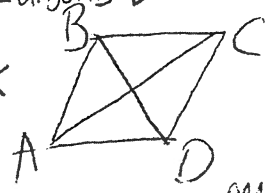
one of the 5 p-gram properties

Which statement is always true?
1) $\overline{SR} \cong \overline{RV}$ consecutive sides ≅ ✗
2) $\overline{RT} \cong \overline{SV}$ congruent diagonals ✗
3) $\overline{SE} \cong \overline{RE}$ diagonals congruent/bisect each other ✗
4) $\overline{RE} \cong \overline{TE}$ diagonals bisect each other ✓

one of the three ways to prove a rhombus

17. If $ABCD$ is a parallelogram, which statement would prove that $ABCD$ is a rhombus?

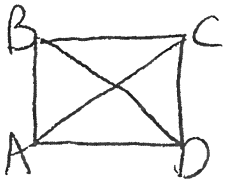
1) $\angle ABC \cong \angle CDA$ opp $\angle$s $\cong$ X
2) $\overline{AC} \cong \overline{BD}$ diagonals $\cong$ X
3) $\overline{AC} \perp \overline{BD}$ ⊥ diagonals ✓
4) $\overline{AB} \perp \overline{CD}$ opp sides ⊥ X



one of the 2 ways to prove rectangle

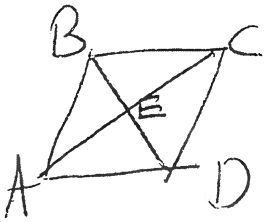
18. If $ABCD$ is a parallelogram, which statement would prove that $ABCD$ is a rectangle?

1) $\angle ABC \cong \angle CDA$
2) $\overline{AC} \cong \overline{BD}$ congruent diagonals
3) $\overline{AC} \perp \overline{BD}$
4) $\overline{AB} \perp \overline{CD}$



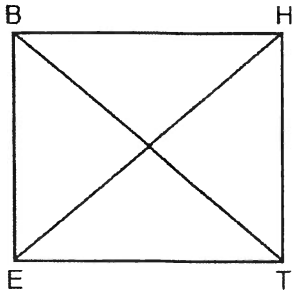
19. In rectangle $ABCD$, diagonals $\overline{AC}$ and $\overline{BD}$ intersect at E. Which statement does *not* prove rectangle $ABCD$ is a square? needs one of the 3 rhombus ~~three~~ ways to prove.

1) $\overline{AC} \cong \overline{DB}$ ≅ diagonals X
2) $\overline{AB} \cong \overline{BC}$ ≅ consecutive sides ✓
3) $\overline{AC} \perp \overline{DB}$ ⊥ diagonals ✓
4) $\overline{AC}$ bisects $\angle DCB$ diagonals bisect the angles ✓



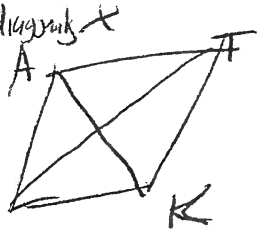
20. Parallelogram $BETH$, with diagonals $\overline{BT}$ and $\overline{HE}$, is drawn below. What additional information is sufficient to prove that $BETH$ is a rectangle? need one of the 2 ways to prove rectangle

1) $\overline{BT} \perp \overline{HE}$ ⊥ diagonals X
2) $\overline{BE} \parallel \overline{HT}$ opp sides ∥ X
3) $\overline{BT} \cong \overline{HE}$ ≅ diagonals ✓
4) $\overline{BE} \cong \overline{ET}$ ≅ consecutive sides X



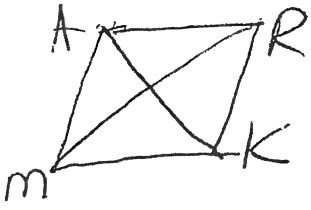
21. Parallelogram $EATK$ has diagonals $\overline{ET}$ and $\overline{AK}$. Which information is always sufficient to prove $EATK$ is a rhombus? need one of the 3 ways to prove rhombus

1) $\overline{EA} \perp \overline{AT}$ consecutive sides ⊥ X
2) $\overline{EA} \cong \overline{AT}$ consecutive sides ≅ ✓
3) $\overline{ET} \cong \overline{AK}$ ≅ diagonals X
4) $\overline{ET} \cong \overline{AT}$ nothing X



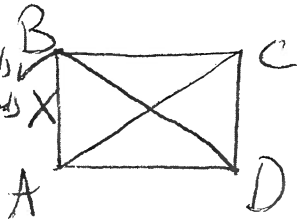
22. Which congruence statement is sufficient to prove parallelogram $MARK$ is a rhombus?

1) $\overline{MA} \cong \overline{MK}$ ≅ consecutive sides ✓
2) $\overline{MA} \cong \overline{KR}$ ≅ opposite sides X
3) $\angle K \cong \angle A$ ≅ opposite $\angle$s X need one of the 3 ways to prove rhombus
4) $\angle R \cong \angle A$ ≅ consecutive $\angle$s X



23. If $ABCD$ is a parallelogram, which additional information is sufficient to prove that $ABCD$ is a rectangle? one of the two ways to prove a rectangle

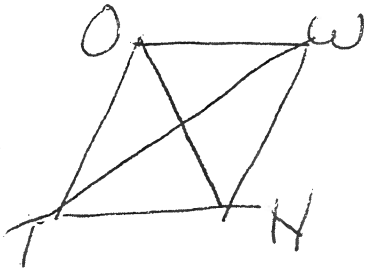
1) $\overline{AB} \cong \overline{BC}$ ≅ consecutive sides ✗
2) $\overline{AB} \parallel \overline{CD}$ opp sides ∥ ✗
3) $\overline{AC} \cong \overline{BD}$ ≅ diagonals ✓
4) $\overline{AC} \perp \overline{BD}$ ⊥ diagonals ✗



proves Rectangle

24. In quadrilateral $TOWN$, $\overline{OW} \cong \overline{TN}$ and $\overline{OT} \cong \overline{WN}$. Which additional piece of information is sufficient to prove quadrilateral $TOWN$ is a rhombus? one of the three ways to prove rhombus

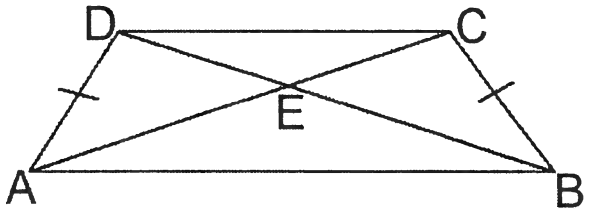
1) $\overline{ON} \perp \overline{TW}$ ⊥ diagonals ✓
2) $\overline{TO} \perp \overline{OW}$ ⊥ consecutive sides ✗
3) $\overline{OW} \parallel \overline{TN}$ ∥ opposite sides ✗
4) $\overline{ON}$ and $\overline{TW}$ bisect each other ✗
diagonals bisect each other



26. In the diagram below, isosceles trapezoid ABCD has diagonals $\overline{AC}$ and $\overline{BD}$ that intersect at point E.

Which expression is *not* always true?

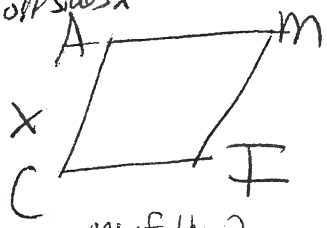
1) $\overline{AC} \cong \overline{DB}$ ≅ diagonals ✓
2) $\overline{DC} \parallel \overline{AB}$ opp sides ∥ ✓
3) $\overline{DE} \cong \overline{AE}$ diagonals ≅/bisect ✗
4) $\overline{AD} \cong \overline{CB}$ ≅ legs ✓



has rectangle, needs one of 3 ways to prove rhombus

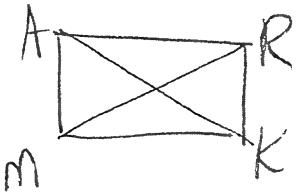
27. Which statement would prove rectangle $CAMI$ is a square?

1) $\overline{CA} \cong \overline{AM}$ ≅ consecutive sides ✓
2) $\overline{CM} \cong \overline{AI}$ ≅ diagonals ✗
3) $\overline{CA} \cong \overline{MI}$ ≅ opp sides ✗
4) $\overline{MA} \perp \overline{AC}$ ⊥ consecutive sides ✗



28. Which statement would prove parallelogram $MARK$ is a rectangle? one of the 2 ways to prove rectangle

1) $\overline{MA} \cong \overline{MK}$ ≅ consecutive sides
2) ~~$\overline{MR} \cong \overline{AK}$ ≅ diagonals~~
2) $\overline{MA} \cong \overline{RK}$ ≅ opposite sides
3) $\overline{MR} \perp \overline{AK}$ ⊥ diagonals
4) $\overline{MA} \perp \overline{AK}$ ⊥ consecutive sides ✓



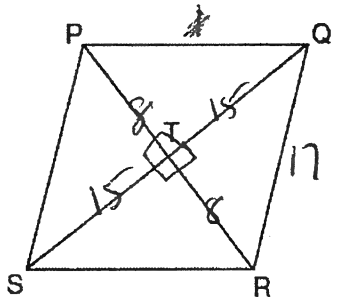
- cut the diagonals in half (diagonals bisect each other)
- use Pythagorean Theorem to find the rhombus side (diagonals ⊥)
- multiply by 4 for the perimeter

Name Schlansky
Mr. Schlansky

Date _____
Geometry

Perimeter of a Rhombus

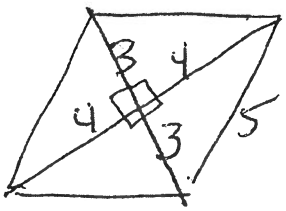
1. In the diagram of rhombus $PQRS$ below, the diagonals $\overline{PR}$ and $\overline{QS}$ intersect at point T, $PR = 16$, and $QS = 30$. Determine and state the perimeter of $PQRS$.



$a^2+b^2=c^2$
$8^2+15^2=c^2$
$64+225=c^2$
$\sqrt{289}=\sqrt{c^2}$
$17=c$

$17(4)=68$

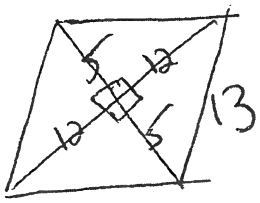
2. A rhombus has diagonals that measure 6 and 8. Find the perimeter of the rhombus.



$a^2+b^2=c^2$
$3^2+4^2=c^2$
$9+16=c^2$
$\sqrt{25}=\sqrt{c^2}$
$5=c$

$5(4)=20$

3. A rhombus has diagonals that measure 10 and 24. Find the perimeter of the rhombus.

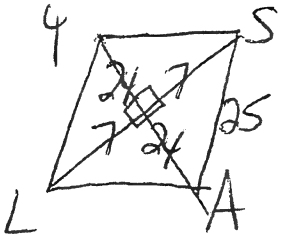


$a^2+b^2=c^2$
$5^2+12^2=c^2$
$25+144=c^2$
$\sqrt{169}=\sqrt{c^2}$
$13=c$

$13(4)=52$

Parallelogram with ≅ consecutive sides must be a rhombus

4. In parallelogram $LYSA$, $\overline{LY} \cong \overline{YS}$. If $\overline{LS} = 14$ and $\overline{YA} = 48$, find the perimeter of $LYSA$.



$a^2+b^2=c^2$

$7^2+24^2=c^2$

$49+576=c^2$

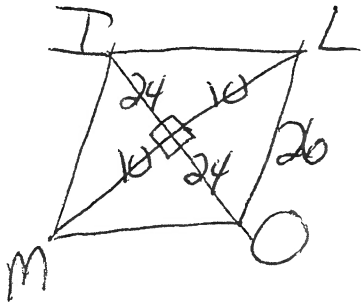
$\sqrt{625}=\sqrt{c^2}$

$25=c$

$25(4)=100$

Parallelogram with diagonals bisect its angles must be a rhombus

5. In parallelogram $MILO$, $\overline{ML}$ bisects $\angle IMO$. If $\overline{ML} = 20$ and $\overline{IO} = 48$, find the perimeter of $MILO$.



$a^2+b^2=c^2$

$10^2+24^2=c^2$

$100+576=c^2$

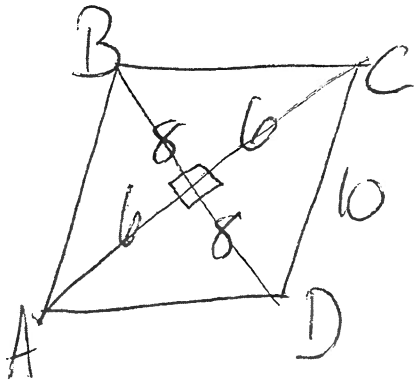
$\sqrt{676}=\sqrt{c^2}$

$26=c$

$26(4)=104$

Parallelogram with perpendicular diagonals must be a rhombus

6. In parallelogram $ABCD$ with $\overline{AC} \perp \overline{BD}$, $AC = 12$ and $BD = 16$. What is the perimeter of $ABCD$?



$a^2+b^2=c^2$

$6^2+8^2=c^2$

$36+64=c^2$

$\sqrt{100}=\sqrt{c^2}$

$10=c$

$10(4)=40$

Triangles/Parallel Lines Cut By a Transversal/Angles of Parallelograms

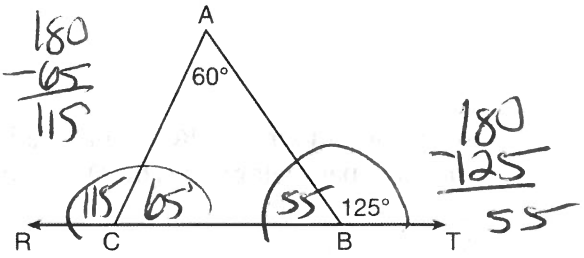
1) The three angles of a triangle add to equal 180°. **Look for triangles.**
   *The four angles of a quadrilateral add to 360°.
2) Linear pairs add to 180°. **Look for linear pairs.**
3) Vertical angles are congruent. Look for an X (intersecting lines).
4) **Given congruent sides:** Isosceles triangle has congruent angles opposite congruent sides.
5) **Given equilateral triangle:** Equilateral triangle has angles 60, 60, 60.
6) **Given angle bisector:** An angle bisector cuts an angle into two congruent halves.
7) **Given parallel**: Extend parallel lines and transversal. Follow the transversal and fill in all 8 angles. If angles are the same (both acute or both obtuse), the angles are congruent. If the angles are different (one acute and one obtuse), the angles are supplementary (add to 180).
8) **Given parallelogram**: Opposite angles are congruent and consecutive angles are supplementary (add to 180)

1. In the diagram below, $\overleftrightarrow{RCBT}$ and $\triangle ABC$ are shown with $m\angle A = 60$ and $m\angle ABT = 125$.

What is $m\angle ACR$?

1) 125
2) 115
3) 65
4) 55

△ABC
x+60+55=180
115+x=180
-115 -115
x=65

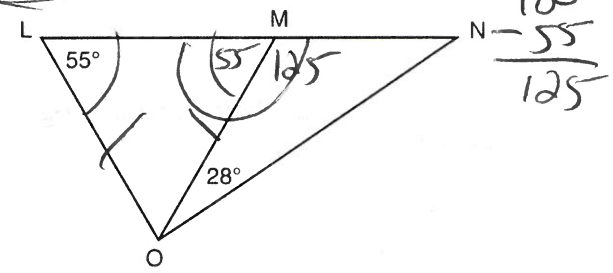


2. In the diagram below, $\triangle LMO$ is isosceles with $LO = MO$.

If $m\angle L = 55$ and $m\angle NOM = 28$, what is $m\angle N$?

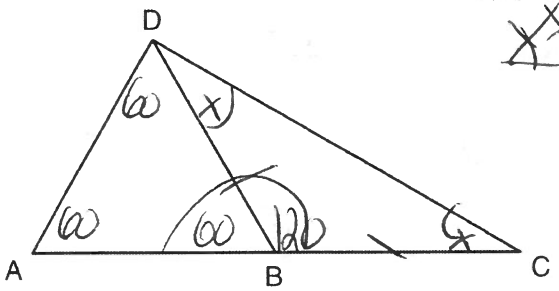
1) 27
2) 28
3) 42
4) 70

△OMN
28+125+x=180
153+x=180
-153 -153
x=27



3. In the diagram below of $\triangle ACD$, B is a point on $\overline{AC}$ such that $\triangle ADB$ is an equilateral triangle, and $\triangle DBC$ is an isosceles triangle with $\overline{DB} \cong \overline{BC}$. Find $m\angle C$.

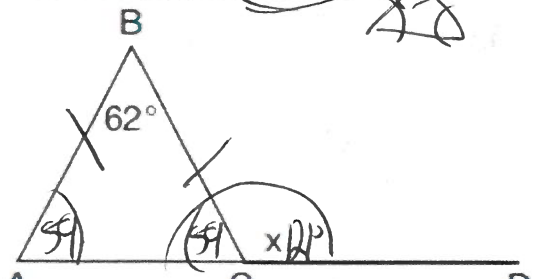
60 60 60



180
-60
120

△DBC
x+x+120=180
2x+120=180
-120 -120
2x=60
2 2
x=30

4. Given $\triangle ABC$ with $m\angle B = 62°$ and side $\overline{AC}$ extended to D, as shown below. Which value of x makes $\overline{AB} \cong \overline{CB}$?



180
−59
121

$\triangle ABC$

$x + x + 62 = 180$

$2x + 62 = 180$

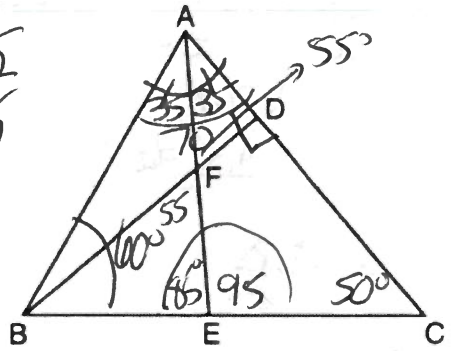
$-62 \quad -62$

$\frac{2x}{2} = \frac{118}{2}$

$x = 59$

$x = 121°$

5. In the diagram of $\triangle ABC$ below, $\overline{AE}$ bisects angle BAC, and altitude $\overline{BD}$ is drawn. If $m\angle C = 50°$ and $m\angle ABC = 60°$, what is $m\angle FEB$?



180
−95
85

$\triangle BAC$

$60 + 50 + x = 180$

$110 + x = 180$

$-110 \quad -110$

$x = 70$

$\triangle ADF$

$35 + 90 + x = 180$

$125 + x = 180$

$-125 \quad -125$

$x = 55$

$\triangle ACE$

$35 + 50 + x = 180$

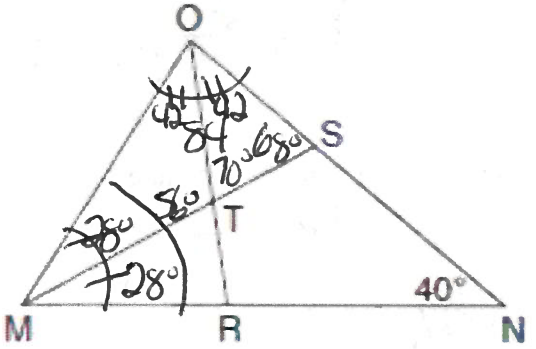
$85 + x = 180$

$-85 \quad -85$

$x = 95$

$\angle FEB = 85°$

6. In the diagram below of triangle MNO, $\angle M$ and $\angle O$ are bisected by $\overline{MS}$ and $\overline{OR}$, respectively. Segments MS and OR intersect at T, and $m\angle N = 40°$. If $m\angle TMR = 28°$, what is the measure of angle OTS?



$\triangle OMN$

$56 + 40 + x = 180$

$96 + x = 180$

$-96 \quad -96$

$x = 84$

$\triangle MOS$

$28 + 84 + x = 180$

$112 + x = 180$

$-112 \quad -112$

$x = 68$

$\triangle OTS$

$42 + 68 + x = 180$

$110 + x = 180$

$-110 \quad -110$

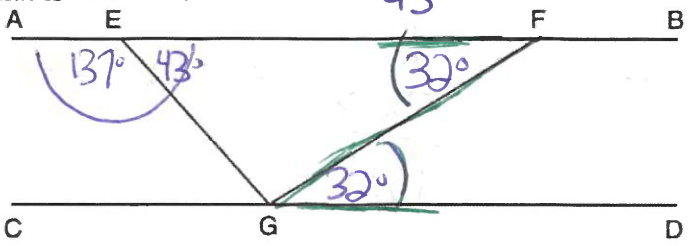
$x = 70$

$\angle OTS = 70°$

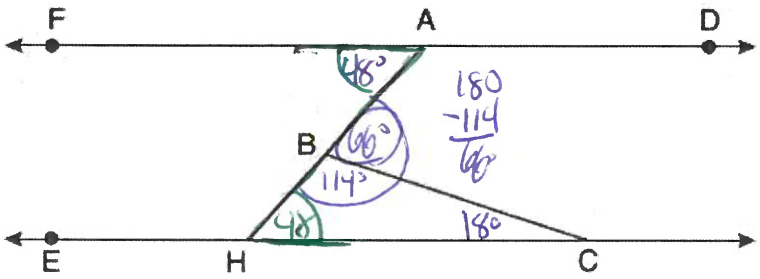
175
~~183~~

7. In the diagram below, $\overline{AEFB} \parallel \overline{CGD}$, and $\overline{GE}$ and $\overline{GF}$ are drawn. If $m\angle EFG = 32°$ and $m\angle AEG = 137°$, what is $m\angle EGF$?

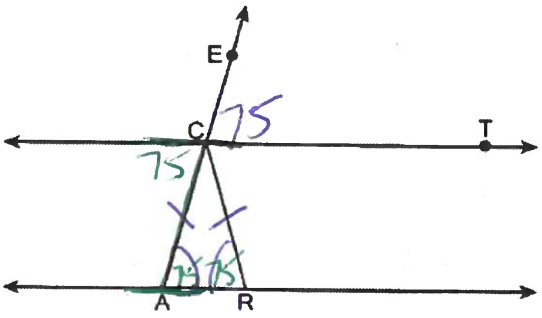
1) 11°
2) 43°
3) 75°
4) 105°



8. In the diagram below, $\overline{FAD} \parallel \overline{EHC}$, and $\overline{ABH}$ and $\overline{BC}$ are drawn. If $m\angle FAB = 48°$ and $m\angle ECB = 18°$, what is $m\angle ABC$?

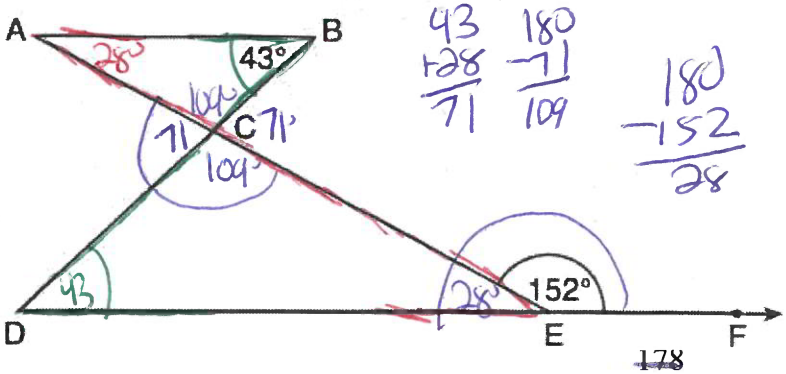


9. In the diagram below, $\overleftrightarrow{CT} \parallel \overleftrightarrow{AR}$, and $\overline{ACE}$ and $\overline{RC}$ are drawn such that $\overline{AC} \cong \overline{RC}$. If $m\angle ECT = 75°$, what is $m\angle ACR$?



10. In the diagram below, $\overline{AB} \parallel \overrightarrow{DEF}$, $\overline{AE}$ and $\overline{BD}$ intersect at C, $m\angle B = 43°$, and $m\angle CEF = 152°$.
Which statement is true?

1) $m\angle D = 28°$
2) $m\angle A = 43°$
3) $m\angle ACD = 71°$
4) $m\angle BCE = 109°$

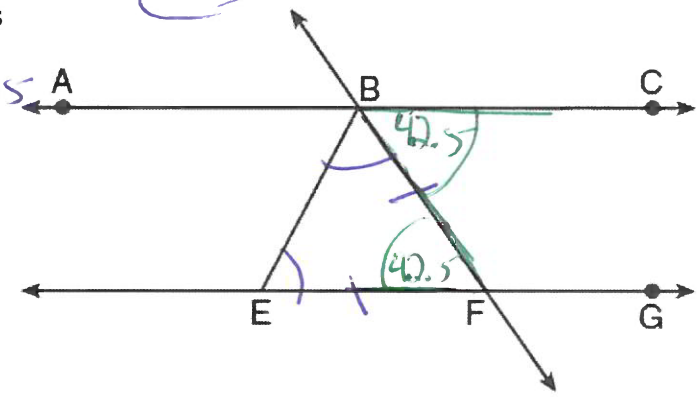


11. As shown in the diagram below, $\overleftrightarrow{ABC} \parallel \overleftrightarrow{EFG}$ and $\overline{BF} \cong \overline{EF}$.
If $m\angle CBF = 42.5°$, then $m\angle EBF$ is

1) 42.5°
2) 68.75°
3) 95°
4) 137.5°

180 − 42.5 = 137.5

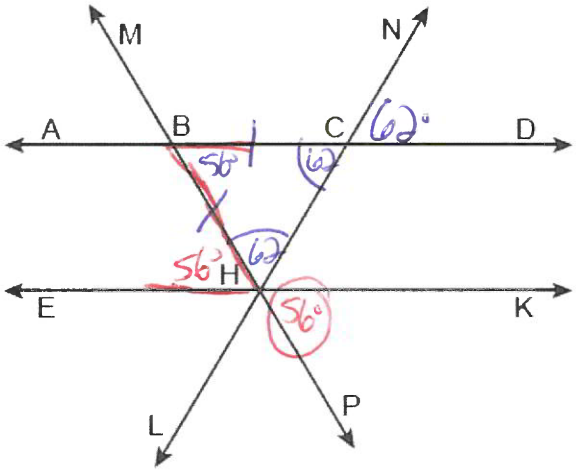
137.5 / 2 = 68.75



12. In the diagram below, $\overleftrightarrow{ABCD} \parallel \overleftrightarrow{EHK}$, and $\overleftrightarrow{MBHP}$ and $\overleftrightarrow{NCHL}$ are drawn such that $\overline{BC} \cong \overline{BH}$. If $m\angle NCD = 62°$, what is $m\angle PHK$?.

62 + 62 = 124

180 − 124 = 56



DE ∥ BC

13. In the diagram below, $\overline{DE}$ divides $\overline{AB}$ and $\overline{AC}$ proportionally, $m\angle C = 26°$, $m\angle A = 82°$, and $\overline{DF}$ bisects $\angle BDE$.
The measure of angle DFB is

1) 36°
2) 54°
3) 72°
4) 82°

△ADE

82 + 26 = 108

180 − 108 = 72

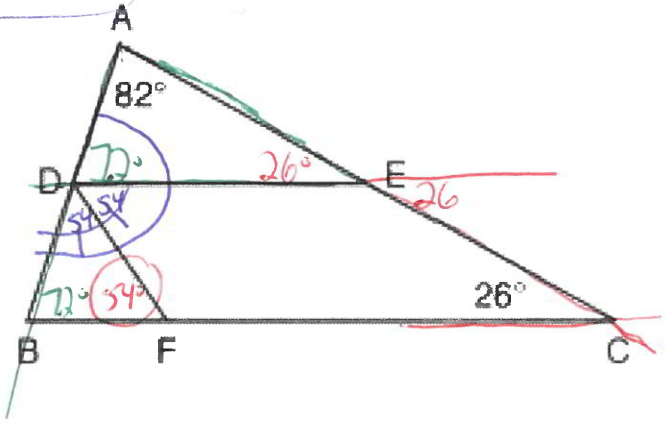
△DBF

72 + 54 = 126

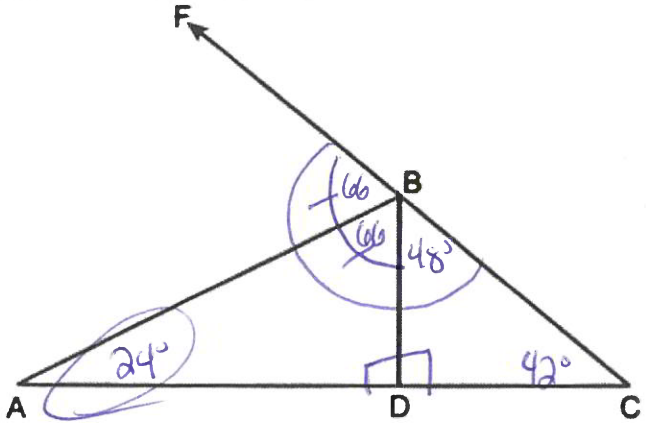
180 − 126 = 54

180 − 72 = 108

108 / 2 = 54



14. In the diagram below of $\triangle ABC$, $\overrightarrow{CBF}$ is drawn, $\overline{AB}$ bisects $\angle FBD$, and $\overline{BD} \perp \overline{AC}$. If $m\angle C = 42°$ what is $m\angle A$?



$\triangle BDC$: $90 + 42 = 132$; $180 - 132 = 48$

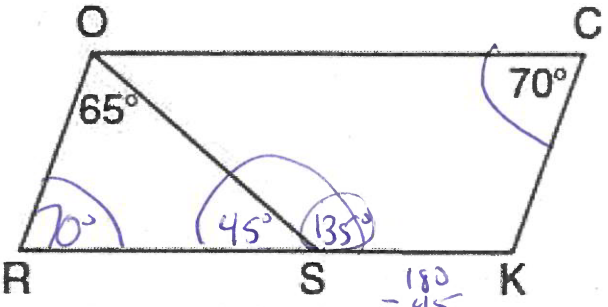
$180 - 48 = 132$; $\frac{132}{2} = 66$

$\triangle DBA$: $90 + 66 = 156$; $180 - 156 = 24$

15. In the diagram below of parallelogram *ROCK*, $m\angle C$ is 70° and $m\angle ROS$ is 65°. What is $m\angle KSO$?

1) 45°
2) 110°
3) 115°
4) 135°

$\triangle ROS$: $70 + 65 = 135$; $180 - 135 = 45$

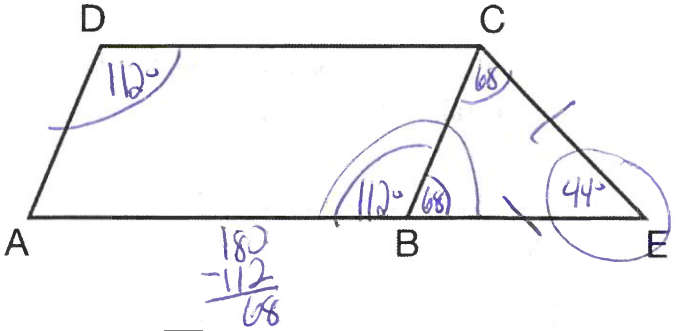


16. In the diagram below, *ABCD* is a parallelogram, $\overline{AB}$ is extended through *B* to *E*, and $\overline{CE}$ is drawn.

If $\overline{CE} \cong \overline{BE}$ and $m\angle D = 112°$, what is $m\angle E$?

1) 44°
2) 56°
3) 68°
4) 112°

$\triangle BCE$: $68 + 68 = 136$; $180 - 136 = 44$



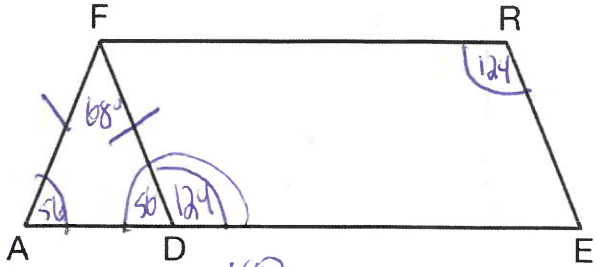
$180 - 112 = 68$

17. In the diagram of parallelogram *FRED* shown below, $\overline{ED}$ is extended to *A*, and $\overline{AF}$ is drawn such that $\overline{AF} \cong \overline{DF}$.

If $m\angle R = 124°$, what is $m\angle AFD$?

1) 124°
2) 112°
3) 68°
4) 56°

$\triangle ADF$: $56 + 56 = 112$; $180 - 112 = 68$

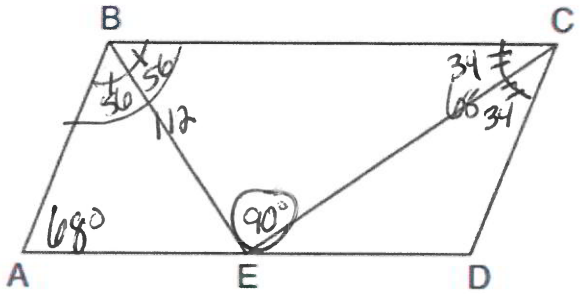


$180 - 124 = 56$

~~180~~ 178

15. In parallelogram $ABCD$ shown below, the bisectors of $\angle ABC$ and $\angle DCB$ meet at E, a point on $\overline{AD}$.
If $m\angle A = 68°$, determine and state $m\angle BEC$.

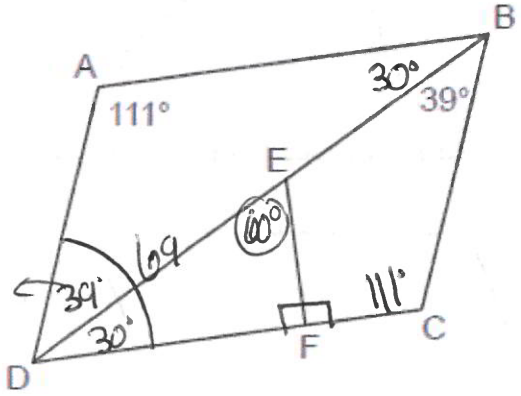
180 − 68 = 112



56 + 34 = 90 180 − 90 = 90

16. In the diagram below of parallelogram $ABCD$, diagonal $\overline{BED}$ and $\overline{EF}$ are drawn, $\overline{EF} \perp \overline{DFC}$, $m\angle DAB = 111°$, and $m\angle DBC = 39°$. What is $m\angle DEF$?

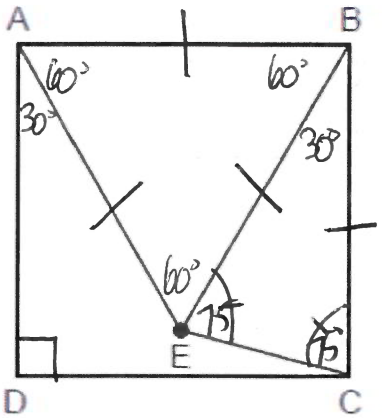
180 − 111 = 69



90 + 30 = 120 180 − 120 = 60

17. In the diagram below, point E is located inside square $ABCD$ such that $\triangle ABE$ is equilateral, and $\overline{CE}$ is drawn. What is $m\angle BEC$? 75°

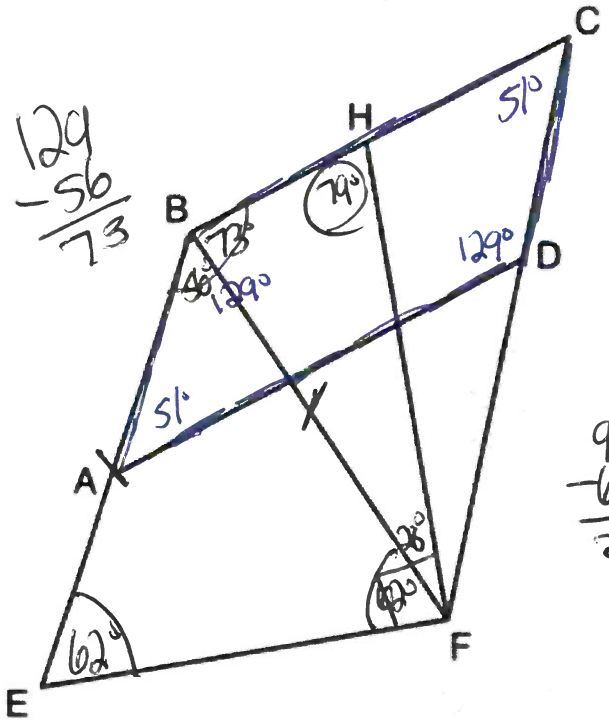
90 − 60 = 30



180 − 30 = 150 150 / 2 = 75

~~132~~ ~~136~~
179

18. Quadrilateral $EBCF$ and $\overline{AD}$ are drawn below, such that $ABCD$ is a parallelogram, $\overline{EB} \cong \overline{FB}$, and $\overline{EF} \perp \overline{FH}$. If $m\angle E = 62°$ and $m\angle C = 51°$, what is $m\angle FHB$?



180 − 51 = 129

73 + 28 = 101

180 − 101 = 79

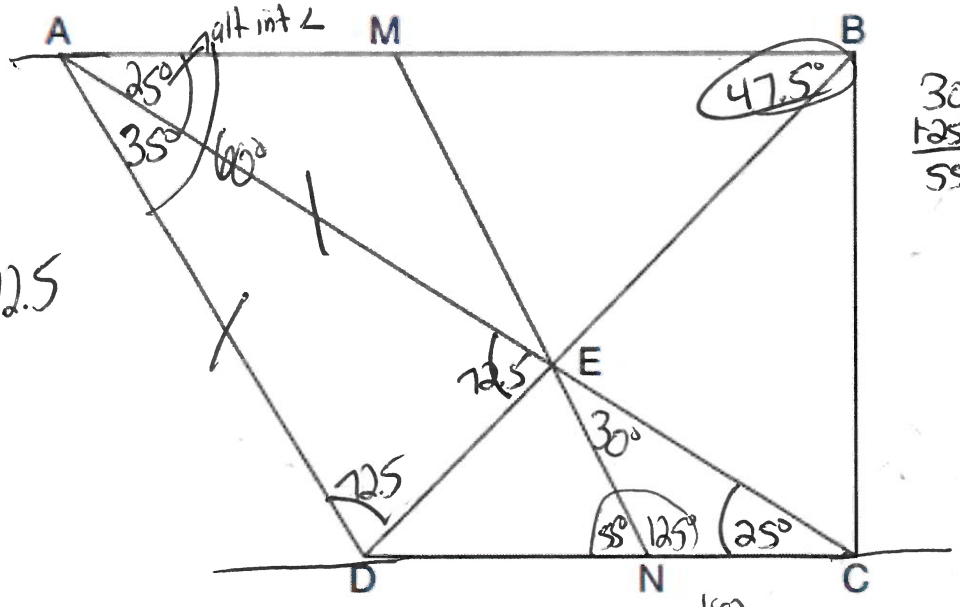
129 − 56 = 73

90 − 62 = 28

62 + 62 = 124

180 − 124 = 56

19. Trapezoid $ABCD$, where $\overline{AB} \parallel \overline{CD}$, is shown below. Diagonals $\overline{AC}$ and $\overline{DB}$ intersect $\overline{MN}$ at E, and $\overline{AD} \cong \overline{AE}$. If $m\angle DAE = 35°$, $m\angle DCE = 25°$, and $m\angle NEC = 30°$, determine and state $m\angle ABD$.



180 − 35 = 145

145 / 2 = 72.5

30 + 125 = 55

180 − 55 = 125

60 + 72.5 = 132.5

180 − 132.5 = 47.5

180 − 125 = 55

$d=\sqrt{\Delta x^2+\Delta y^2}$

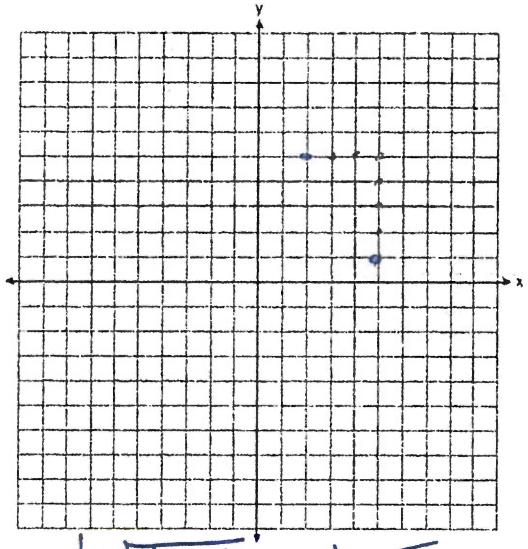
Name Schlansky
Mr. Schlansky

Date _____
Geometry

Calculating Distance

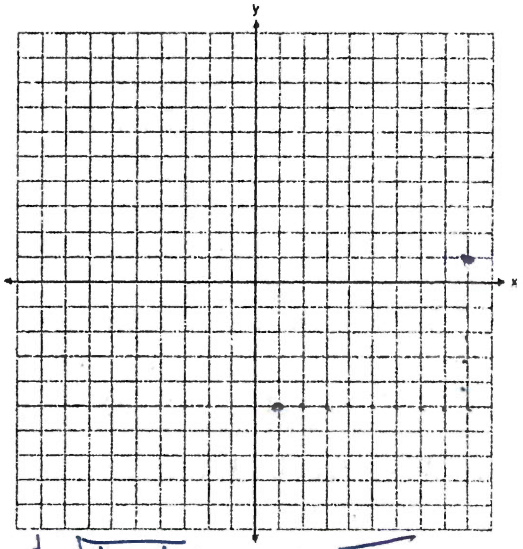
Calculate the distance between the following sets of points. Express in simplest radical form

1. (5,1) and (2,5)



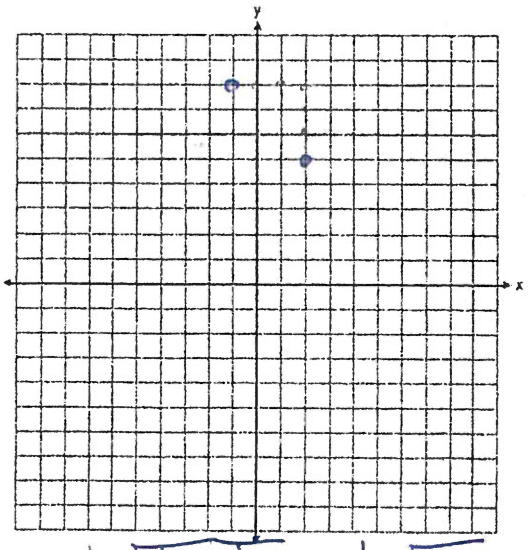
$d=\sqrt{\Delta x^2+\Delta y^2}$
$d=\sqrt{3^2+4^2}$
$d=\sqrt{9+16}$
$d=\sqrt{25}$
$d=5$

2. (9,1) and (1,-5)



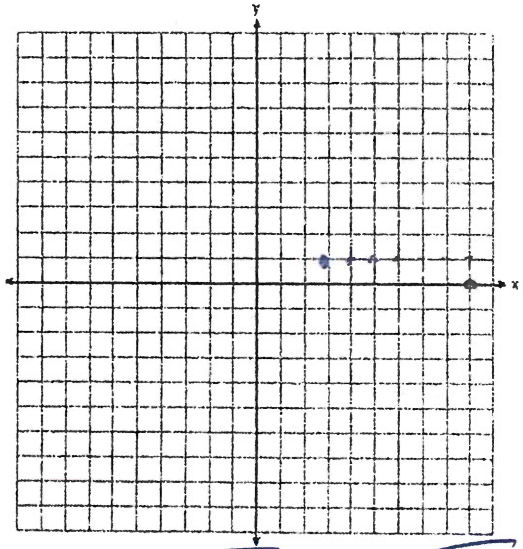
$d=\sqrt{\Delta x^2+\Delta y^2}$
$d=\sqrt{8^2+6^2}$
$d=\sqrt{64+36}$
$d=\sqrt{100}$
$d=10$

3. (2,5) and (-1,8)



$d=\sqrt{\Delta x^2+\Delta y^2}$
$d=\sqrt{3^2+3^2}$
$d=\sqrt{9+9}$
$d=\sqrt{18}$
$\sqrt{9}\sqrt{2}$
$3\sqrt{2}$

4. (3,1) and (9,0)



$d=\sqrt{\Delta x^2+\Delta y^2}$
$d=\sqrt{6^2+1^2}$
$d=\sqrt{36+1}$
$d=\sqrt{37}$

181
~~174~~

Adding Radicals
Reduce first so they have the same radicand
Add coefficients, keep radicand

Express each of the following in simplest radical form

1. $\sqrt{50} + \sqrt{50}$

$\sqrt{25}\sqrt{2}\ \sqrt{25}\sqrt{2}$
$5\sqrt{2} + 5\sqrt{2}$
$10\sqrt{2}$

2. $\sqrt{32} + \sqrt{32}$

$\sqrt{16}\sqrt{2}\ \sqrt{16}\sqrt{2}$
$4\sqrt{2}+4\sqrt{2}$
$8\sqrt{2}$

3. $\sqrt{63} + \sqrt{28}$

$\sqrt{9}\sqrt{7}\ \sqrt{4}\sqrt{7}$
$3\sqrt{7}+2\sqrt{7}$
$5\sqrt{7}$

4. $\sqrt{45} + \sqrt{125}$

$\sqrt{9}\sqrt{5}\ \sqrt{25}\sqrt{5}$
$3\sqrt{5}+5\sqrt{5}$
$8\sqrt{5}$

5. $3\sqrt{18} + 2\sqrt{72}$

$3\sqrt{9}\sqrt{2}\ 2\sqrt{36}\sqrt{2}$
$3(3)\sqrt{2}+2(6)\sqrt{2}$
$9\sqrt{2}+12\sqrt{2}$
$21\sqrt{2}$

6. $5\sqrt{27} + 2\sqrt{75}$

$5\sqrt{9}\sqrt{3}\ 2\sqrt{25}\sqrt{3}$
$5(3)\sqrt{3}+2(5)\sqrt{3}$
$15\sqrt{3}+10\sqrt{3}$
$25\sqrt{3}$

Perfect Squares

1
4
9
16
25
36
49
64
81
100

7. $2\sqrt{200} + 2\sqrt{18}$

$2\sqrt{100}\sqrt{2}\ 2\sqrt{9}\sqrt{2}$

$2(10)\sqrt{2} + 2(3)\sqrt{2}$

$20\sqrt{2} + 6\sqrt{2}$

$26\sqrt{2}$

8. $4\sqrt{12} + 3\sqrt{48}$

$4\sqrt{4}\sqrt{3}\ 3\sqrt{16}\sqrt{3}$

$4(2)\sqrt{3} + 3(4)\sqrt{3}$

$8\sqrt{3} + 12\sqrt{3}$

$20\sqrt{3}$

9. $4\sqrt{80} + 2\sqrt{45} + 12$

$4\sqrt{16}\sqrt{5} + 2\sqrt{9}\sqrt{5} + 12$

$4(4)\sqrt{5} + 2(3)\sqrt{5} + 12$

$16\sqrt{5} + 6\sqrt{5} + 12$

$22\sqrt{5} + 12$

10. $4\sqrt{75} + 8 + 3\sqrt{24}$

$4\sqrt{25}\sqrt{3} + 8 + 2\sqrt{4}\sqrt{6}$

$4(5)\sqrt{3} + 8 + 2(2)\sqrt{6}$

$20\sqrt{3} + 8 + 4\sqrt{6}$

11. $3\sqrt{50} + 2\sqrt{75} + 4\sqrt{8}$

$3\sqrt{25}\sqrt{2}\ 2\sqrt{25}\sqrt{3}\ 4\sqrt{4}\sqrt{2}$

$3(5)\sqrt{2} + 2(5)\sqrt{3} + 4(2)\sqrt{2}$

$10\sqrt{2} + 10\sqrt{3} + 8\sqrt{2}$

$18\sqrt{2} + 10\sqrt{3}$

12. $2\sqrt{294} + 3\sqrt{216} + 2\sqrt{180} + 6$

$2\sqrt{49}\sqrt{6} + 3\sqrt{36}\sqrt{6} + 2\sqrt{36}\sqrt{5} + 6$

$2(7)\sqrt{6} + 3(6)\sqrt{6} + 2(6)\sqrt{5} + 6$

$14\sqrt{6} + 18\sqrt{6} + 12\sqrt{5} + 6$

$32\sqrt{6} + 12\sqrt{5} + 6$

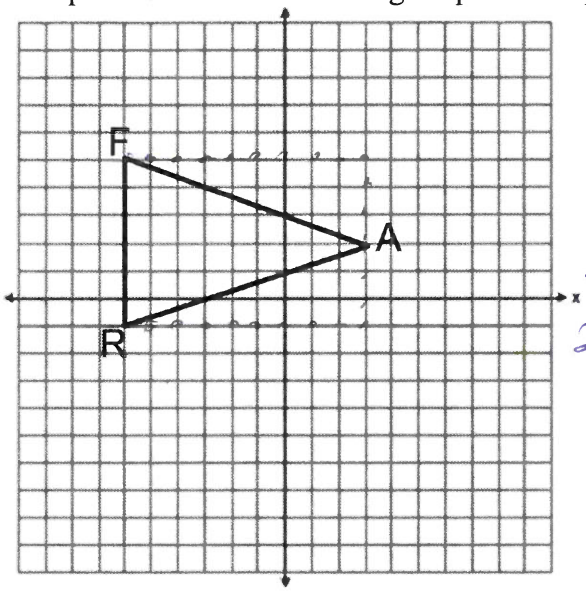
Perimeter with Coordinate Geometry

Find the length of each side. If side is straight, count the boxes. If side is sloped, use distance formula.

To add them together, reduce the radicals, then combine coefficients. Do not combine unlike terms.

Find the perimeter of the following shapes in simplest radical form.

1.



$d\overline{AF} = \sqrt{9^2+3^2} = \sqrt{81+9} = \sqrt{90}$

$d\overline{AR} = \sqrt{9^2+3^2} = \sqrt{81+9} = \sqrt{90}$

$d\overline{RF} = 6$

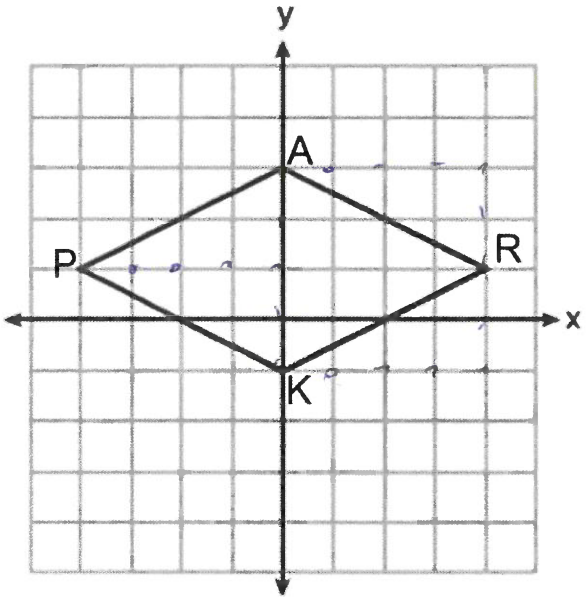
$2\sqrt{90}+6$

$2\sqrt{9}\sqrt{10}$

$2(3)\sqrt{10}+6$

$6\sqrt{10}+6$

2.



$d\overline{PA} = \sqrt{4^2+2^2} = \sqrt{16+4} = \sqrt{20}$

$d\overline{AR} = \sqrt{4^2+2^2} = \sqrt{16+4} = \sqrt{20}$

$d\overline{RK} = \sqrt{4^2+2^2} = \sqrt{16+4} = \sqrt{20}$

$d\overline{KP} = \sqrt{4^2+2^2} = \sqrt{16+4} = \sqrt{20}$

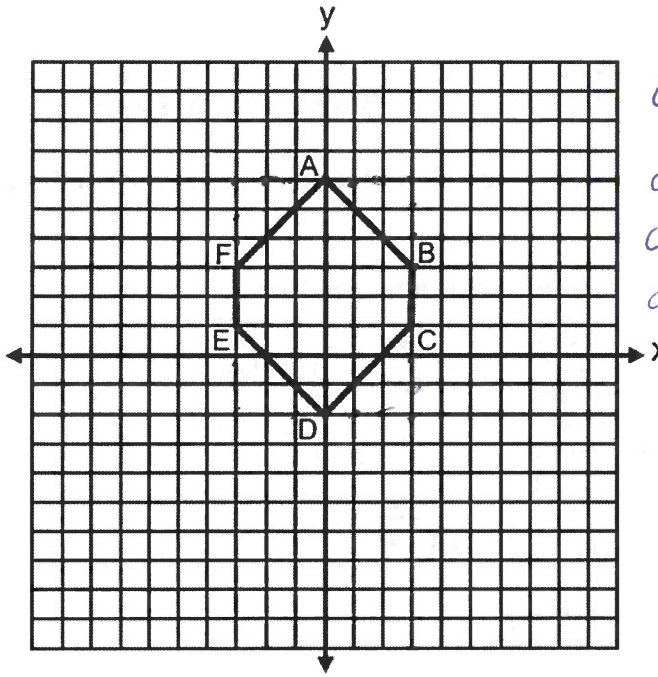
$4\sqrt{20}$

$4\sqrt{4}\sqrt{5}$

$4(2)\sqrt{5}$

$8\sqrt{5}$

3.



$d\overline{AB} = \sqrt{3^2+3^2} = \sqrt{9+9} = \sqrt{18}$

$d\overline{BC} = 2$

$d\overline{CD} = \sqrt{3^2+3^2} = \sqrt{9+9} = \sqrt{18}$

$d\overline{DE} = \sqrt{3^2+3^2} = \sqrt{9+9} = \sqrt{18}$

$d\overline{EF} = 2$

$d\overline{FA} = \sqrt{3^2+3^2} = \sqrt{9+9} = \sqrt{18}$

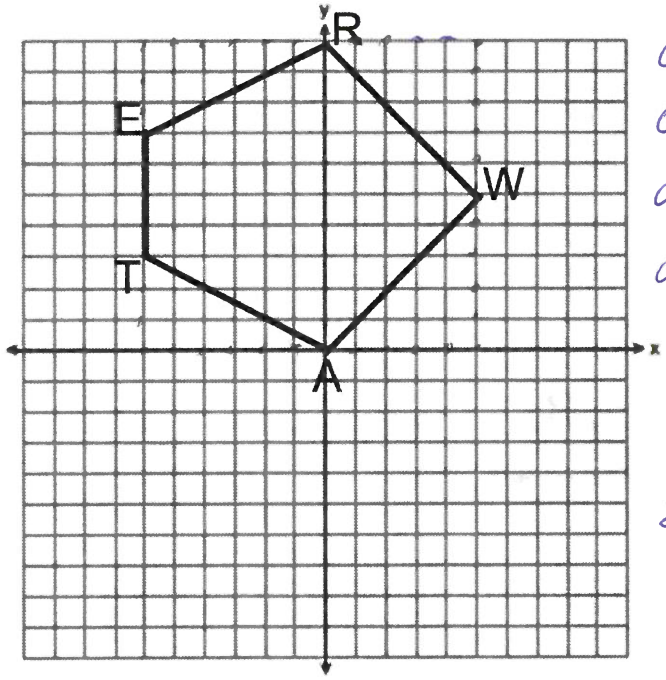
$4\sqrt{18}+4$

$4\sqrt{9}\sqrt{2}+4$

$4(3)\sqrt{2}+4$

$12\sqrt{2}+4$

4.



$d\overline{RW} = \sqrt{5^2+5^2} = \sqrt{25+25} = \sqrt{50}$

$d\overline{WA} = \sqrt{5^2+5^2} = \sqrt{25+25} = \sqrt{50}$

$d\overline{AT} = \sqrt{6^2+3^2} = \sqrt{36+9} = \sqrt{45}$

$d\overline{TE} = 4$

$d\overline{ER} = \sqrt{6^2+3^2} = \sqrt{36+9} = \sqrt{45}$

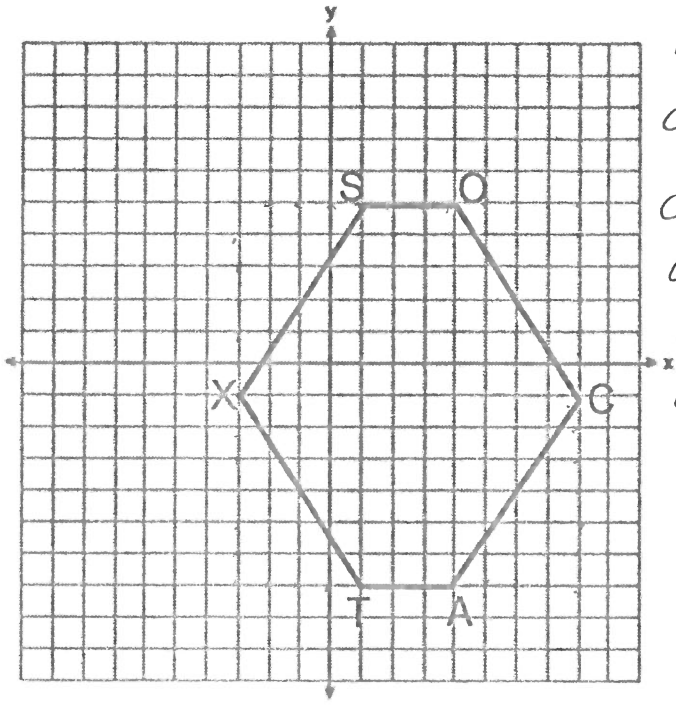
$2\sqrt{50}+2\sqrt{45}+4$

$2\sqrt{25}\sqrt{2}\quad 2\sqrt{9}\sqrt{5}$

$2(5)\sqrt{2}+2(3)\sqrt{5}+4$

$10\sqrt{2}+6\sqrt{5}+4$

5.



$d\overline{SO} = 3$

$d\overline{OC} = \sqrt{4^2+6^2} = \sqrt{16+36} = \sqrt{52}$

$d\overline{CA} = \sqrt{4^2+6^2} = \sqrt{16+36} = \sqrt{52}$

$d\overline{AT} = 3$

$d\overline{TX} = \sqrt{4^2+6^2} = \sqrt{16+36} = \sqrt{52}$

$d\overline{XS} = \sqrt{4^2+6^2} = \sqrt{16+36} = \sqrt{52}$

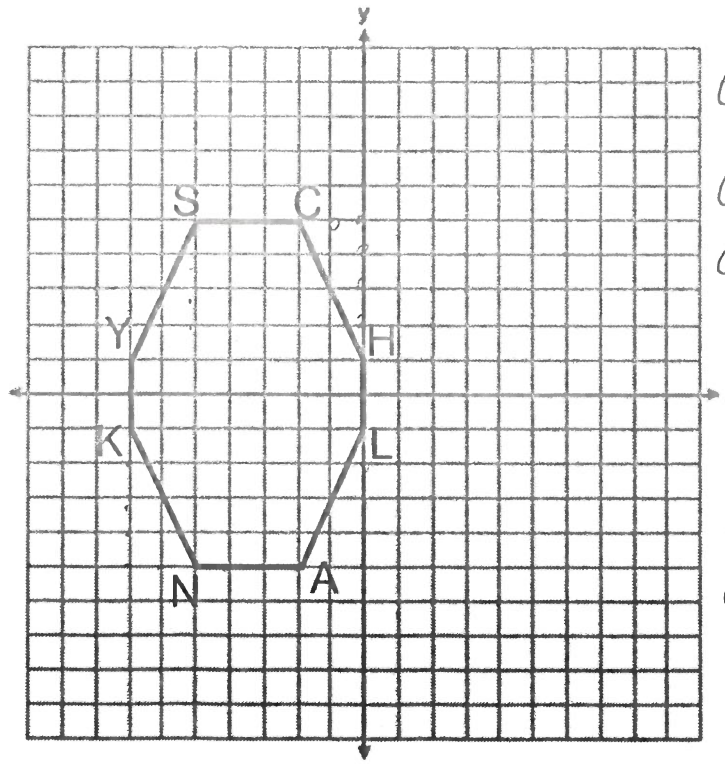
$4\sqrt{52} + 6$

$4\sqrt{4}\sqrt{13}$

$4(2)\sqrt{13} + 6$

$8\sqrt{13} + 6$

6.



$d\overline{SC} = 3$

$d\overline{CH} = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20}$

$d\overline{HL} = 2$

$d\overline{LA} = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20}$

$d\overline{AN} = 3$

$d\overline{NK} = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20}$

$d\overline{KY} = 2$

$d\overline{YS} = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20}$

$4\sqrt{20} + 10$

$4\sqrt{4}\sqrt{5}$

$4(2)\sqrt{5} + 10$

$8\sqrt{5} + 10$

~~188~~ 186

$m = \frac{\Delta y}{\Delta x}$

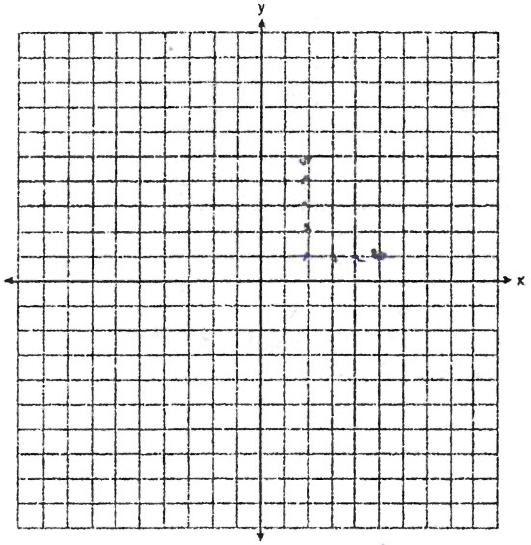
Name Schlansky
Mr. Schlansky

Date _____
Geometry

Calculating Slope

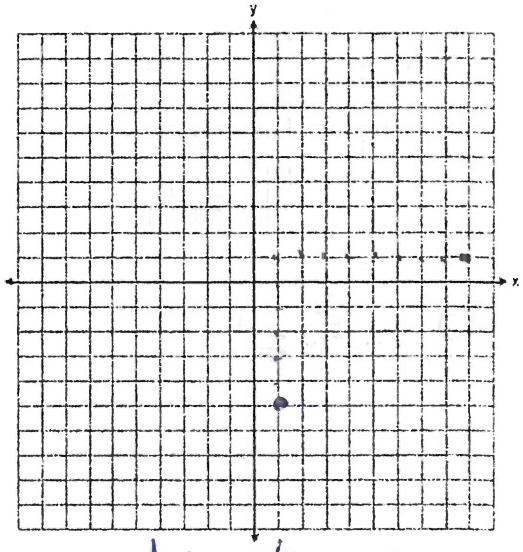
Calculate the slopes between the following sets of points. Express in simplest terms

1. (5,1) and (2,5)



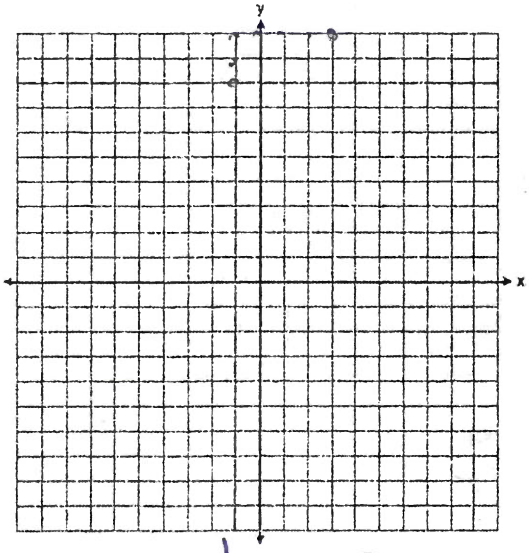
$m = \frac{\Delta y}{\Delta x} = \frac{-4}{3}$

2. (9,1) and (1,-5)



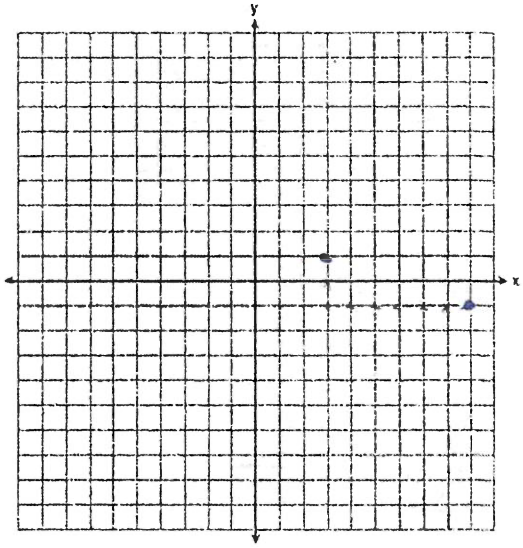
$m = \frac{\Delta y}{\Delta x} = \frac{6}{8} = \frac{3}{4}$

3. (3,10) and (-1,8)



$m = \frac{\Delta y}{\Delta x} = \frac{2}{4} = \frac{1}{2}$

4. (3,1) and (9,-1)



$m = \frac{\Delta y}{\Delta x} = \frac{-2}{6} = \frac{-1}{3}$

187
~~176~~

Coordinate Geometry Proofs

Distance (Length) = $\sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

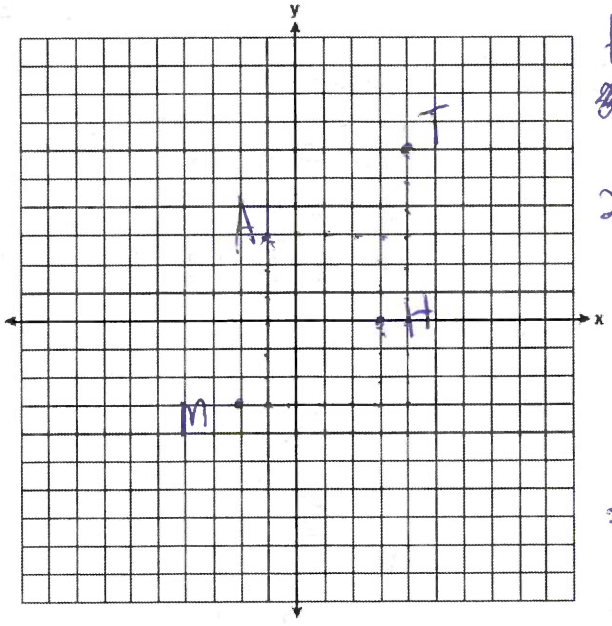
Slope = $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Midpoint = (*average x, average y*) = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

How do you prove…?

…an **isosceles triangle**? (2 Distances)
Two Congruent Sides
. … a **right triangle**? (3 Distances)
Show the sides fit into Pythagorean Theorem
… a **parallelogram**? (4 Distances)
Two Pairs of Opposite Sides Congruent
… a **rhombus**? (4 Distances)
All Sides Congruent
… a **rectangle**? (6 Distances)
1) Two Pairs of Opposite Sides Congruent
2) Diagonals Congruent
… a **square**? (6 Distances)
1) All Sides Congruent
2) Diagonals Congruent
…a **trapezoid**? (4 Slopes)
1) 1 pair of opposite sides parallel
…an **isosceles trapezoid**? (4 Slopes, 2 Distances)
1) 1 pair of opposite sides parallel
2) Congruent Legs

1. Graph the quadrilateral MATH: M(-2, -3) A(-1, 3) T(4, 6), H(3, 0). Prove that MATH **IS** a parallelogram but is **NOT** a rectangle.

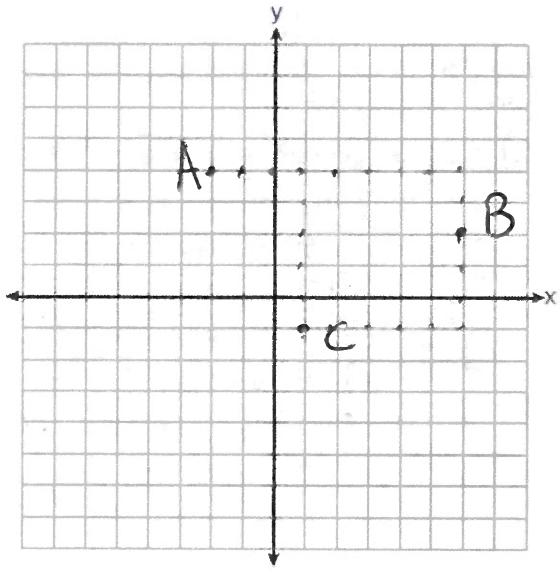


1) MATH is a parallelogram because it has two pairs of opposite sides congruent.
It is not a rectangle because the diagonals are not congruent.

2) $d\overline{MA} = \sqrt{1^2+6^2} = \sqrt{1+36} = \sqrt{37}$
$d\overline{AT} = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$
$d\overline{TH} = \sqrt{1^2+6^2} = \sqrt{1+36} = \sqrt{37}$
$d\overline{HM} = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$
$d\overline{AH} = \sqrt{4^2+3^2} = \sqrt{16+9} = \sqrt{25}$
$d\overline{MT} = \sqrt{6^2+9^2} = \sqrt{36+81} = \sqrt{117}$

3) $\overline{MA} \cong \overline{TH}$, $\overline{AT} \cong \overline{MH}$ because they have the same distance.
$\overline{MT} \ncong \overline{AH}$ because they don't have the same distance

2. A triangle has vertices $A(-2,4)$, $B(6,2)$, and $C(1,-1)$. Prove that $\triangle ABC$ is an isosceles right triangle. [The use of the set of axes below is optional.]



1) $\triangle ABC$ is an isosceles right triangle because it has two congruent sides and its sides fit into Pythagorean Theorem.

2) $d\overline{AC} = \sqrt{3^2+5^2} = \sqrt{9+25} = \sqrt{34}$

$d\overline{CB} = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$

$d\overline{BA} = \sqrt{8^2+2^2} = \sqrt{64+4} = \sqrt{68}$

3) $\overline{AC} \cong \overline{CB}$ because they have the same distance.

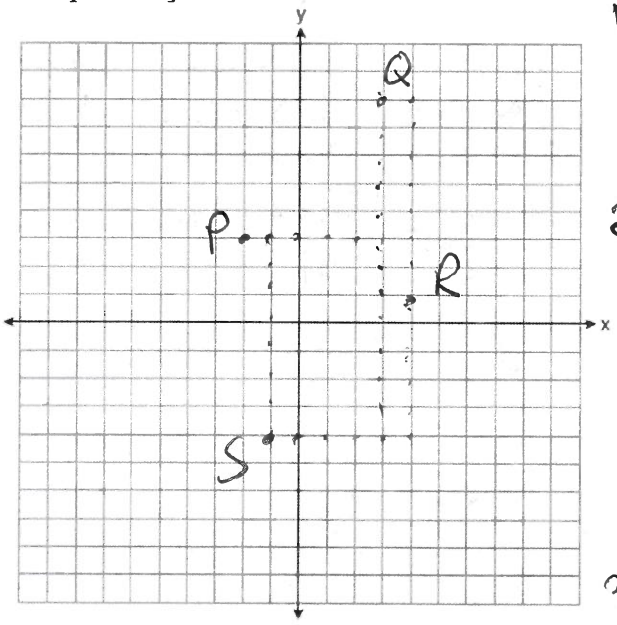
$a^2+b^2=c^2$

$\sqrt{34}^2+\sqrt{34}^2=\sqrt{68}^2$

$34+34=68$

$68=68$ ✓

3. Quadrilateral $PQRS$ has vertices $P(-2,3)$, $Q(3,8)$, $R(4,1)$, and $S(-1,-4)$. Prove that $PQRS$ is a rhombus. Prove that $PQRS$ is *not* a square. [The use of the set of axes below is optional.]



1) PQRS is a rhombus because all sides are congruent. It is not a square because diagonals are not congruent.

2) $d\overline{PQ} = \sqrt{5^2+5^2} = \sqrt{25+25} = \sqrt{50}$

$d\overline{QR} = \sqrt{1^2+7^2} = \sqrt{1+49} = \sqrt{50}$

$d\overline{RS} = \sqrt{5^2+5^2} = \sqrt{25+25} = \sqrt{50}$

$d\overline{SP} = \sqrt{1^2+7^2} = \sqrt{1+49} = \sqrt{50}$

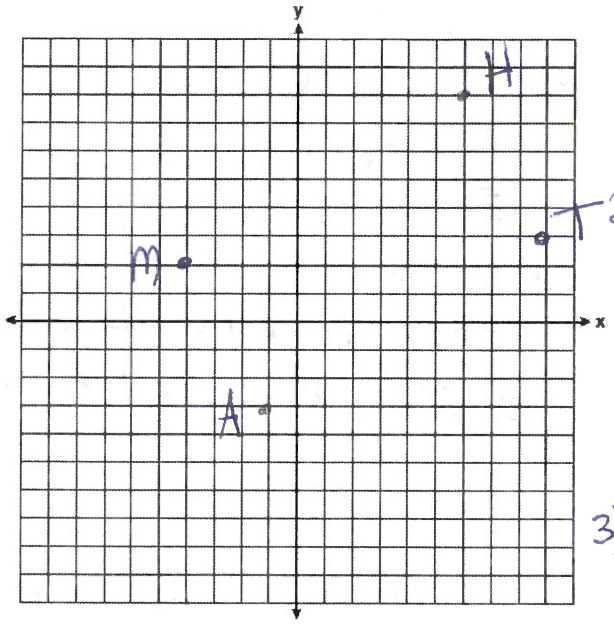
$d\overline{PR} = \sqrt{6^2+2^2} = \sqrt{36+4} = \sqrt{40}$

$d\overline{QS} = \sqrt{4^2+12^2} = \sqrt{16+144} = \sqrt{160}$

3) $\overline{PQ} \cong \overline{QR} \cong \overline{RS} \cong \overline{SP}$ because they have the same distance.

$\overline{PR} \ncong \overline{QS}$ because they don't have the same distance.

4. The vertices of quadrilateral *MATH* have coordinates $M(-4, 2)$, $A(-1, -3)$, $T(9, 3)$, and $H(6, 8)$. Prove that quadrilateral *MATH* is a rectangle but not a square. [The use of the set of axes below is optional.]

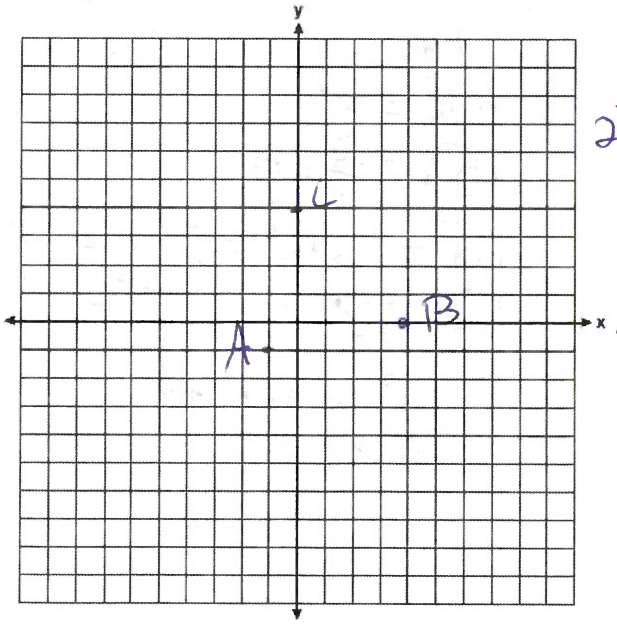


1) MATH is a rectangle because it has 2 pairs of opposite sides congruent and diagonals congruent. It is not a square because consecutive sides are not congruent.

2) $d\overline{MH} = \sqrt{10^2+6^2} = \sqrt{100+36} = \sqrt{136}$
$d\overline{HT} = \sqrt{3^2+5^2} = \sqrt{9+25} = \sqrt{34}$
$d\overline{TA} = \sqrt{10^2+6^2} = \sqrt{100+36} = \sqrt{136}$
$d\overline{AM} = \sqrt{3^2+5^2} = \sqrt{9+25} = \sqrt{34}$
$d\overline{MT} = \sqrt{13^2+1^2} = \sqrt{169+1} = \sqrt{170}$
$d\overline{AH} = \sqrt{7^2+11^2} = \sqrt{49+121} = \sqrt{170}$

3) $\overline{MH} \cong \overline{AT}$, $\overline{MA} \cong \overline{HT}$, $\overline{MT} \cong \overline{AH}$ because they have the same distance
$\overline{MA} \ncong \overline{AT}$ because they don't have the same distance.

5. Triangle *ABC* has vertices with coordinates $A(-1, -1)$, $B(4, 0)$, and $C(0, 4)$. Prove that $\triangle ABC$ is an isosceles triangle but *not* an equilateral triangle. [The use of the set of axes below is optional.]

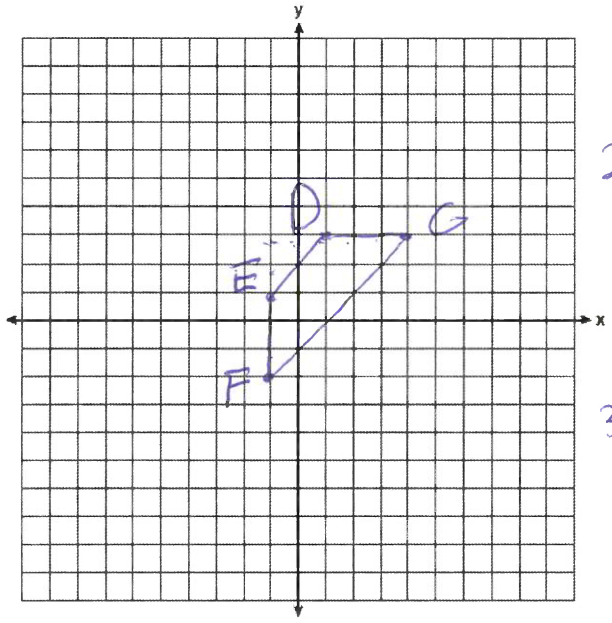


1) $\triangle ABC$ is isosceles because it has two congruent sides. It is not equilateral because not all sides are congruent.

2) $d\overline{AC} = \sqrt{1^2+5^2} = \sqrt{1+25} = \sqrt{26}$
$d\overline{AB} = \sqrt{5^2+1^2} = \sqrt{25+1} = \sqrt{26}$
$d\overline{BC} = \sqrt{4^2+4^2} = \sqrt{16+16} = \sqrt{32}$

3) $\overline{AC} \cong \overline{AB}$ because they have the same distance. $\overline{AB} \ncong \overline{BC}$ because they don't have the same distance.

6. Quadrilateral DEFG has vertices D(1,3) E(-1,1) F(-1,-2) G(4,3). Prove that DEFG is an isosceles trapezoid.



1) DEFG is an isosceles trapezoid because it has 1 pair of opposite sides parallel and congruent legs

2) $m\overline{ED} = \frac{2}{2} = 1$

$m\overline{FG} = \frac{5}{5} = 1$

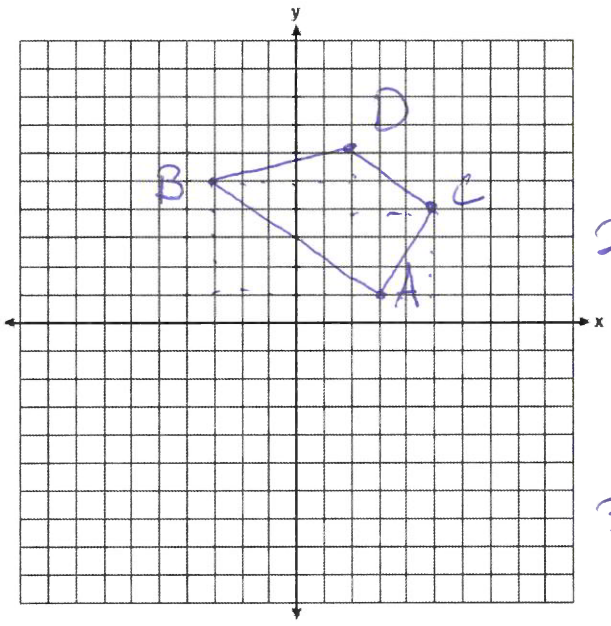
$d\overline{EF} = 3$

$d\overline{DG} = 3$

3) $\overline{ED} \parallel \overline{FG}$ because they have the same slope

$\overline{EF} \cong \overline{DG}$ because they have the same distance.

7. Quadrilateral ABCD has vertices A(3,1) B(-3,5) C(5,4) and D(2,6). Prove quadrilateral ABCD is a trapezoid but *not* an isosceles trapezoid.



1) ABCD is a trapezoid because it has 1 pair of opposite sides parallel. It is not isosceles because it does not have congruent legs.

2) $m\overline{DC} = \frac{-2}{3}$

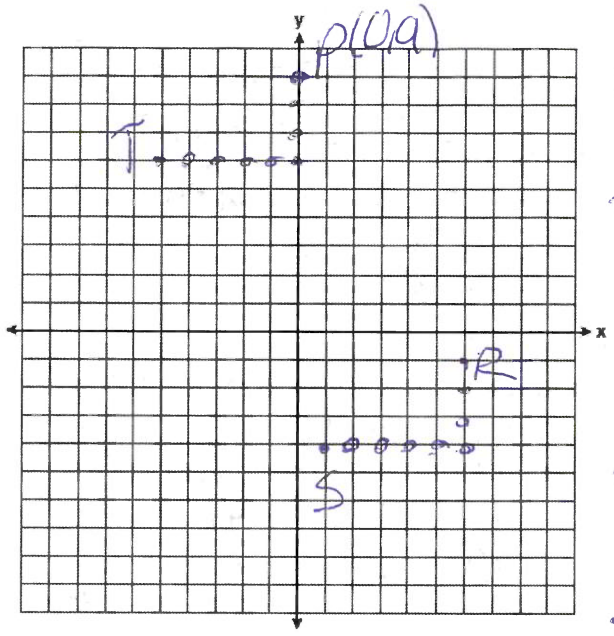
$m\overline{BA} = \frac{-4}{6} = \frac{-2}{3}$

$d\overline{BD} = \sqrt{5^2+1^2} = \sqrt{25+1} = \sqrt{26}$

$d\overline{CA} = \sqrt{2^2+3^2} = \sqrt{4+9} = \sqrt{13}$

3) $\overline{DC} \parallel \overline{BA}$ because they have the same slope.

8. In the coordinate plane, the vertices of $\triangle RST$ are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$. Prove that $\triangle RST$ is a right triangle. State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle. Prove that your quadrilateral $RSTP$ is a rectangle. [The use of the set of axes below is optional.]



1) $\triangle RST$ is a right triangle because the sides fit into Pythagorean Theorem.

2) $d\overline{ST} = \sqrt{6^2+10^2} = \sqrt{36+100} = \sqrt{136}$

$d\overline{SR} = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$

$d\overline{RT} = \sqrt{11^2+7^2} = \sqrt{121+49} = \sqrt{170}$

3) $a^2+b^2=c^2$

$\sqrt{136}^2+\sqrt{34}^2=\sqrt{170}^2$

$170=170$

1) RSTP is a rectangle because it has 2 pairs of opposite sides congruent and diagonals are congruent.

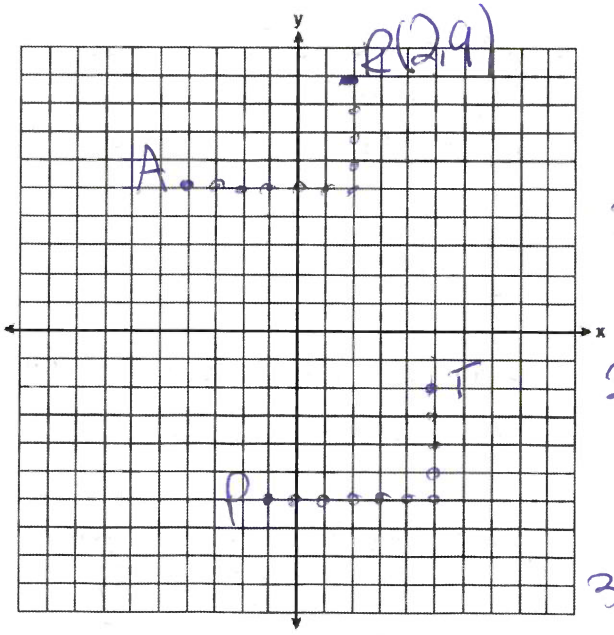
2) $d\overline{TP} = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$

$d\overline{PR} = \sqrt{6^2+10^2} = \sqrt{36+100} = \sqrt{136}$

$d\overline{PS} = \sqrt{1^2+13^2} = \sqrt{1+169} = \sqrt{170}$

3) $\overline{TP}\cong\overline{SR}$, $\overline{TS}\cong\overline{PR}$, $\overline{TR}\cong\overline{PS}$ because they have the same distance.

9. In the coordinate plane, the vertices of triangle PAT are $P(-1,-6)$, $A(-4,5)$, and $T(5,-2)$. Prove that $\triangle PAT$ is an isosceles triangle. [The use of the set of axes below is optional.] State the coordinates of R so that quadrilateral $PART$ is a parallelogram. Prove that quadrilateral $PART$ is a parallelogram.



1) $\triangle PAT$ is isosceles because two sides are congruent.

2) $d\overline{AP} = \sqrt{3^2+11^2} = \sqrt{9+121} = \sqrt{130}$

$d\overline{AT} = \sqrt{9^2+7^2} = \sqrt{81+49} = \sqrt{130}$

3) $\overline{AP}\cong\overline{AT}$ because they have the same distance.

1) PART is a parallelogram because two pairs of opposite sides are congruent.

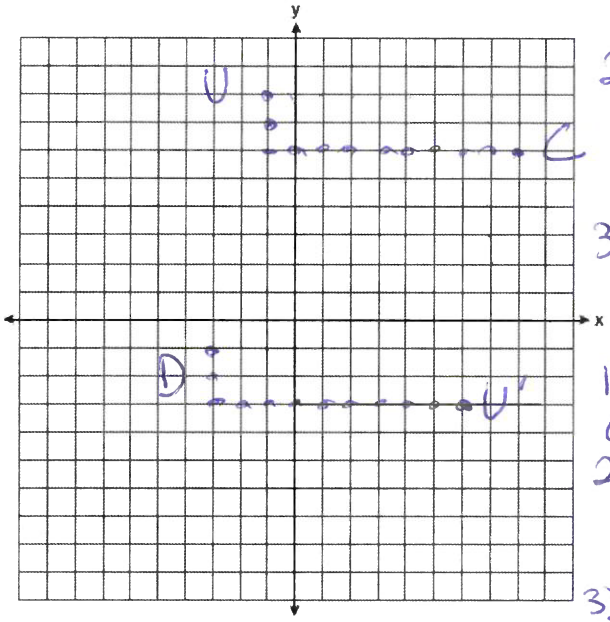
2) $d\overline{AR} = \sqrt{6^2+4^2} = \sqrt{36+16} = \sqrt{52}$

$d\overline{TR} = \sqrt{3^2+11^2} = \sqrt{9+121} = \sqrt{130}$

$d\overline{PT} = \sqrt{6^2+4^2} = \sqrt{36+16} = \sqrt{52}$

3) $\overline{AR}\cong\overline{PT}$, $\overline{AP}\cong\overline{RT}$ because they have the same distance.

10. Given: Triangle DUC with coordinates $D(-3,-1)$, $U(-1,8)$, and $C(8,6)$
Prove: $\triangle DUC$ is a right triangle
Point U is reflected over $\overline{DC}$ to locate its image point, U', forming quadrilateral $DUCU'$.
Prove quadrilateral $DUCU'$ is a square.
[The use of the set of axes below is optional.]



1) $\triangle DUC$ is a right triangle because its sides fit into Pythagorean theorem.

2) $d\overline{DU} = \sqrt{2^2+9^2} = \sqrt{4+81} = \sqrt{85}$

$d\overline{UC} = \sqrt{9^2+2^2} = \sqrt{81+4} = \sqrt{85}$

$d\overline{CD} = \sqrt{11^2+7^2} = \sqrt{121+49} = \sqrt{170}$

3) $a^2+b^2=c^2$

$\sqrt{85}^2 + \sqrt{85}^2 = \sqrt{170}^2$

$170 = 170$

1) $DUCU'$ is a square because all sides are congruent and diagonals are congruent

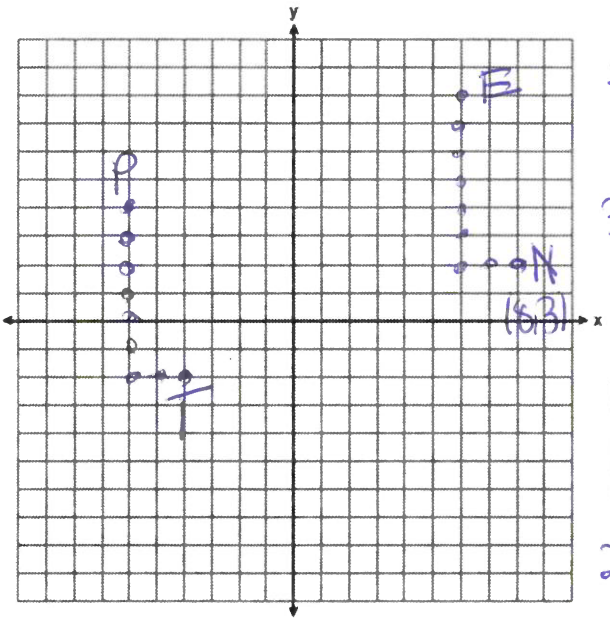
2) $d\overline{DU'} = \sqrt{9^2+2^2} = \sqrt{81+4} = \sqrt{85}$

$d\overline{UU'} = \sqrt{7^2+11^2} = \sqrt{49+121} = \sqrt{170}$

$d\overline{CU'} = \sqrt{2^2+9^2} = \sqrt{4+81} = \sqrt{85}$

3) $\overline{DU} \cong \overline{UC} \cong \overline{CU'} \cong \overline{U'D}$ and $\overline{UU'} \cong \overline{DC}$ because they have the same distance.

11. Triangle PET has vertices with coordinates $P(-6,4)$, $E(6,8)$, and $T(-4,-2)$. Prove $\triangle PET$ is a right triangle. State the coordinates of N, the image of P, after a 180° rotation centered at $(1,3)$. Prove $PENT$ is a rectangle. [The use of the set of axes below is optional.]



1) $\triangle PET$ is a right triangle because its sides fit into Pythagorean theorem.

2) $d\overline{PE} = \sqrt{12^2+4^2} = \sqrt{144+16} = \sqrt{160}$

$d\overline{ET} = \sqrt{10^2+10^2} = \sqrt{100+100} = \sqrt{200}$

$d\overline{TP} = \sqrt{2^2+6^2} = \sqrt{4+36} = \sqrt{40}$

3) $a^2+b^2=c^2$

$\sqrt{40}^2 + \sqrt{160}^2 = \sqrt{200}^2$

$40+160 = 200$

$200 = 200$

1) $PENT$ is a rectangle because it has two pairs of opposite sides congruent and diagonals congruent.

2) $d\overline{TN} = \sqrt{12^2+4^2} = \sqrt{144+16} = \sqrt{160}$

$d\overline{EN} = \sqrt{2^2+6^2} = \sqrt{4+36} = \sqrt{40}$

$d\overline{PN} = \sqrt{14^2+2^2} = \sqrt{196+4} = \sqrt{200}$

3) $\overline{PE} \cong \overline{TN}$, $\overline{PT} \cong \overline{EN}$, $\overline{PN} \cong \overline{TE}$ because they have the same distance

You can just find the 4th point the same way as the previous page by counting!

Coordinate Geometry Applications

Slope: $m = \frac{\Delta y}{\Delta x}$

Distance: $d = \sqrt{\Delta x^2 + \Delta y^2}$

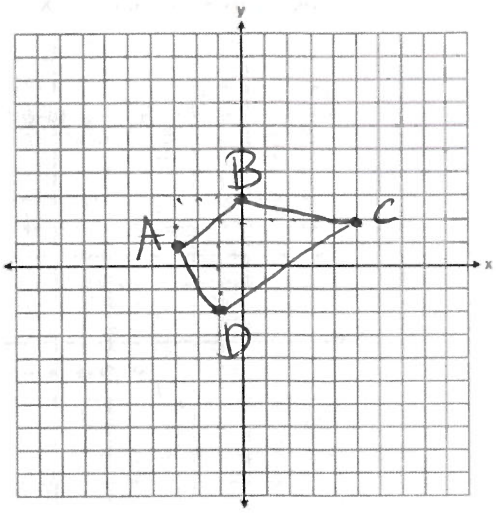
1. A quadrilateral has vertices with coordinates $(-3, 1)$, $(0, 3)$, $(5, 2)$, and $(-1, -2)$. Which type of quadrilateral is this?

1) rhombus
2) rectangle
3) square
4) trapezoid

$m\overline{AB} = \frac{2}{3}$

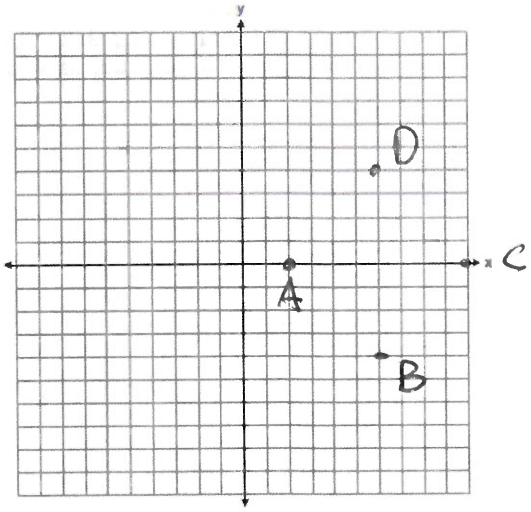
$m\overline{DC} = \frac{4}{6} = \frac{2}{3}$

$\overline{AB} \parallel \overline{DC}$



2. Quadrilateral ABCD has coordinates A(2,0), B(6,-4), C(10,0), and D(6,4). ABCD *cannot* be

1) rhombus
2) rectangle
3) square
4) trapezoid

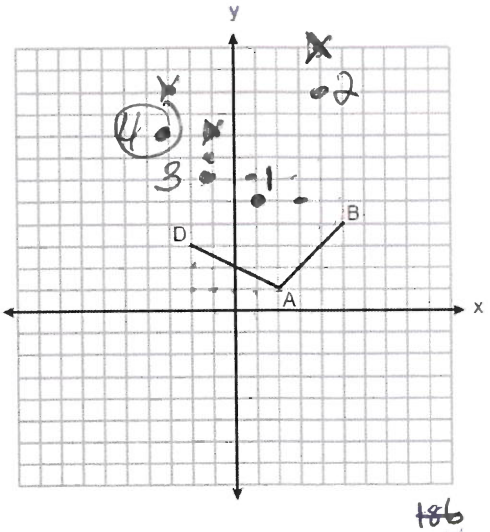


3. On the set of axes below, the coordinates of three vertices of trapezoid *ABCD* are $A(2, 1)$, $B(5, 4)$, and $D(-2, 3)$.
Which point could be vertex *C*?

1) $(1, 5)$
2) $(4, 10)$
3) $(-1, 6)$
4) $(-3, 8)$

$\overline{DA}$ must be $\parallel$ to $\overline{BC}$ (same slope)

$m\overline{DA} = \frac{-2}{4} = \frac{-1}{2}$



~~186~~
144

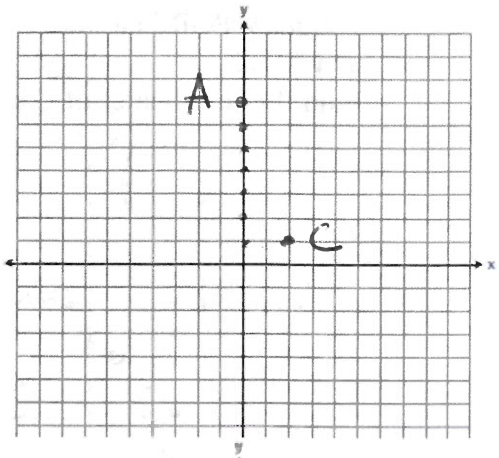
4. Parallelogram $ABCD$ has coordinates $A(0,7)$ and $C(2,1)$. Which statement would prove that $ABCD$ is a rhombus?

1) The midpoint of $\overline{AC}$ is $(1,4)$.
2) The length of $\overline{BD}$ is $\sqrt{40}$.
3) The slope of $\overline{BD}$ is $\frac{1}{3}$.
4) The slope of $\overline{AB}$ is $\frac{1}{3}$.

⊥ diagonals
negative reciprocal slopes

$m\overline{AC} = \frac{-6}{2} = -3$

$m\perp = \frac{1}{3}$



5. Parallelogram QRST has coordinates Q(-3,2) and S(6,0). Which statement would prove that QRST is a rectangle?

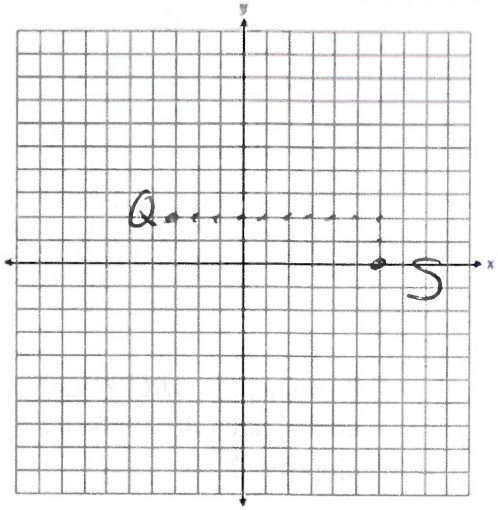
1) The slope of $\overline{RT}$ is $\frac{9}{2}$
2) The length of $\overline{RT}$ is $\sqrt{85}$
3) The midpoint of $\overline{RT}$ is (1.5,1)
4) $\overline{QR} \cong \overline{ST}$

≅ diagonals
same distance

$d\overline{QS} = \sqrt{9^2+2^2}$

$d\overline{QS} = \sqrt{81+4}$

$d\overline{QS} = \sqrt{85}$



6. The diagonals of rhombus $TEAM$ intersect at $P(2,1)$. If the equation of the line that contains diagonal $\overline{TA}$ is $y = -x + 3$, what is the equation of a line that contains diagonal EM?

1) $y = x - 1$
2) $y = x - 3$
3) $y = -x - 1$
4) $y = -x - 3$

⊥ diagonals
negative reciprocal slopes

$m\overline{TA} = -1$

$m\overline{EM} = 1$

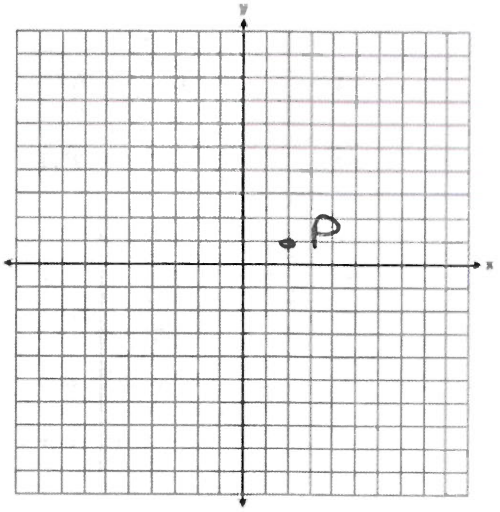
$m\perp = 1$
$x_1 = 2$
$y_1 = 1$

$y - y_1 = m(x - x_1)$

$y - 1 = 1(x - 2)$

$y - 1 = x - 2$
$+1 \quad +1$

$y = x - 1$

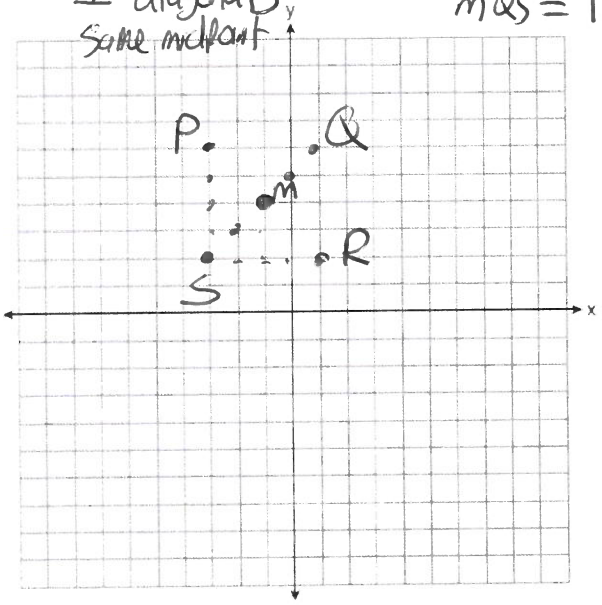


7. Square PQRS has diagonal $\overline{PR}$ with P(-3,6) and R(1,2). Find the coordinates of Q and S.

(1,6) (-3,2) $m\overline{PR} = \frac{-4}{4} = -1$

⊥ diagonals $m\overline{QS} = 1$

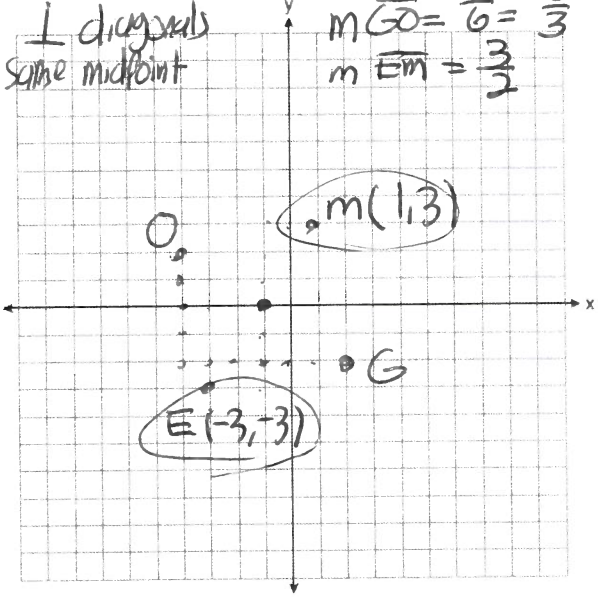
Same midpoint



8. In square *GEOM*, the coordinates of G are (2, −2) and the coordinates of *O* are (−4, 2). Determine and state the coordinates of vertices *E* and *M*.

⊥ diagonals $m\overline{GO} = \frac{-4}{6} = \frac{-2}{3}$

Same midpoint $m\overline{Em} = \frac{3}{2}$



9. Rectangle *ABCD* has two vertices at coordinates *A*(−1, −3) and *B*(6, 5). The slope of $\overline{BC}$ is

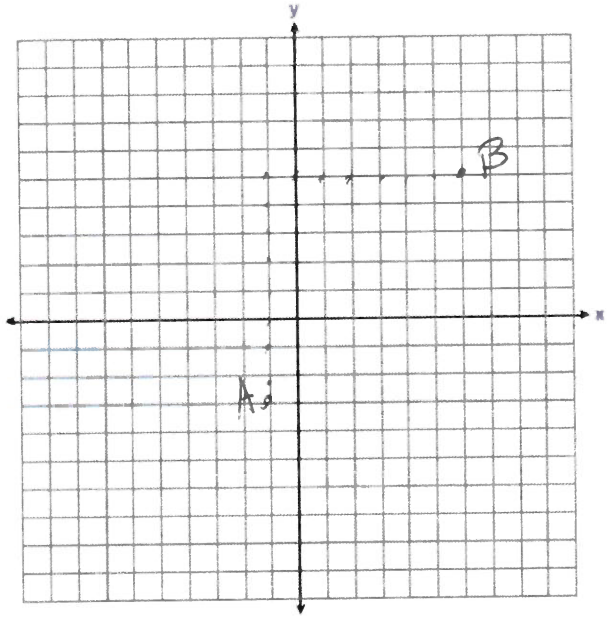
Consecutive sides ⊥ negative reciprocal slopes

1) $-\frac{7}{8}$ 3) $-\frac{8}{7}$

2) $\frac{7}{8}$ 4) $\frac{8}{7}$

$m\overline{AB} = \frac{8}{7}$

$m\perp = -\frac{7}{8}$



10. Triangle *RST* has vertices with coordinates *R*(−3, −2), *S*(3, 2) and *T*(4, −4). Determine and state an equation of the line parallel to $\overline{RT}$ that passes through point *S*.

$m\perp = \frac{-2}{7}$

$x_1 = 3$

$y_1 = 2$

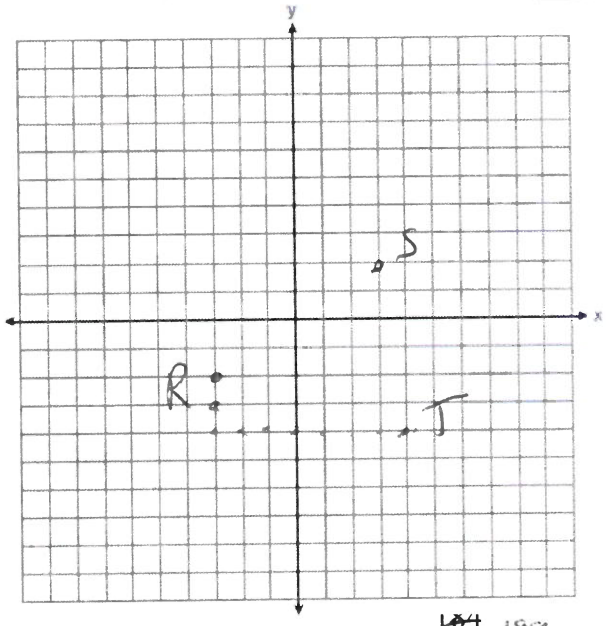
(3,2)

Same slope

$m\overline{RT} = \frac{-2}{7}$

$y - y_1 = m(x - x_1)$

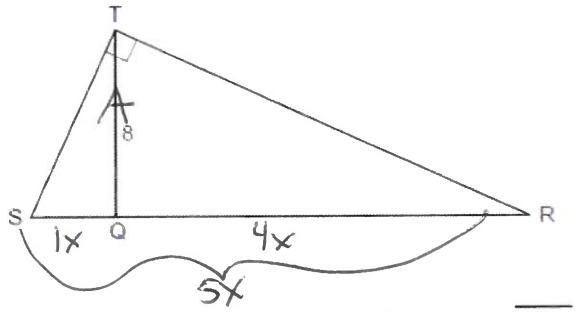
$y - 2 = \frac{-2}{7}(x - 3)$



Ratios

If you see a ratio, put an x behind each number!

1. Right triangle STR is shown below, with $m\angle T = 90°$. Altitude $\overline{TQ}$ is drawn to $\overline{SQR}$, and $TQ = 8$. If the ratio $SQ:QR$ is 1:4, determine and state the length of $\overline{SR}$.



$\frac{S}{A} = \frac{A}{S}$ $\quad \frac{4x^2}{4} = \frac{64}{4}$ $\quad \sqrt{x^2} = \sqrt{16}$ $\quad x = 4$

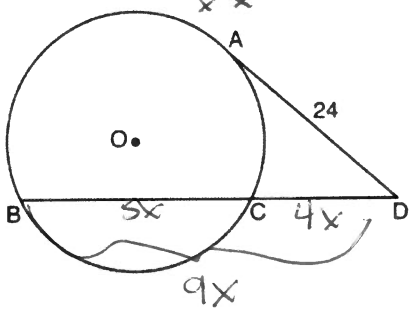
$SR = 5x$

$SR = 5(4)$

$SR = 20$

2. Circle O is drawn below with secant $\overline{BCD}$. The length of tangent $\overline{AD}$ is 24. If the ratio of $DC:CB$ is 4:5, what is the length of $\overline{CB}$?

1) 36
2) 20
3) 16
4) 4



$W \cdot E = W \cdot E$

$24 \cdot 24 = 9x \cdot 4x$

$\frac{576}{36} = \frac{36x^2}{36}$ $\quad \sqrt{16} = \sqrt{x^2}$ $\quad 4 = x$

$\overline{CB} = 5x$

$\overline{CB} = 5(4)$

$\overline{CB} = 20$

3. The ratio of the measures of the angles of a triangle is 2:3:5. Find the measure of the *smallest* angle of the triangle.

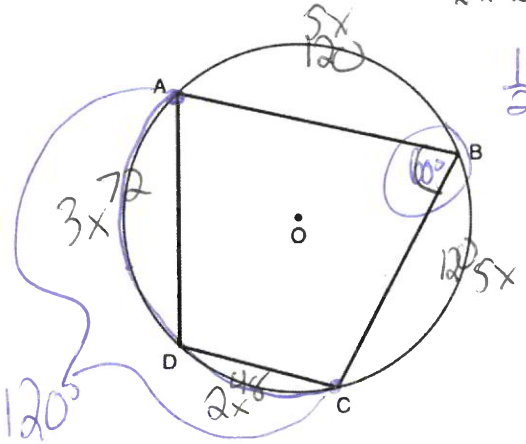
$2x + 3x + 5x = 180$

$\frac{10x}{10} = \frac{180}{10}$ $\quad x = 18$

$2x$

$2(18) = 36$

4. In the diagram below, quadrilateral $ABCD$ is inscribed in circle O, and $m\overset{\frown}{CD} : m\overset{\frown}{DA} : m\overset{\frown}{AB} : m\overset{\frown}{BC} = 2:3:5:5$. Determine and state $m\angle B$.



$\frac{1}{2}(120) = 60$

$5x + 5x + 3x + 2x = 360$

$\frac{15x}{15} = \frac{360}{15}$

$x = 24$

$5(24) = 120$

$3(24) = 72$

$2(24) = 48$

Reference Sheet for Geometry (NGLS)

Volume	Cylinder	$V = Bh$ where B is the area of the base
	General Prism	$V = Bh$ where B is the area of the base
	Sphere	$V = \frac{4}{3}\pi r^3$
	Cone	$V = \frac{1}{3}Bh$ where B is the area of the base
	Pyramid	$V = \frac{1}{3}Bh$ where B is the area of the base

Rectangular Prism $V = lwh$

Triangular Prism $V = \frac{1}{2}lwh$

Pyramid $V = \frac{1}{3}lwh$

Cylinder $V = \pi r^2 h$

Cone $V = \frac{1}{3}\pi r^2 h$