

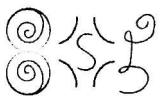
Name:

# **Proofs Regents Review!**

**Mr. Schlansky**



2025



## Triangle Proofs:

If it is not specified, prove triangles are congruent

To prove triangles are congruent, prove 3 pairs of sides/angles are congruent

To prove segments or angles, use CPCTC

\*If you get stuck, make something up and keep on going!

### 1) Do a mini proof with your givens

Line bisector creates two congruent segments

Midpoint creates two congruent segments

Angle bisector creates two congruent angles

Perpendicular lines create two congruent right angles

Parallel lines cut by a transversal create congruent alternate interior angles

\*Perpendicular bisector is perpendicular and line bisector (1 pair of congruent right angles, 1 pair of congruent segs)

\*If segments bisect each other, they are both cut in half (2 pairs of congruent segments)

\*A median intersects a segment at its midpoint

\*An altitude is perpendicular to the base

### 2) Use additional tools:

Vertical Angles are congruent (Look for an X)

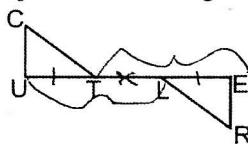
Reflexive Property (A side/angle is in both triangles and is congruent to itself)

Isosceles Triangles (In a triangle, congruent angles are opposite congruent sides)

Addition and Subtraction Property (If you need more or less of a shared side)

\*You must use three congruent statements to get one congruent statement for the triangles. The two that you are adding/subtracting and the one that you want to prove in the triangle.

Statements	Reasons
① $\overline{UL} \cong \overline{TE}$	(1) Given
② $\overline{TL} \cong \overline{TL}$	(2) Reflexive property
③ $\overline{UT} \cong \overline{LE}$	(3) Subtraction property
$UL - TL \cong TE - TL$	



Parallelogram Theorems	Circle Theorems (Look for inscribed angles)
<b>A parallelogram/rectangle/rhombus/square has:</b> Two pairs of opposite sides congruent Two pairs of opposite sides parallel Diagonals that bisect each other Opposite angles congruent	<b>Angles inscribed to the same arc are congruent</b> <b>An angle inscribed to a semicircle is a right angle</b> <b>A tangent and a radius/diameter form a right angles</b> All radii/diameters of a circle are congruent Congruent arcs have congruent chords have congruent central angles Parallel Lines intercept congruent arcs Tangents drawn from the same point are congruent
<b>A rectangle/square has:</b> Congruent right angles Congruent diagonals	
<b>A rhombus/square has:</b> Consecutive sides congruent Perpendicular diagonals Diagonals that bisect the angles	

### To prove triangles are SIMILAR, prove AA $\cong$ AA

If asked to prove a proportion/multiplication:

1) Prove triangles are similar

2) Corresponding Sides of Similar Triangle are In Proportion (CSSTIP)

3) Cross Products are Equal

Work Backwards!

$$\begin{aligned} & \text{3) } \triangle AED \sim \triangle CEB \\ & \text{4) } \frac{AE}{ED} = \frac{CE}{EB} \\ & \text{5) } AE \cdot EB = CE \cdot ED \end{aligned}$$

$$\begin{aligned} & \text{3) } AA \cong AA \\ & \text{4) } CSSTIP \\ & \text{5) } \text{Cross products are equal!} \end{aligned}$$

## Mini Proofs

If it is not specified, prove triangles are congruent

To prove triangles are congruent, prove 3 pairs of sides/angles are congruent

To prove segments or angles, use CPCTC

\*If you get stuck, make something up and keep on going!

1) Do a mini proof with your givens

Altitude creates congruent right angles

Median creates congruent segments

Line bisector creates congruent segments

Midpoint creates congruent segments

Angle bisector creates congruent angles

Perpendicular lines create congruent right angles

When given parallel lines:

Corresponding angles are congruent OR Alternate interior angles are congruent OR

Alternate exterior angles are congruent

2) Use additional tools:

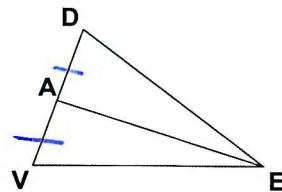
Vertical Angles are congruent (Look for an X)

Reflexive Property (A side/angle is congruent to itself)



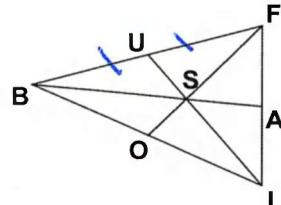
1. Given: A is the midpoint of  $\overline{DV}$

Statements	Reasons
① A is the midpoint of $\overline{DV}$	① given
② $\overline{DA} \cong \overline{AV}$	② A midpoint creates two congruent segments



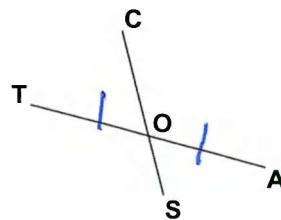
2. Given: U is the midpoint of  $\overline{BF}$

Statements	Reasons
① U is the midpoint of $\overline{BF}$	① given
② $\overline{BU} \cong \overline{UF}$	② A midpoint creates two congruent segments



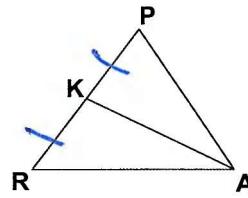
3. Given:  $\overline{CS}$  bisects  $\overline{TA}$

Statements	Reasons
① $\overline{CS}$ bisects $\overline{TA}$	① given
② $\overline{TO} \cong \overline{OA}$	② A line bisector creates two congruent segments



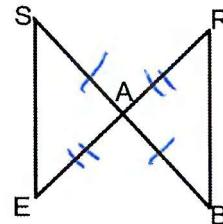
4. Given:  $\overline{KA}$  bisects  $\overline{PR}$

<u>Statements</u>	<u>Reasons</u>
(1) $\overline{KA}$ bisects $\overline{PR}$	(1) given
(2) $\overline{PK} \cong \overline{KR}$	(2) A line bisector creates two congruent segments



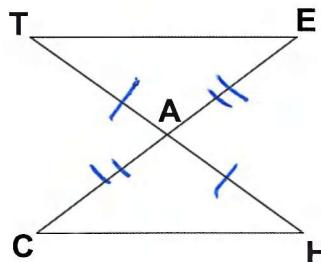
5. Given:  $\overline{SB}$  and  $\overline{RE}$  bisect each other

<u>Statements</u>	<u>Reasons</u>
(1) $\overline{SB}$ and $\overline{RE}$ bisect each other	(1) given
(2) $\overline{SA} \cong \overline{AB}$ $\overline{EA} \cong \overline{AR}$	(2) A line bisector creates two congruent segments



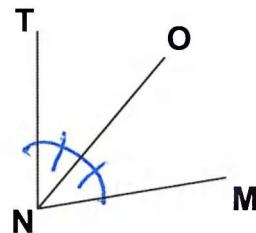
6. Given:  $\overline{TH}$  and  $\overline{CE}$  bisect each other

<u>Statements</u>	<u>Reasons</u>
(1) $\overline{TH}$ and $\overline{CE}$ bisect each other	(1) given
(2) $\overline{TA} \cong \overline{AH}$ $\overline{CA} \cong \overline{AE}$	(2) A line bisector creates two congruent segments



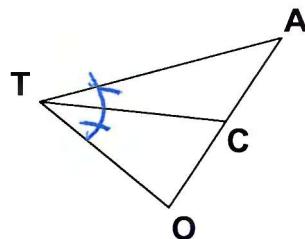
7. Given:  $\overline{ON}$  bisects  $\angle TNM$

<u>Statements</u>	<u>Reasons</u>
(1) $\overline{ON}$ bisects $\angle TNM$	(1) given
(2) $\angle TNO \cong \angle MNO$	(2) An angle bisector creates two congruent angles



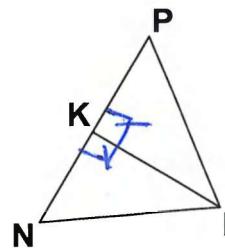
8. Given:  $\overline{CT}$  bisects  $\angle ATO$

<u>Statements</u>	<u>Reasons</u>
(1) $\overline{CT}$ bisects $\angle ATO$	(1) given
(2) $\angle ATC \cong \angle CTO$	(2) An angle bisector creates two congruent angles



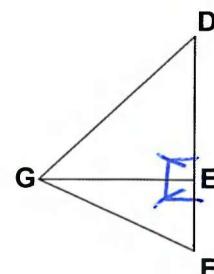
9. Given:  $\overline{IK} \perp \overline{PN}$

<u>Statements</u>	<u>Reasons</u>
① $\overline{IK} \perp \overline{PN}$	① given
② $\angle PKI \cong \angle NKI$	② Perpendicular lines form congruent right angles



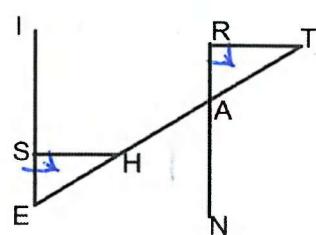
10. Given:  $\overline{GE} \perp \overline{DF}$

<u>Statements</u>	<u>Reasons</u>
① $\overline{GE} \perp \overline{DF}$	① given
② $\angle FEG \cong \angle DEG$	② Perpendicular lines form congruent right angles



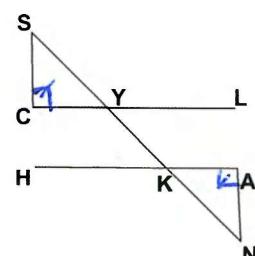
11. Given:  $\overline{IE} \perp \overline{SH}$ ,  $\overline{RN} \perp \overline{RT}$

<u>Statements</u>	<u>Reasons</u>
① $\overline{IE} \perp \overline{SH}$ , $\overline{RN} \perp \overline{RT}$	① given
② $\angle HSE \cong \angle TRA$	② Perpendicular lines form congruent right angles



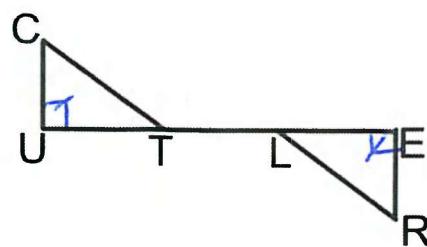
12. Given:  $\overline{CL} \perp \overline{CS}$ ,  $\overline{HA} \perp \overline{AN}$

<u>Statements</u>	<u>Reasons</u>
① $\overline{CL} \perp \overline{CS}$ , $\overline{HA} \perp \overline{AN}$	① given
② $\angle SCY \cong \angle KAN$	② Perpendicular lines form congruent right angles

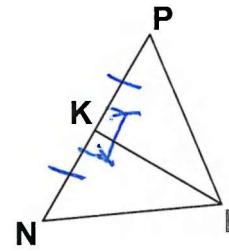


13. Given:  $\overline{CU} \perp \overline{UE}$ ,  $\overline{RE} \perp \overline{UE}$

<u>Statements</u>	<u>Reasons</u>
① $\overline{CU} \perp \overline{UE}$ , $\overline{RE} \perp \overline{UE}$	① given
② $\angle CUT \cong \angle RBL$	② Perpendicular lines form congruent right angles



14.  $\overline{IK}$  is the perpendicular bisector of  $\overline{NP}$



Statements | Reasons

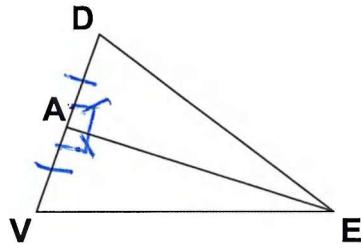
- |                                                                      |                                                     |
|----------------------------------------------------------------------|-----------------------------------------------------|
| (1) $\overline{IK}$ is the perpendicular bisector of $\overline{NP}$ | (1) Given                                           |
| (2) $\angle PKI \cong \angle NKI$                                    | (2) Perpendicular lines form congruent right angles |

- |                   |                                                    |
|-------------------|----------------------------------------------------|
| (3) $PK \cong KN$ | (3) A line bisector creates two congruent segments |
|-------------------|----------------------------------------------------|

15.  $\overline{EA}$  is the perpendicular bisector of  $\overline{DV}$

Statements | Reasons

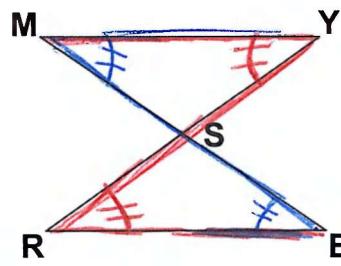
- |                                                                      |                                                     |
|----------------------------------------------------------------------|-----------------------------------------------------|
| (1) $\overline{EA}$ is the perpendicular bisector of $\overline{DV}$ | (1) Given                                           |
| (2) $\angle DAE \cong \angle VAE$                                    | (2) Perpendicular lines form congruent right angles |
| (3) $DA \cong AV$                                                    | (3) A line bisector creates two congruent segments  |



16. Given:  $\overline{MY} \parallel \overline{RE}$

Statements | Reasons

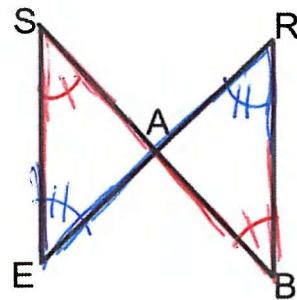
- |                                                        |                                                                                  |
|--------------------------------------------------------|----------------------------------------------------------------------------------|
| (1) $\overline{MY} \parallel \overline{RE}$            | (1) Given                                                                        |
| (2) $\angle M \cong \angle E, \angle Y \cong \angle R$ | (2) Parallel lines cut by a transversal form congruent alternate interior angles |



17. Given:  $\overline{SE} \parallel \overline{RB}$

Statements | Reasons

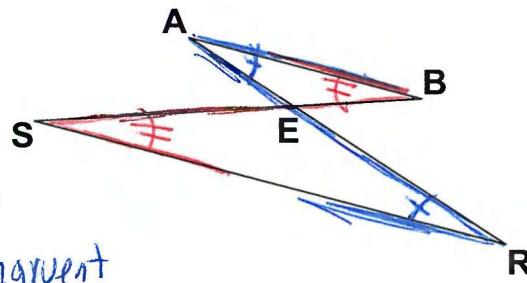
- |                                                        |                                                                                  |
|--------------------------------------------------------|----------------------------------------------------------------------------------|
| (1) $\overline{SE} \parallel \overline{RB}$            | (1) Given                                                                        |
| (2) $\angle S \cong \angle B, \angle E \cong \angle R$ | (2) Parallel lines cut by a transversal form congruent alternate interior angles |



18. Given:  $\overline{SR} \parallel \overline{AB}$

Statements | Reasons

- |                                                        |                                                                                    |
|--------------------------------------------------------|------------------------------------------------------------------------------------|
| (1) $\overline{SR} \parallel \overline{AB}$            | (1) Given                                                                          |
| (2) $\angle A \cong \angle R, \angle S \cong \angle B$ | (2) Parallel lines cut by a transversal create congruent alternate interior angles |

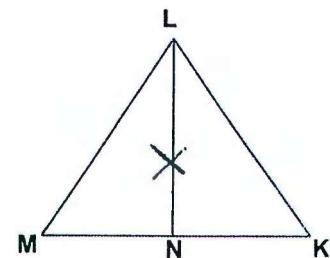


## Reflexive Property and Vertical Angles

1. Given: None

Prove:  $\triangle LNM \cong \triangle LNK$

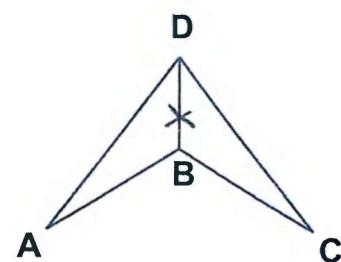
<u>Statements</u>	<u>Reasons</u>
$\triangle LNLN$	(1) Reflexive Property



2. Given: None

Prove:  $\triangle DBA \cong \triangle DBC$

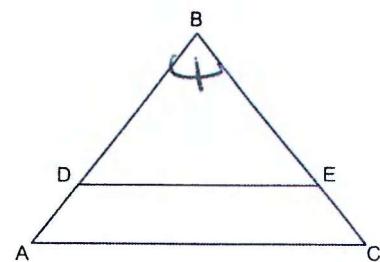
<u>Statements</u>	<u>Reasons</u>
(1) $DB \cong DB$	(1) Reflexive Property



3. Given: None

Prove:  $\triangle BDE \sim \triangle BAC$

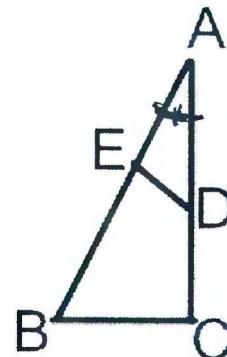
<u>Statements</u>	<u>Reasons</u>
(1) $LB \cong LB$	(1) Reflexive Property



4. Given: None

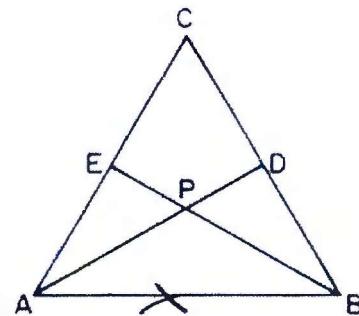
Prove:  $\triangle ABC \sim \triangle ADE$

<u>Statements</u>	<u>Reasons</u>
(1) $LA \cong LA$	(1) Reflexive Property



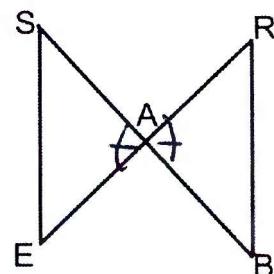
5. Given: None  
 Prove:  $\triangle AEB \cong \triangle BDA$

<u>Statements</u>	<u>Reasons</u>
(1) $\overline{AB} \cong \overline{AB}$	(1) Reflexive Property



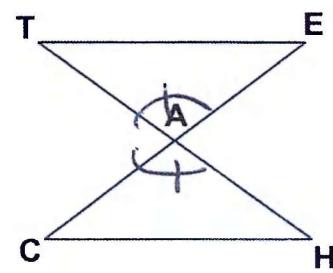
6. Given: None  
 Prove:  $\triangle SAE \cong \triangle RAB$

<u>Statements</u>	<u>Reasons</u>
(1) $\angle SAE \cong \angle RAB$	(1) Vertical angles are congruent



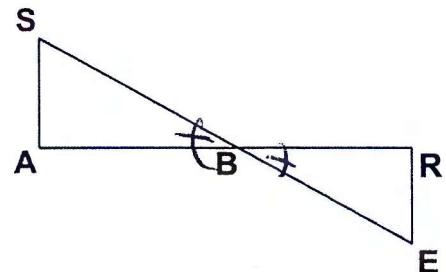
7. Given: None  
 Prove:  $\triangle TAE \cong \triangle CAH$

<u>Statements</u>	<u>Reasons</u>
(1) $\angle TAE \cong \angle CAH$	(1) Vertical angles are congruent



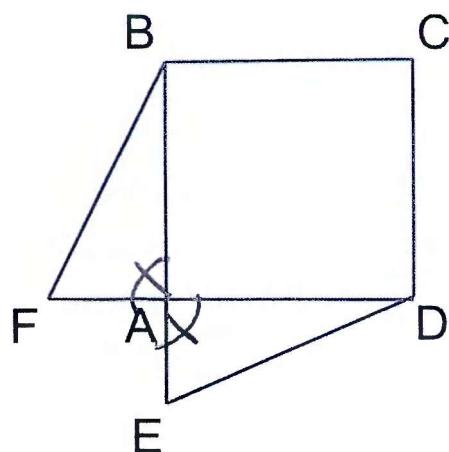
8. Given: None  
 Prove:  $\triangle SBA \cong \triangle EBR$

<u>Statements</u>	<u>Reasons</u>
(1) $\angle SBA \cong \angle EBR$	(1) Vertical angles are congruent

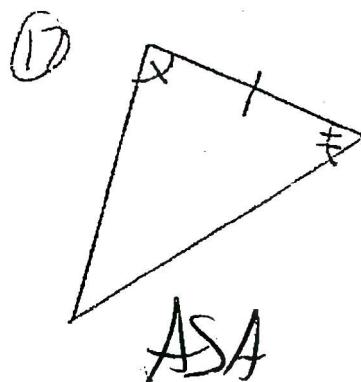
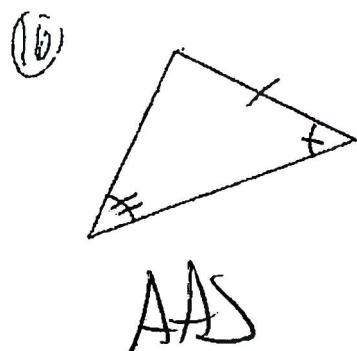
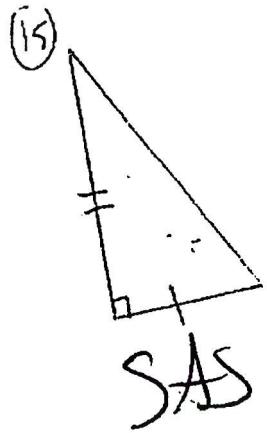
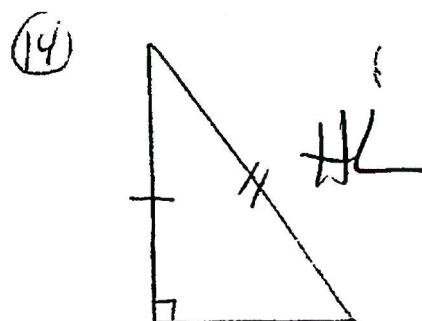
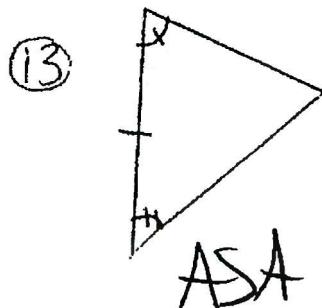
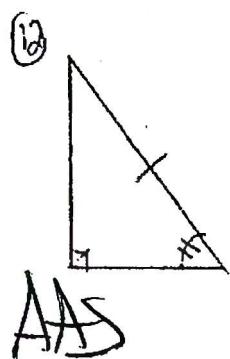
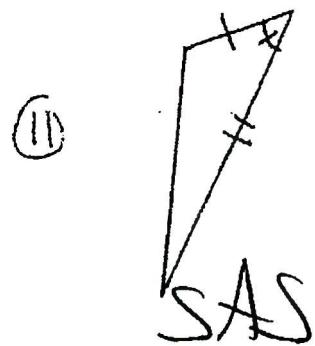
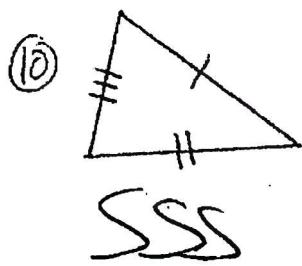
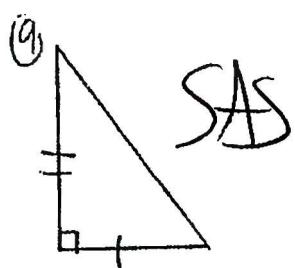
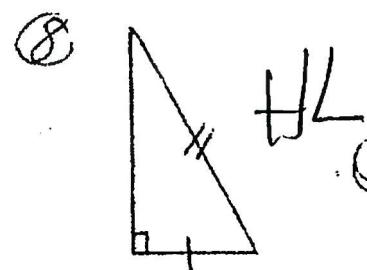
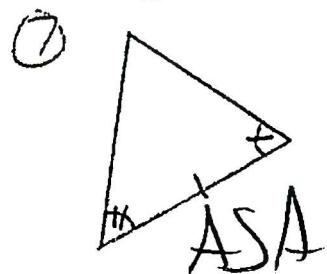
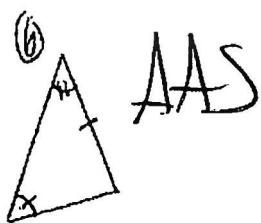


9. Given: None  
 Prove:  $\triangle BAF \cong \triangle DAE$

<u>Statements</u>	<u>Reasons</u>
(1) $\angle BAF \cong \angle DAE$	(1) Vertical angles are congruent



## Methods for Proving Triangles are Congruent



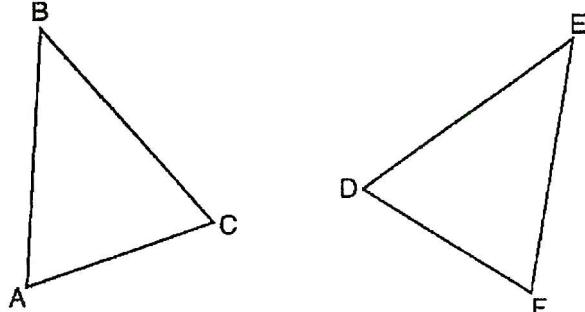
## Congruent Triangle Methods with Sequences of Rigid Motions

If a sequence of rigid motions is performed, the image is CONGRUENT to the original!

1. Which statement is sufficient evidence that  $\triangle DEF$  is congruent to  $\triangle ABC$ ?

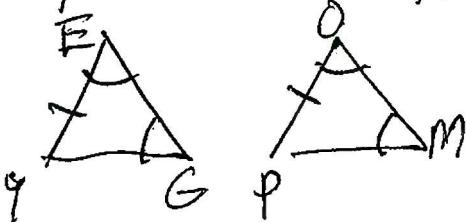
- 1)  $AB = DE$  and  $BC = EF$  **SS**
- 2)  $\angle D \cong \angle A$ ,  $\angle B \cong \angle E$ ,  $\angle C \cong \angle F$  **AAA**
- 3) There is a sequence of rigid motions that maps  $\overline{AB}$  onto  $\overline{DE}$ ,  $\overline{BC}$  onto  $\overline{EF}$ , and  $\overline{AC}$  onto  $\overline{DF}$ . **SSS**
- 4) There is a sequence of rigid motions that maps point  $A$  onto point  $D$ ,  $\overline{AB}$  onto  $\overline{DE}$ , and  $\angle B$  onto  $\angle E$ . **SA**

This is  
nothing



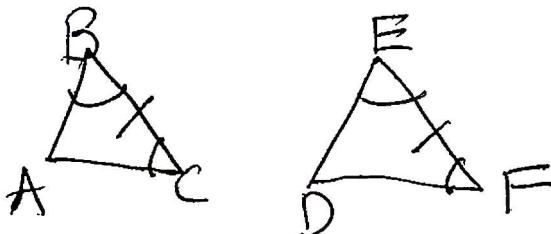
2. Triangles  $YEG$  and  $POM$  are two distinct non-right triangles such that  $\angle G \cong \angle M$ . Which statement is sufficient to prove  $\triangle YEG$  is always congruent to  $\triangle POM$ ?

- 1)  $\angle E \cong \angle O$  and  $\angle Y \cong \angle P$  **AAA**
- 2)  $\overline{YG} \cong \overline{PM}$  and  $\overline{YE} \cong \overline{PO}$  **ASS**
- 3) There is a sequence of rigid motions that maps  $\angle E$  onto  $\angle O$  and  $\overline{YE}$  onto  $\overline{PO}$ . **AA S**
- 4) There is a sequence of rigid motions that maps point  $Y$  onto point  $P$  and  $\overline{YG}$  onto  $\overline{PM}$ . **SA**



3. In the two distinct acute triangles  $ABC$  and  $DEF$ ,  $\angle B \cong \angle E$ . Triangles  $ABC$  and  $DEF$  are congruent when there is a sequence of rigid motions that maps

- 1)  $\angle A$  onto  $\angle D$ , and  $\angle C$  onto  $\angle F$  **AAA**      3)  $\angle C$  onto  $\angle F$ , and  $\overline{BC}$  onto  $\overline{EF}$  **ASA**
- 2)  $\overline{AC}$  onto  $\overline{DF}$ , and  $\overline{BC}$  onto  $\overline{EF}$  **ASS**      4) point  $A$  onto point  $D$ , and  $\overline{AB}$  onto  $\overline{DE}$  **AS**

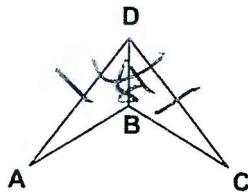


### Proving Triangles are Congruent

1. Given:  $\overline{BD}$  bisects  $\angle ADC$

$$\overline{AD} \cong \overline{DC}$$

Prove:  $\overline{AB} \cong \overline{BC}$



Statements

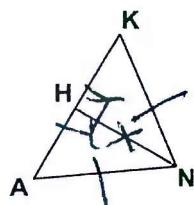
- ①  $\overline{BD}$  bisects  $\angle ADC$
- ②  $\angle ADB \cong \angle CDB$
- ③  $\overline{AD} \cong \overline{DC}$
- ④  $\overline{DB} \cong \overline{DB}$
- ⑤  $\triangle ADB \cong \triangle CDB$
- ⑥  $\overline{AB} \cong \overline{BC}$

Reasons

- (1) given
- (2) An angle bisector creates congruent angles
- (3) given
- (4) Reflexive Property
- (5) SAS
- (6) CPCTC

2. Given:  $\overline{HN} \perp \overline{KA}$ ,  $\overline{KN} \cong \overline{AN}$

Prove:  $\angle HAN \cong \angle HKN$



Statements

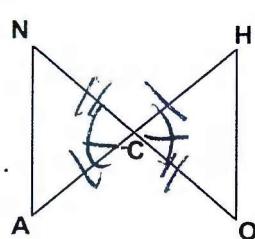
- ①  $\overline{HN} \perp \overline{KA}$
- ②  $\angle HKN \cong \angle ANH$
- ③  $\overline{KN} \cong \overline{AN}$
- ④  $\overline{HN} \cong \overline{HN}$
- ⑤  $\triangle ANH \cong \triangle KNH$
- ⑥  $\angle HAN \cong \angle HKN$

Reasons

- (1) given
- (2) Perpendicular lines form congruent right angles
- (3) given
- (4) Definition of Perp
- (5) HL
- (6) CPCTC

3. Given:  $\overline{NO}$  and  $\overline{HA}$  bisect each other

Prove:  $\overline{NA} \cong \overline{HO}$



Statements

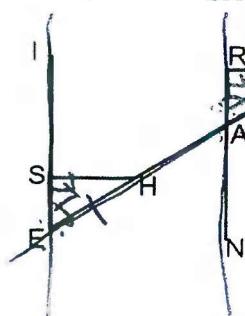
- ①  $\overline{NO}$  and  $\overline{HA}$  bisect each other
- ②  $\overline{AC} \cong \overline{CH}$ ,  $\overline{NC} \cong \overline{CO}$
- ③  $\angle NCA \cong \angle HCO$
- ④  $\triangle ANC \cong \triangle HOC$
- ⑤  $\overline{NA} \cong \overline{HO}$

Reasons

- (1) given
- (2) A line bisector creates congruent segments
- (3) Vertical angles are congruent
- (4) SAS
- (5) CPCTC

4. Given:  $\overline{IE} \parallel \overline{RN}$ ,  $\overline{TR} \perp \overline{RN}$ ,  $\overline{HS} \perp \overline{IE}$ ,  $\overline{EH} \cong \overline{AT}$

Prove:  $\overline{SH} \cong \overline{RT}$



Statements

- ①  $\overline{IE} \parallel \overline{RN}$
- ②  $\angle TAE \cong \angle HES$
- ③  $\overline{TR} \perp \overline{RN}$ ,  $\overline{HS} \perp \overline{IE}$
- ④  $\angle TIR \cong \angle HSI$
- ⑤  $\angle HS \cong \angle AT$
- ⑥  $\angle HSE \cong \angle TRA$
- ⑦  $\overline{SH} \cong \overline{RT}$

Reasons

- (1) given
- (2) Parallel lines cut by a transversal create congruent corresponding angles
- (3) given
- (4) Perpendicular lines form congruent right angles
- (5) given
- (6) AAS
- (7) CPCTC

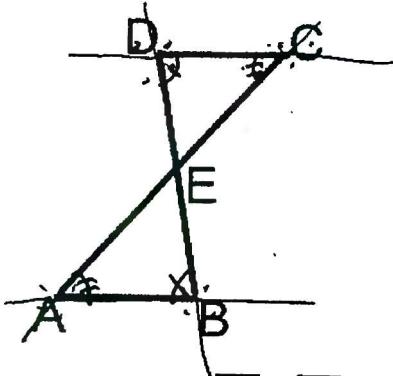
To prove triangles are SIMILAR, prove AA  $\cong$  AA  
 If asked to prove a proportion/multiplication:

- 1) Prove triangles are similar
- 2) Corresponding Sides of Similar Triangle are In Proportion (CSSTIP)
- 3) Cross Products are Equal

Work Backwards!

1. Given  $\overline{AB} \parallel \overline{DC}$

Prove:  $\frac{\overline{DC}}{\overline{DE}} \cdot \frac{\overline{EB}}{\overline{AB}} = \frac{\overline{AB}}{\overline{DE}} \cdot \frac{\overline{DC}}{\overline{EB}}$



Statements

$$\textcircled{1} \overline{AB} \parallel \overline{DC}$$

$$\textcircled{2} \angle D \cong \angle B$$

$$\angle C \cong \angle A$$

Reasons

$\textcircled{1}$  given

$\textcircled{2}$  Parallel lines cut by a transversal create congruent alternate interior angles

$$\textcircled{3} \triangle DCE \sim \triangle BAE$$

$$\textcircled{3} AA \cong AA$$

$$\textcircled{4} \frac{\overline{DC}}{\overline{DE}} = \frac{\overline{AB}}{\overline{EB}}$$

$$\textcircled{4} CSSTIP$$

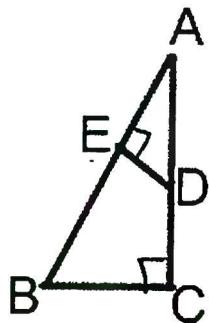
$$\textcircled{5} \overline{DC} \cdot \overline{EB} = \overline{AB} \cdot \overline{DE}$$

$$\textcircled{5} \text{ Cross products are equal}$$

2. Given:  $\overline{BC} \perp \overline{AC}$

$\overline{DE} \perp \overline{AB}$

Prove:  $\frac{\overline{AC}}{\overline{AD}} \cdot \frac{\overline{AB}}{\overline{AE}} = \frac{\overline{AB}}{\overline{AD}} \cdot \frac{\overline{AC}}{\overline{AE}}$



Statements

$$\textcircled{1} \overline{BC} \perp \overline{AC}, \overline{DE} \perp \overline{AB}$$

$$\textcircled{2} \angle ACB \cong \angle AED$$

$$\textcircled{3} \angle A \cong \angle A$$

Reasons

$\textcircled{1}$  given

$\textcircled{2}$  Perpendicular lines form congruent right angles

$\textcircled{3}$  Reflexive Property

$$\textcircled{4} \triangle ACB \sim \triangle AED$$

$$\textcircled{4} AA \cong AA$$

$$\textcircled{5} \frac{\overline{AC}}{\overline{AB}} = \frac{\overline{AE}}{\overline{AD}}$$

$$\textcircled{5} CSSTIP$$

$$\textcircled{6} \overline{AC} \cdot \overline{AD} = \overline{AE} \cdot \overline{AB}$$

$$\textcircled{6} \text{ Cross products are equal}$$

3. Given:  $\angle HCE \cong \angle LIE$

Prove:  $\overline{CE} \cdot \overline{IL} = \overline{CH} \cdot \overline{EI}$

Statements

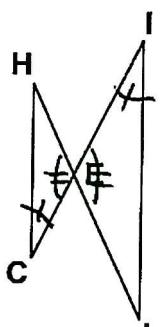
$$\textcircled{1} \angle HCE \cong \angle LIE$$

$$\textcircled{2} \angle HEC \cong \angle LIE$$

Reasons

$\textcircled{1}$  given

$\textcircled{2}$  Vertical angles are congruent



$$\textcircled{3} \triangle HCE \sim \triangle LIE$$

$$\textcircled{3} AA \cong AA$$

$$\textcircled{4} \frac{\overline{CE}}{\overline{EI}} = \frac{\overline{CH}}{\overline{IL}}$$

$$\textcircled{4} CSSTIP$$

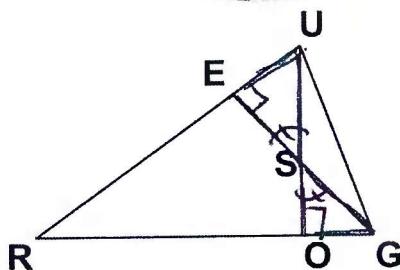
$$\textcircled{5} \overline{CE} \cdot \overline{IL} = \overline{CH} \cdot \overline{EI}$$

$$\textcircled{5} \text{ Cross products are equal}$$

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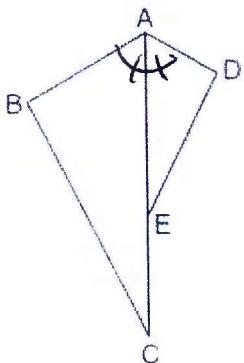
4. Given:  $\overline{UO} \perp \overline{RG}$ ,  $\overline{UR} \perp \overline{EG}$

$$\text{Prove: } \frac{\overline{US}}{\overline{SO}} = \frac{\overline{EU}}{\overline{OG}}$$



5. Given:  $\overline{CA}$  bisects  $\angle BAD$ ,  $\angle ABC \cong \angle ADE$

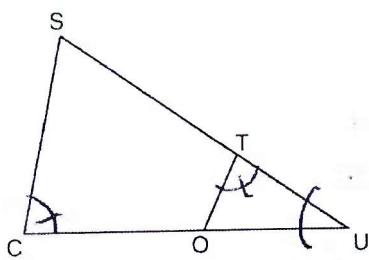
$$\text{Prove: } \overline{BC} \cdot \overline{AE} = \overline{DE} \cdot \overline{AC}$$



6. In  $\triangle SCU$  shown below, points  $T$  and  $O$  are

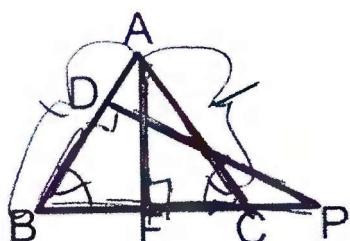
on  $\overline{SU}$  and  $\overline{CU}$ , respectively. Segment  $OT$  is drawn so that  $\angle C \cong \angle OTU$ .

$$\text{Prove: } \overline{SC} \cdot \overline{OU} = \overline{OT} \cdot \overline{SU}$$



7. Given:  $\overline{AB} \cong \overline{AC}$ ,  $\overline{AF} \perp \overline{BC}$ ,  $\overline{PD} \perp \overline{AB}$

$$\text{Prove: } \overline{FC} \cdot \overline{PB} = \overline{DB} \cdot \overline{AC}$$



### Statements

- ①  $\overline{UO} \perp \overline{RG}$ ,  $\overline{UR} \perp \overline{EG}$
- ②  $\angle USE \cong \angle SOG$
- ③  $\angle USE \cong \angle SOG$
- ④  $\triangle USE \sim \triangle SOG$
- ⑤  $\frac{\overline{US}}{\overline{SO}} = \frac{\overline{EU}}{\overline{OG}}$

### Reasons

- (1) given
- (2) perpendicular lines form congruent right angles
- (3) vertical angles are congruent
- (4) AA  $\cong$  AA
- (5) CSSTIP

### Statements

- ①  $\overline{CA}$  bisects  $\angle BAD$
- ②  $\angle BAC \cong \angle DAE$
- ③  $\angle ABC \cong \angle ADE$

### Reasons

- (1) given
- (2) an angle bisector creates  $\cong$  angles
- (3) given

- ④  $\triangle BAC \sim \triangle DAE$

- (4) AA  $\cong$  AA

$$\text{⑤ } \frac{\overline{BC}}{\overline{AC}} = \frac{\overline{DE}}{\overline{AE}}$$

- (5) CSSTIP

$$\text{⑥ } \overline{BC} \cdot \overline{AE} = \overline{DE} \cdot \overline{AC}$$

- (6) cross products are equal

### Statements

- ①  $\angle C \cong \angle OTU$
- ②  $\angle U \cong \angle U$

### Reasons

- (1) given
- (2) Reflexive Property

- ③  $\triangle SCU \sim \triangle OTU$

- (3) AA  $\cong$  AA

$$\text{④ } \frac{\overline{SC}}{\overline{SU}} = \frac{\overline{OT}}{\overline{OU}}$$

- (4) CSSTIP

$$\text{⑤ } \overline{SC} \cdot \overline{OU} = \overline{OT} \cdot \overline{SU}$$

- (5) cross products are equal

### Statements

- ①  $\overline{AB} \cong \overline{AC}$

### Reasons

- (1) given

- ②  $\angle B \cong \angle C$

- (2) in a triangle, congruent angles are opposite congruent sides

- ③ given

- ④ perpendicular lines form congruent right angles

- (4) given

- ⑤  $\triangle FCA \sim \triangle DBP$

- (5) AA  $\cong$  AA

$$\text{⑥ } \frac{\overline{FC}}{\overline{AC}} = \frac{\overline{DB}}{\overline{PB}}$$

- (6) CSSTIP 181 153

$$\text{⑦ } \overline{FC} \cdot \overline{PB} = \overline{DB} \cdot \overline{AC}$$

- (7) cross products are equal

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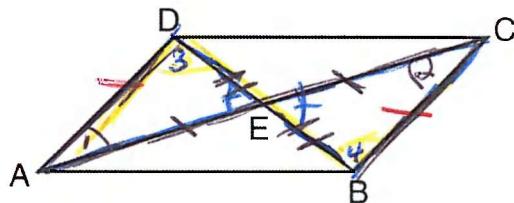


## Euclidean Proofs with Parallelogram and Circle Theorems

Parallelogram Theorems	Circle Theorems
<b>Parallelogram Properties</b>	<b>All radii/diameters of a circle are congruent</b> <b>Angles inscribed to the same arc are congruent</b> <b>An angle inscribed to a semicircle is a right angle</b> <b>A tangent and a radius/diameter form a right angles</b> <b>Congruent arcs have congruent chords have congruent central angles</b> <b>Parallel Lines intercept congruent arcs</b> <b>Tangents drawn from the same point are congruent</b>

1. Given: Parallelogram  $ABCD$ .

Prove:  $\triangle AED \cong \triangle CEB$



3. Given: ABCD is a rectangle, M is the midpoint of  $\overline{AC}$

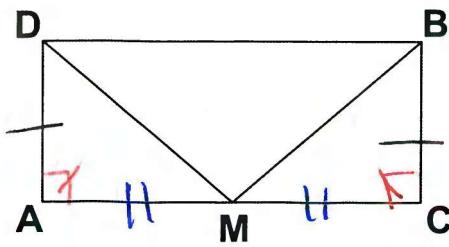
Prove:  $\overline{DM} \cong \overline{BM}$

Statements

- (1) ABCD is a rectangle
- (2)  $\angle A \cong \angle C$
- (3)  $\overline{DA} \cong \overline{BC}$
- (4) M is the midpoint of  $\overline{AC}$
- (5)  $\overline{AM} \cong \overline{MC}$
- (6)  $\triangle DAM \cong \triangle BCM$
- (7)  $\overline{DM} \cong \overline{BM}$

Reasons

- (1) given
- (2) A rectangle has congruent right angles
- (3) A rectangle has opposite sides  $\cong$
- (4) given
- (5) A midpoint creates two congruent segments
- (6) SAS  $\cong$  SAS
- (7) CPCTC



4. Given: SACK is a parallelogram,  $\overline{HL} \perp \overline{SA}$ ,  $\overline{YN} \perp \overline{KC}$ ,  $\overline{HL} \cong \overline{NY}$

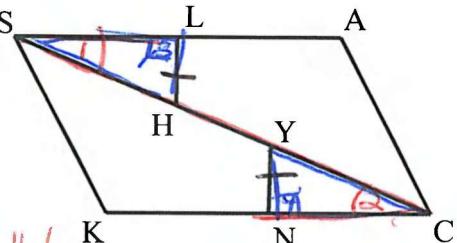
Prove:  $\overline{SL} \cong \overline{CN}$

Statements

- (1) SACK is a parallelogram
- (2)  $\angle 1 \cong \angle 2$
- (3)  $\overline{HL} \perp \overline{SA}$ ,  $\overline{YN} \perp \overline{KC}$
- (4)  $\angle 3 \cong \angle 4$
- (5)  $\overline{HL} \cong \overline{NY}$
- (6)  $\triangle SLH \cong \triangle CNY$
- (7)  $\overline{SL} \cong \overline{CN}$

Reasons

- (1) given
- (2) A parallelogram has parallel lines cut by a transversal which create  $\cong$  alternate interior angles
- (3) given
- (4) perpendicular lines form congruent right angles
- (5) given
- (6) AAS  $\cong$  AAS
- (7) CPCTC



5. Given: Parallelogram  $ABCD$ ,  $\overline{BF} \perp \overline{AFD}$ , and  $\overline{DE} \perp \overline{BEC}$

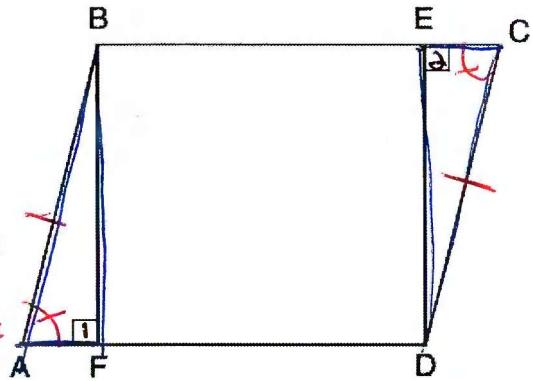
Prove:  $\overline{AF} \cong \overline{EC}$

Statements

- (1) Parallelogram  $ABCD$
- (2)  $\overline{AB} \cong \overline{DC}$
- (3)  $\angle A \cong \angle C$
- (4)  $\overline{BF} \perp \overline{AFD}$ ,  $\overline{DE} \perp \overline{BEC}$
- (5)  $\angle 1 \cong \angle 2$
- (6)  $\triangle AFB \cong \triangle CED$
- (7)  $\overline{AF} \cong \overline{EC}$

Reasons

- (1) Given
- (2) A parallelogram has opposite sides congruent
- (3) A parallelogram has opposite angles congruent
- (4) Given
- (5) Perpendicular lines form congruent right angles
- (6) AAS  $\cong$  AAS
- (7) CPCTC



6. Given:  $ABCD$  is a parallelogram,  $\overline{BE} \perp \overline{AC}$ , and  $\overline{DF} \perp \overline{AC}$ .

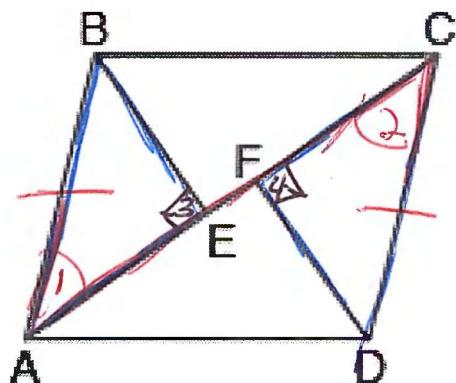
Prove:  $\angle ABE \cong \angle CDF$

Statements

- (1)  $ABCD$  is a parallelogram
- (2)  $\angle 1 \cong \angle 2$
- (3)  $\overline{AB} \cong \overline{DC}$
- (4)  $\overline{BE} \perp \overline{AC}$ ,  $\overline{DF} \perp \overline{AC}$
- (5)  $\angle 3 \cong \angle 4$
- (6)  $\triangle BEA \cong \triangle DCF$
- (7)  $\angle ABE \cong \angle CDF$

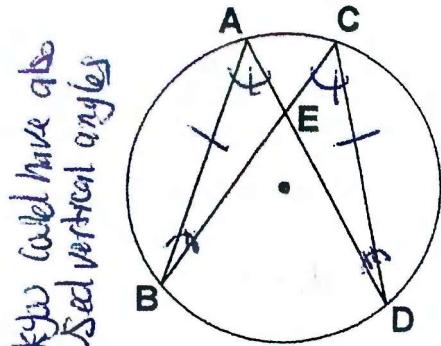
Reasons

- (1) Given
- (2) A parallelogram has parallel lines cut by a transversal which create congruent alternate interior angles
- (3) A parallelogram has opposite sides congruent
- (4) Given
- (5) Perpendicular lines form congruent right angles
- (6) AAS  $\cong$  AAS
- (7) CPCTC



# \*Look for inscribed angles

7. Given: Chords  $\overline{AD}$  and  $\overline{BC}$  of circle O intersect at E,  $\overline{AB} \cong \overline{CD}$   
 Prove:  $\overline{BC} \cong \overline{AD}$



Statements

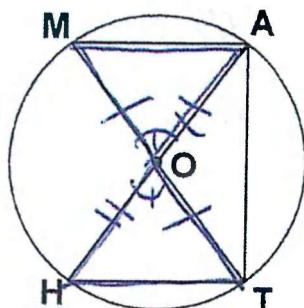
- ①  $\overline{AB} \cong \overline{CD}$
- ②  $\angle BAE \cong \angle DCE$   
 $\angle ABE \cong \angle CDE$
- ③  $\triangle BAE \cong \triangle DCE$
- ④  $\overline{BC} \cong \overline{AD}$

Reasons

- ① Given
- ② Angles inscribed to the same arc are congruent
- ③ SAS  $\cong$  SAS
- ④ CACTE

8. Given: Circle O with diameters  $\overline{MOT}$  and  $\overline{AOH}$ .

Prove:  $\overline{MA} \cong \overline{HT}$



Statements

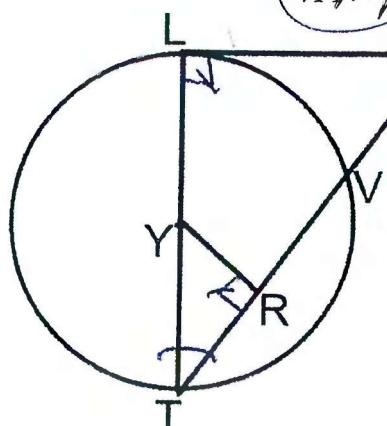
- ①  $\overline{MO} \cong \overline{OT}, \overline{AO} \cong \overline{OH}$
- ②  $\angle MOA \cong \angle HOA$
- ③  $\angle MOT \cong \angle TOH$
- ④  $\overline{MA} \cong \overline{HT}$

Reasons

- ① All radii of a circle are  $\cong$
- ② Vertical angles are  $\cong$
- ③ SAS  $\cong$  SAS
- ④ CACTE

9. In circle Y, tangent  $\overline{LE}$  is drawn to diameter  $\overline{TYL}$  and  $\overline{YR} \perp \overline{TE}$ . Prove that  $\overline{TR} \parallel \overline{TL}$ ,  $TE \cdot TR = TL \cdot TY$

(Tangent  $\overline{LE}$  is drawn to diameter  $\overline{TYL}$ , look backwards)  
 $\overline{YR} \perp \overline{TE}$ ,  $TR \cdot TE = TL \cdot TY$



Statements

- ① Tangent  $\overline{LE}$  is drawn to diameter  $\overline{TYL}$
- ②  $\overline{YR} \perp \overline{TE}$
- ③  $\angle RTY \cong \angle RTY$

Reasons

- ① Given
- ② An angle formed by a tangent and diameter and perpendicular lines form congruent right angles
- ③ Reflexive Property

④  $\triangle TLE \cong \triangle TRY$   
 SIE  $\frac{TL}{TR} = \frac{TY}{TR}$   
 $TY \cdot TE \cdot TR = TL \cdot TY$

⑤  $\triangle A \cong \triangle A$

⑥ SSS TIP

∴ cross products are equal

11. In the diagram below, secant  $\overline{ACD}$  and tangent  $\overline{AB}$  are drawn from external point  $A$  to circle  $O$ .

Prove the theorem: If a secant and a tangent are drawn to a circle from an external point, the product of the lengths of the secant segment and its external segment equals the length of the tangent segment squared. ( $AC \cdot AD = AB^2$ ) *work backwardly*

Statements	Reasons
$\overline{AB} \perp \overline{CD}$	(1) <del>An <math>\angle</math> inscribed in a semi-circle is a right angle</del> can be defined
$\angle BAC \cong \angle BAC$	(2) Reflexive Property
$\angle BDC \cong \angle ABC$	(3) Angles inscribed to the same arc are $\cong$
$\triangle ACB \sim \triangle ABD$	$\triangle A \cong \triangle A$
$\frac{AC}{AB} = \frac{AB}{AD}$	CSSS TPP
$AC \cdot AD = AB^2$	(4) Cross products are equal

12. Given: Circle  $O$ , chords  $\overline{AB}$  and  $\overline{CD}$  intersect at  $E$

Theorem: If two chords intersect in a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord. Prove this theorem by proving  $AE \cdot EB = CE \cdot ED$ . *work backwardly*

Statements	Reasons
$\overline{AB} \cap \overline{CD}$	(1) <del>An <math>\angle</math> inscribed in a semi-circle is a right angle</del> can be defined
$\angle BCO \cong \angle DAB$ $\angle CBE \cong \angle ADE$	(2) Angles inscribed to the same arc are $\cong$
$\triangle AED \sim \triangle CEB$	$\triangle A \cong \triangle A$
$\frac{AE}{ED} = \frac{CE}{EB}$	CSSS TPP
$AE \cdot EB = CE \cdot ED$	(4) Cross products are equal

\*you could have also used  
vertical angles

Name Schlansky  
Mr. Schlansky

Date \_\_\_\_\_  
Geometry

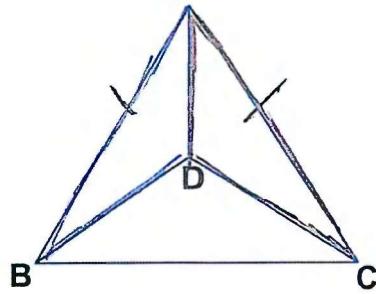


## Proving Triangles Isosceles Mini Proofs

1. Given:  $\triangle ADB \cong \triangle ADC$

Prove:  $\triangle BAC$  is isosceles

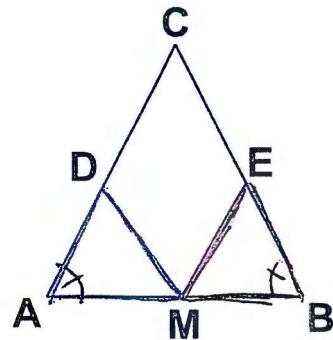
Statements	Reasons
(1) $\triangle ADB \cong \triangle ADC$	(1) Given
(2) $\overline{AB} \cong \overline{AC}$	(2) CPCTC
(3) $\triangle BAC$ is isosceles	(3) An isosceles triangle has two congruent sides.



2. Given:  $\triangle ADM \cong \triangle BEM$

Prove:  $\triangle ACB$  is isosceles

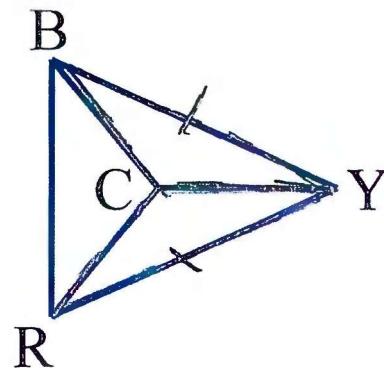
Statements	Reasons
(1) $\triangle ADM \cong \triangle BEM$	(1) Given
(2) $\angle DAM \cong \angle EBM$	(2) CPCTC
(3) $\triangle ACB$ is isosceles	(3) An isosceles triangle has two congruent angles



3. Given:  $\triangle YCB \cong \triangle YCR$

Prove:  $\triangle BYR$  is isosceles

Statements	Reasons
(1) $\triangle YCB \cong \triangle YCR$	(1) Given
(2) $\overline{YB} \cong \overline{YR}$	(2) CPCTC
(3) $\triangle BYR$ is isosceles	(3) An isosceles triangle has two congruent sides



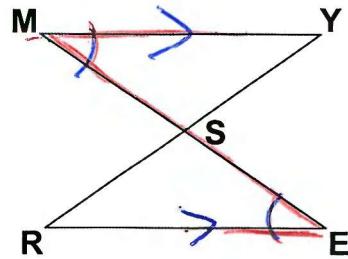
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## Proving Parallel Mini Proofs (PR2)

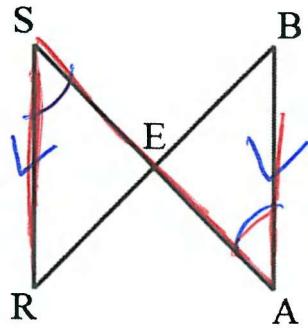
1. Given:  $\angle M \cong \angle E$

Statements	Reasons
① $\angle M \cong \angle E$	① given
② $\overline{MY} \parallel \overline{RE}$	② parallel lines cut by a transversal form congruent alternate interior angles



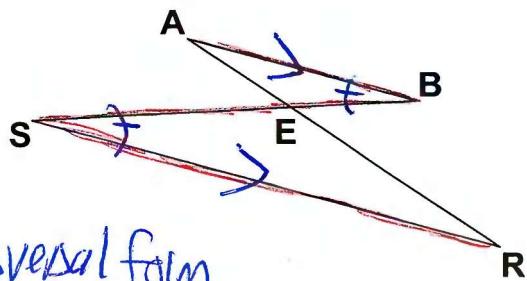
2. Given:  $\angle S \cong \angle A$

Statements	Reasons
① $\angle S \cong \angle A$	① given
② $\overline{SR} \parallel \overline{BA}$	② parallel lines cut by a transversal form congruent alternate interior angles



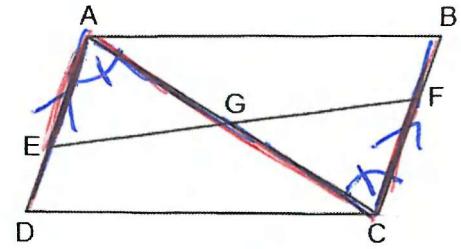
3. Given:  $\angle S \cong \angle B$

Statements	Reasons
① $\angle S \cong \angle B$	① given
② $\overline{AB} \parallel \overline{SR}$	② parallel lines cut by a transversal form congruent alternate interior angles



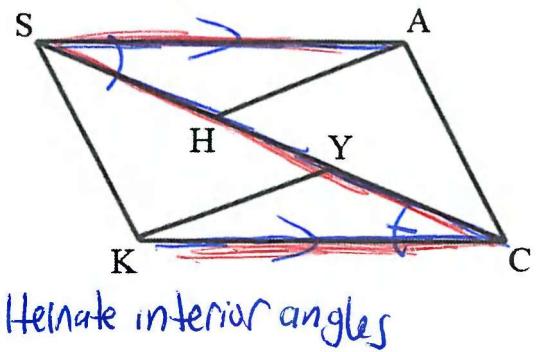
4. Given:  $\angle EAG \cong \angle FCG$

<u>Statements</u>	<u>Reasons</u>
(1) $\angle EAG \cong \angle FCG$	(1) given
(2) $\overline{DA} \parallel \overline{CB}$	(2) Parallel lines cut by a transversal create congruent alternate interior angles



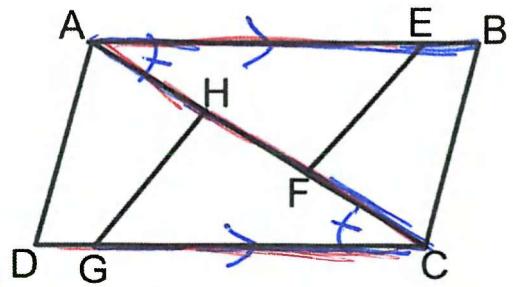
5. Given:  $\angle ASH \cong \angle KCY$

<u>Statements</u>	<u>Reasons</u>
(1) $\angle ASH \cong \angle KCY$	(1) given
(2) $\overline{SA} \parallel \overline{KC}$	(2) Parallel lines cut by a transversal create congruent alternate interior angles



6. Given:  $\angle EAF \cong \angle GCH$

<u>Statements</u>	<u>Reasons</u>
(1) $\angle EAF \cong \angle GCH$	(1) given
(2) <del><math>\overline{AD} \parallel \overline{BC}</math></del>	(2) Parallel lines cut by a transversal create congruent alternate interior angles



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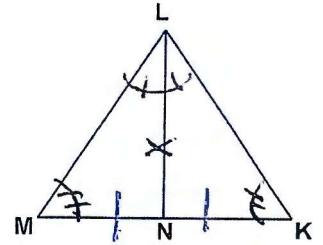
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## Triangle Proofs Using CPCTC

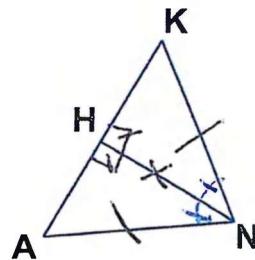
1. Given:  $\overline{LN}$  bisects  $\angle KLM$   
 $\angle LKM \cong \angle LMK$

Prove: N is the midpoint of  $\overline{MK}$



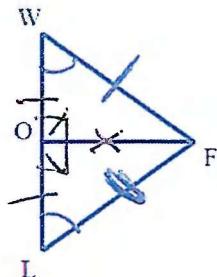
Statements	Reasons
(1) $\overline{LN}$ bisects $\angle KLM$	(1) Given
(2) $\angle MLN \cong \angle LKN$	(2) An angle bisector creates two congruent angles
(3) $\angle LKM \cong \angle LMK$	(3) Given
(4) $\overline{LN} \cong \overline{LN}$	(4) Reflexive Property
(5) $\triangle MLN \cong \triangle KLN$	(5) AAS $\cong$ AAS
(6) $MN \cong KN$	(6) CPCTC
(7) N is the midpoint of $\overline{MK}$	(7) A midpoint creates two congruent segments.

2. Given:  $\overline{HN} \perp \overline{KA}$ ,  $\overline{KN} \cong \overline{AN}$   
 Prove:  $\overline{HN}$  bisects  $\angle KNA$



Statements	Reasons
(1) $\overline{HN} \perp \overline{KA}$	(1) Given
(2) $\angle KHN \cong \angle ANH$	(2) Perpendicular lines form congruent right angles.
(3) $\overline{KN} \cong \overline{AN}$	(3) Given
(4) $\overline{HN} \cong \overline{HN}$	(4) Reflexive Property
(5) $\triangle KHN \cong \triangle ANH$	(5) HL $\cong$ HL
(6) $\angle KNH \cong \angle ANH$	(6) CPCTC
(7) $\overline{HN}$ bisects $\angle KNA$	(7) An angle bisector creates two congruent angles

3. Given:  $\overline{OF}$  is the perpendicular bisector of  $\overline{WL}$   
 Prove:  $\triangle WFL$  is isosceles

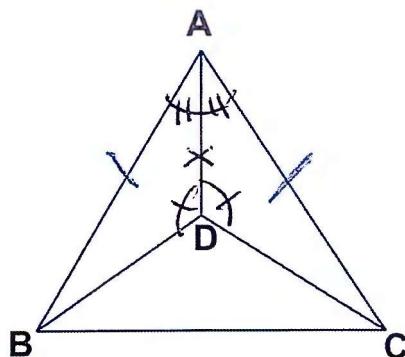


Statements	Reasons
(1) $\overline{OF}$ is the perpendicular bisector of $\overline{WL}$	(1) given
(2) $\overline{WO} \cong \overline{OL}$	(2) A line bisector creates two congruent segments
(3) $\angle WOF \cong \angle LOF$	(3) Perpendicular lines form congruent right angles
(4) $\overline{OF} \cong \overline{OF}$	(4) Reflexive Property
(5) $\triangle WOF \cong \triangle LOF$	(5) SAS $\cong$ SAS
(6) $\overline{WF} \cong \overline{LF}$ (7) $\angle W \cong \angle L$	(6) CPCTC (7) An isosceles triangle has two congruent sides
(7) $\triangle WFL$ is isosceles	

4. Given:  $\angle ADB \cong \angle ADC$

$\overline{AD}$  bisects  $\angle BAC$

Prove:  $\triangle ABC$  is isosceles

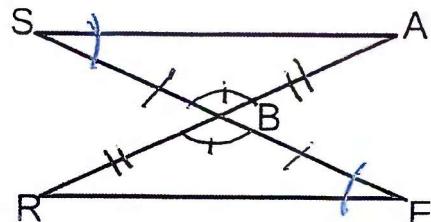


Statements	Reasons
(1) $\angle ADB \cong \angle ADC$	(1) given
(2) $\overline{AD}$ bisects $\angle BAC$	(2) given
(3) $\angle BAD \cong \angle CAD$	(3) An angle bisector creates two congruent angles
(4) $\overline{AD} \cong \overline{AD}$	(4) Reflexive Property
(5) $\triangle ADB \cong \triangle ADC$	(5) <del>AAA</del> ASA $\cong$ ASA
(6) $\overline{AB} \cong \overline{AC}$	(6) CPCTC
(7) $\triangle ABC$ is isosceles	(7) An isosceles triangle has two congruent sides

5. Given:  $\overline{SE}$  and  $\overline{AR}$  bisect each other.

Prove that  $\overline{SA} \parallel \overline{RE}$

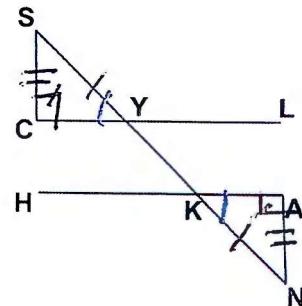
<u>statements</u>	<u>Reasons</u>
① $\overline{SE}$ and $\overline{AR}$ bisect each other	① given ② A line bisector creates two congruent sides
② $\overline{SB} \cong \overline{BE}, \overline{RB} \cong \overline{BA}$	③ Vertical angles are congruent
③ $\angle SBA \cong \angle EBR$	④ SAS $\cong$ SAS
④ $\triangle SBA \cong \triangle EBR$	⑤ CPCTC
⑤ $\angle S \cong \angle E$	⑥ Parallel lines cut by a transversal create congruent alternate interior angles.
⑥ $\overline{SA} \parallel \overline{RE}$	



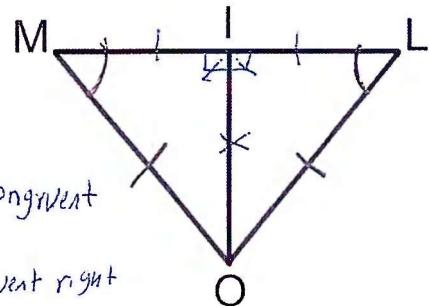
6. Given:  $\overline{SC} \perp \overline{CL}$ ,  $\overline{HA} \perp \overline{AN}$ ,  $\overline{SY} \cong \overline{KN}$ , and  $\overline{SC} \cong \overline{AN}$ .

Prove  $\overline{CL} \parallel \overline{HA}$

<u>statements</u>	<u>Reasons</u>
① $\overline{SC} \perp \overline{CL}, \overline{HA} \perp \overline{AN}$	① given
② $\angle SCY \cong \angle NAK$	② Perpendicular lines form congruent right angles
③ $\overline{SY} \cong \overline{KN}$	③ given
④ $\overline{SC} \cong \overline{AN}$	④ given
⑤ $\triangle SCY \cong \triangle NAK$	⑤ HL $\cong$ HL
⑥ $\angle SYC \cong \angle NKA$	⑥ CPCTC
⑦ $\overline{CL} \parallel \overline{HA}$	⑦ Parallel lines cut by a transversal create congruent alternate exterior angles.

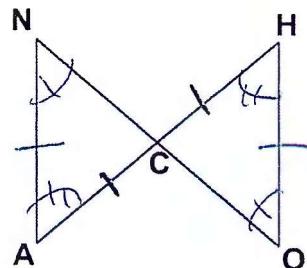


7. Given:  $\overline{OI}$  is the perpendicular bisector of  $\overline{ML}$   
 Prove:  $\triangle MLO$  is isosceles



Statements	Reasons
(1) $\overline{OI}$ is the perpendicular bisector of $\overline{ML}$	(1) given
(2) $\overline{MI} \cong \overline{IL}$	(2) A line bisector creates two congruent segments
(3) $\angle MIO \cong \angle LIO$	(3) Perpendicular lines create congruent right angles.
(4) $\overline{IO} \cong \overline{IO}$	(4) Reflexive Property
(5) $\triangle MIO \cong \triangle LIO$	(5) SAS $\cong$ SAS
(6) $\angle IMO \cong \angle LIO$ or $\angle MOI \cong \angle LIO$	(6) CPCTC
(7) $\triangle MLO$ is isosceles	(7) An isosceles triangle has two congruent angles

8. Given:  $\overline{NA} \parallel \overline{HO}$ ,  $\overline{NA} \cong \overline{HO}$   
 Prove:  $\overline{NO}$  bisects  $\overline{HA}$



Statements	Reasons
(1) $\overline{NA} \parallel \overline{HO}$	(1) given
(2) $\angle N \cong \angle O$ , $\angle A \cong \angle H$	(2) Parallel lines cut by a transversal create congruent alternate interior angles.
(3) $\overline{NA} \cong \overline{HO}$	(3) given
(4) $\triangle NAC \cong \triangle OHC$	(4) ASA $\cong$ ASA
(5) $\overline{AC} \cong \overline{CH}$	(5) CPCTC
(6) $\overline{NO}$ bisects $\overline{HA}$	(6) A line bisector creates two congruent segments

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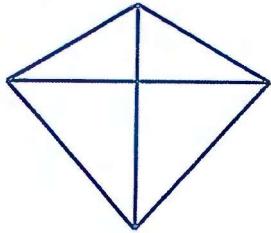
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## Perpendicular Bisector Multiple Choice

Perpendicular bisector creates

- two pairs of congruent triangles so all of their corresponding parts are congruent due to CPCTC
- two isosceles triangles

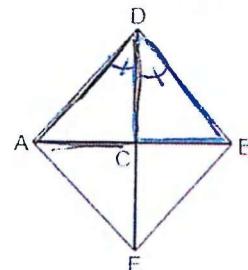


The top 2 small triangles are congruent and the top big triangle is isosceles  
The bottom 2 small triangles are congruent and the bottom big triangle is isosceles

1. In the diagram below of quadrilateral  $ADBE$ ,  $\overline{DE}$  is the perpendicular bisector of  $\overline{AB}$ . Which statement is always true?

- (1)  $\angle ADC \cong \angle BDC$   
(2)  $\angle EAC \cong \angle DAC$

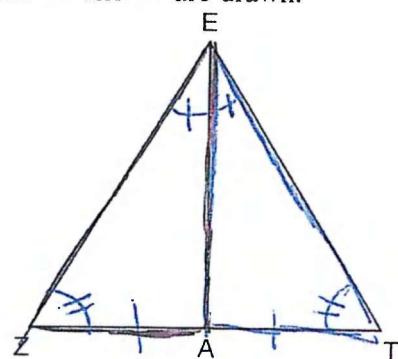
- 3)  $\overline{AD} \cong \overline{BE}$   
4)  $\overline{AE} \cong \overline{AD}$



2. Line segment  $EA$  is the perpendicular bisector of  $\overline{ZT}$ , and  $\overline{ZE}$  and  $\overline{TE}$  are drawn.

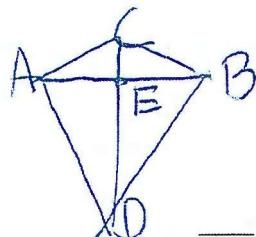
Which conclusion can *not* be proven?

- 1)  $\overline{EA}$  bisects angle  $ZET$ . ✓  
(2) Triangle  $EZT$  is equilateral. ✗  
3)  $\overline{EA}$  is a median of triangle  $EZT$ . ✓  
4) Angle  $Z$  is congruent to angle  $T$ . ✓



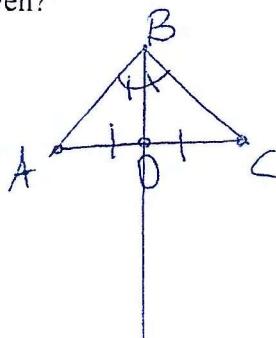
3. Segment  $CD$  is the perpendicular bisector of  $\overline{AB}$  at  $E$ . Which pair of segments does *not* have to be congruent?

- 1)  $\overline{AD} \cong \overline{BD}$  ✓
- 2)  $\overline{AC} \cong \overline{BC}$  ✓
- 3)  $\overline{AE} \cong \overline{BE}$  ✓
- 4)  $\overline{DE} \cong \overline{CE}$  X



4. In  $\triangle ABC$ ,  $\overline{BD}$  is the perpendicular bisector of  $\overline{AC}$ . Based upon this information, which statements below can be proven?

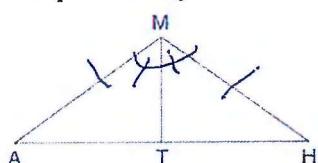
- I.  $\overline{BD}$  is a median.
  - II.  $\overline{BD}$  bisects  $\angle ABC$ .
  - III.  $\triangle ABC$  is isosceles.
- 1) I and II, only
  - 2) I and III, only
  - 3) II and III, only
  - 4) I, II, and III



5. In triangle  $MAH$  below,  $\overline{MT}$  is the perpendicular bisector of  $\overline{AH}$ .

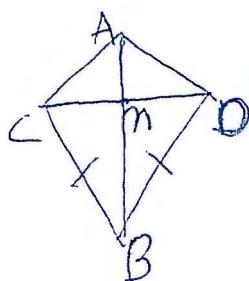
Which statement is *not* always true?

- 1)  $\triangle MAH$  is isosceles.
- 2)  $\triangle MAT$  is isosceles.
- 3)  $\overline{MT}$  bisects  $\angle AMH$ .
- 4)  $\angle A$  and  $\angle TMH$  are complementary.



6. Segment  $AB$  is the perpendicular bisector of  $\overline{CD}$  at point  $M$ . Which statement is always true?

- 1)  $\overline{CB} \cong \overline{DB}$  ✓
- 2)  $\overline{CD} \cong \overline{AB}$  X
- 3)  $\triangle ACD \cong \triangle BCD$  X
- 4)  $\triangle ACM \cong \triangle BCM$  X



◎ 15  
◎ 16



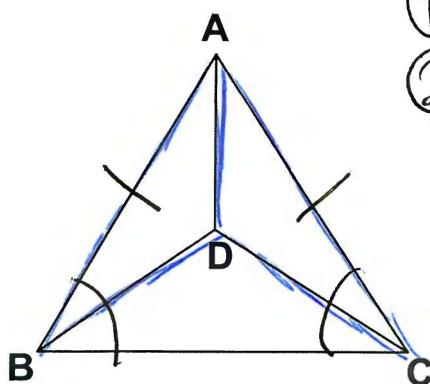
### Isosceles Triangle Theorem Mini Proofs

In a triangle, congruent angles are opposite congruent sides

If the given sides/angles are not sides/angles of the triangles you are trying to prove, if they make an isosceles triangle. Conclude the sides/angles opposite the ones you are given.

I. Given:  $\angle ABC \cong \angle ACB$

Prove:  $\triangle ADB \cong \triangle ADC$



Statements

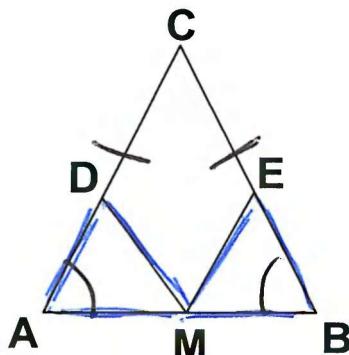
- (1)  $\angle ABC \cong \angle ACB$
- (2)  $\overline{AB} \cong \overline{AC}$

Reasons

- (1) given
- (2) Isosceles Triangle Theorem

2. Given:  $\overline{CA} \cong \overline{CB}$

Prove:  $\triangle ADM \cong \triangle BEM$



Statements

- (1)  $\overline{CA} \cong \overline{CB}$
- (2)  $\angle DAM \cong \angle EBM$

Reasons

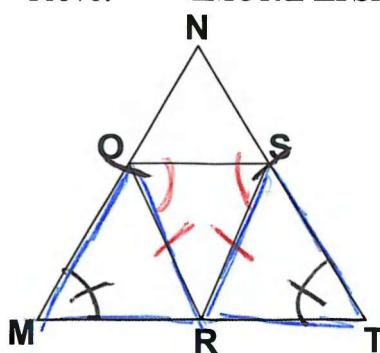
- (1) given
- (2) Isosceles Triangle Theorem

3. Given:

$\overline{MN} \cong \overline{NT}$ ,  $\angle ROS \cong \angle RSO$

Prove:

$\triangle MOR \cong \triangle TSR$



Statements

- (1)  $\overline{MN} \cong \overline{NT}$
- (2)  $\angle ROM \cong \angle STR$
- (3)  $\angle ROS \cong \angle RSO$
- (4)  $\overline{RO} \cong \overline{OS}$

Reasons

- (1) given
- (2) Isosceles Triangle Theorem
- (3) given
- (4) Isosceles triangle theorem

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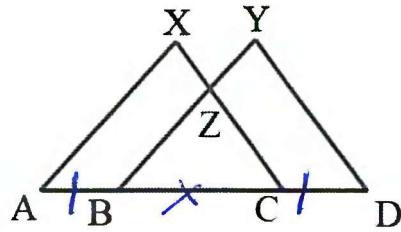
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## Addition and Subtraction Property Mini Proofs (PR4)

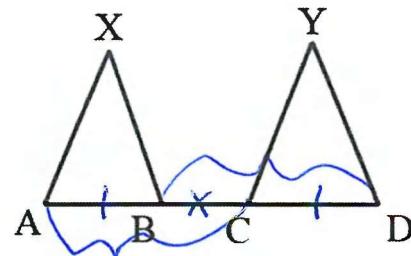
1. Given:  $\overline{AB} \cong \overline{CD}$   
Prove:  $\Delta AXC \cong \Delta BYD$

Statements	Reasons
(1) $\overline{AB} \cong \overline{CD}$	(1) given
(2) $\overline{BC} \cong \overline{BC}$	(2) reflexive property
(3) $\overline{AC} \cong \overline{BD}$ or $\overline{AB} + \overline{BC} = \overline{CD} + \overline{BC}$	(3) addition property



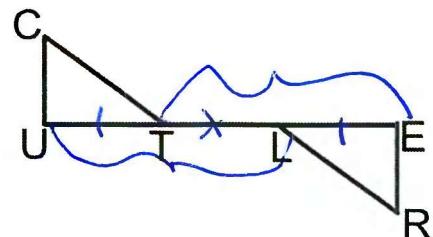
2. Given:  $\overline{AC} \cong \overline{BD}$   
Prove:  $\Delta AXB \cong \Delta DYC$

Statements	Reasons
(1) $\overline{AC} \cong \overline{BD}$	(1) given
(2) $\overline{BC} \cong \overline{BC}$	(2) reflexive property
(3) $\overline{AB} \cong \overline{CD}$ or $\overline{AC} - \overline{BC} = \overline{BD} - \overline{BC}$	(3) subtraction property



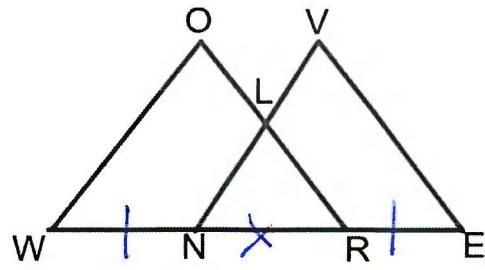
3. Given:  $\overline{UL} \cong \overline{TE}$   
Prove:  $\Delta CUT \cong \Delta REL$

Statements	Reasons
(1) $\overline{UL} \cong \overline{TE}$	(1) given
(2) $\overline{TL} \cong \overline{TL}$	(2) reflexive property
(3) $\overline{UT} \cong \overline{EL}$ or $\overline{UL} - \overline{TL} = \overline{TE} - \overline{TL}$	(3) subtraction property



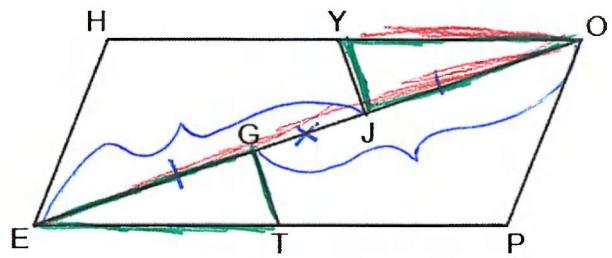
4. Given:  $\overline{WN} \cong \overline{RE}$   
 Prove:  $\triangle WOR \cong \triangle NVE$

Statements	Reasons
(1) $\overline{WN} \cong \overline{RE}$	(1) given
(2) $\overline{NR} \cong \overline{NR}$	(2) reflexive property
(3) $\overline{WR} \cong \overline{NE}$ or $WN + NR = NR + RE$	(3) addition property



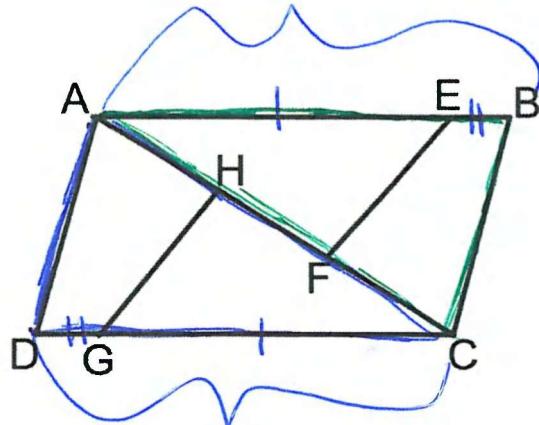
5. Given:  $\overline{EJ} \cong \overline{GO}$   
 Prove:  $\triangle TGE \cong \triangle YJO$

Statements	Reasons
(1) $\overline{EJ} \cong \overline{GO}$	(1) given
(2) $\overline{GJ} \cong \overline{GJ}$	(2) reflexive property
(3) $\overline{EG} \cong \overline{JO}$ or $EG - GJ = GO - GJ$	(3) subtraction property



6. Given:  $\overline{AE} \cong \overline{GC}$ ,  $\overline{EB} \cong \overline{DG}$   
 Prove:  $\triangle AFE \cong \triangle CHG$   $\triangle ABC \cong \triangle CDA$

Statements	Reasons
(1) $\overline{AE} \cong \overline{GC}$	(1) given
(2) $\overline{EB} \cong \overline{DG}$	(2) given
(3) $\overline{AB} \cong \overline{DC}$ or $AE + EB = GC + DG$	(3) addition property



7. Given:  $\overline{SY} \cong \overline{HC}$

Prove:  $\triangle ASH \cong \triangle KCY$

Statements | Reasons

$$\text{(1)} \overline{SY} \cong \overline{HC}$$

(1) Given

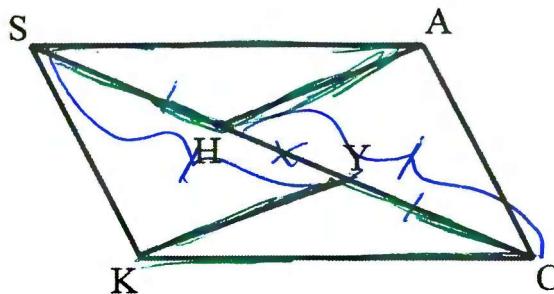
$$\text{(2)} \overline{HY} \cong \overline{HY}$$

(2) Reflexive Property

$$\text{(3)} \overline{SH} \cong \overline{YC}$$

(3) Subtraction Property  
or

$$\overline{SY} - \overline{HY} = \overline{HC} - \overline{HY}$$



8. Given:  $\overline{CE} \cong \overline{AH}$ ,  $\overline{ED} \cong \overline{BH}$

Prove:  $\triangle AXC \cong \triangle BYD$   $\triangle CDA \cong \triangle ABC$

Statements | Reasons

$$\text{(1)} \overline{CE} \cong \overline{AH}$$

(1) Given

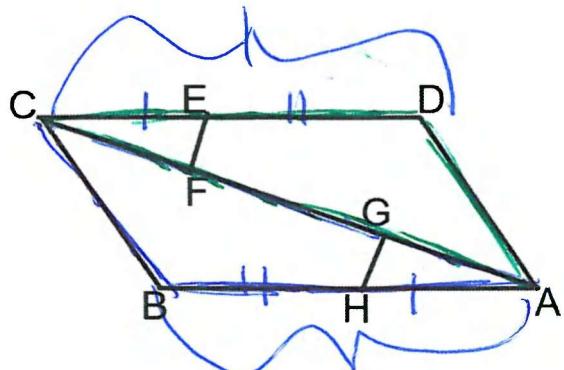
$$\text{(2)} \overline{ED} \cong \overline{BH}$$

(2) Given

$$\text{(3)} \overline{CD} \cong \overline{BA}$$

(3) Addition Property  
or

$$CE + ED = AH + BH$$



9. Given:  $\overline{AH} \cong \overline{FC}$

Prove:  $\triangle AFE \cong \triangle CHG$

Statements | Reasons

$$\text{(1)} \overline{AH} \cong \overline{FC}$$

(1) Given

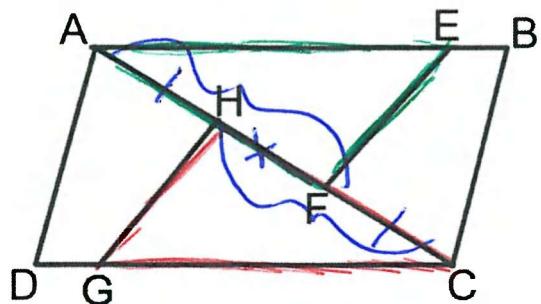
$$\text{(2)} \overline{HF} \cong \overline{HF}$$

(2) Reflexive Property

$$\text{(3)} \overline{AF} \cong \overline{HC}$$

(3) Addition Property  
or

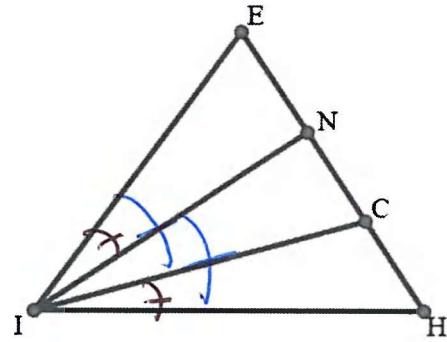
$$AH + HF = FC + HF$$



10. Given:  $\angle EIN \cong \angle HIC$

Prove:  $\angle EIC \cong \angle HIN$

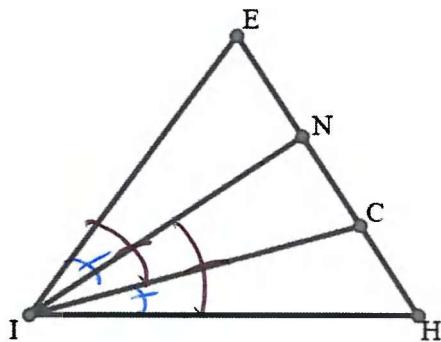
Statements	Reasons
(1) $\angle EIN \cong \angle HIC$	(1) given
(2) $\angle NIC \cong \angle NIC$	(2) reflexive property
(3) $\angle EIC \cong \angle HIN$ or $\angle EIN + \angle NIC = \angle HIC + \angle NIC$	(3) addition property



11. Given:  $\angle EIC \cong \angle HIN$

Prove:  $\angle EIN \cong \angle HIC$

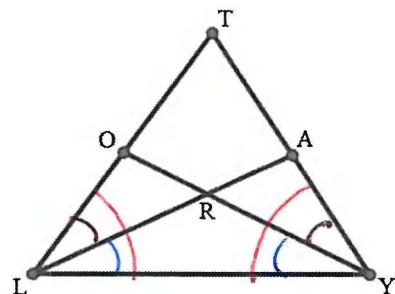
Statements	Reasons
(1) $\angle EIC \cong \angle HIN$	(1) given
(2) $\angle NIC \cong \angle NIC$	(2) reflexive property
(3) $\angle EIN \cong \angle HIC$ or $\angle EIC - \angle NIC = \angle HIN - \angle NIC$	(3) subtraction property



12. Given:  $\angle TLA \cong \angle TYO$ ,  $\angle ALY \cong \angle OYL$

Prove:  $\angle TLY \cong \angle TYL$

Statements	Reasons
(1) $\angle TLA \cong \angle TYO$	(1) given
(2) $\angle ALY \cong \angle OYL$	(2) given
(3) $\angle TLY \cong \angle TYL$ or $\angle TLA + \angle ALY = \angle TYO + \angle OYL$	(3) addition property



## Euclidean Triangle Proofs with Additional Tools

Vertical Angles are congruent (Look for an X)

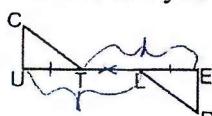
Reflexive Property (A side/angle is congruent to itself)

Isosceles Triangles (In a triangle, congruent angles are opposite congruent sides)

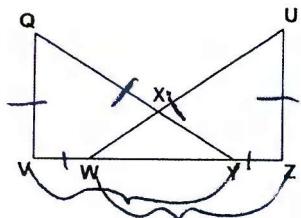
Addition and Subtraction Property (If you need more or less of a shared side)

\*You must use three congruent statements to get one congruent statement for the triangles. The two that you are adding/subtracting and the one that you want to prove in the triangle.

7. Given: $\overline{UT} \cong \overline{TE}$ Prove: $\overline{UT} \cong \overline{TE}$	
Statement	Reasons
① $\overline{UL} \cong \overline{TE}$	Given
② $\overline{TL} \cong \overline{TL}$	Reflexive Property
③ $\overline{UT} \cong \overline{LE}$	Subtraction Property ④ $\overline{UL} - \overline{TL} = \overline{TE} - \overline{TL}$



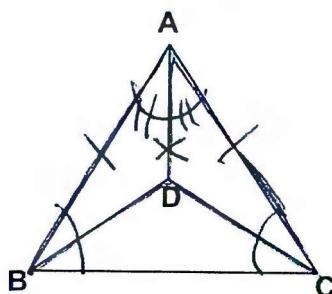
1. Given:  $\overline{QV} \cong \overline{UZ}$ ,  $\overline{VW} \cong \overline{YZ}$ ,  $\overline{YQ} \cong \overline{WU}$  Statements
- Prove:  $\angle Q \cong \angle U$



<u>Reasons</u>	
① $\overline{WV} \cong \overline{UZ}$	Given
② $\overline{VW} \cong \overline{YZ}$	Given
③ $\overline{WY} \cong \overline{WY}$	Reflexive Property
④ $\overline{VY} \cong \overline{WZ}$ or $\overline{VW} + \overline{WY} = \overline{YZ} + \overline{WY}$	Addition Property
⑤ $\overline{YQ} \cong \overline{WU}$	Given
⑥ $\triangle QVY \cong \triangle UWY$	SSS $\cong$ SSS
⑦ $\angle Q \cong \angle U$	CPTC

2. Given:  $\angle ABC \cong \angle ACB$ ,  $\overline{AD}$  bisects  $\angle BAC$  Statements

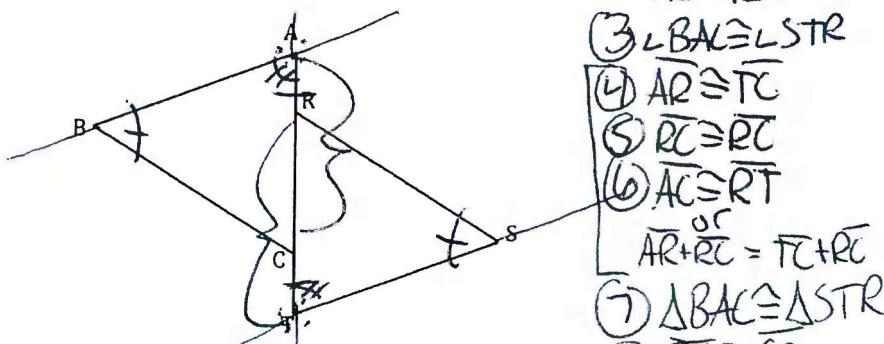
Prove:  $\overline{BD} \cong \overline{DC}$



<u>Reasons</u>	
① $\angle ABC \cong \angle ACB$	Given
② $\overline{AB} \cong \overline{AC}$	In a triangle, congruent angles give opposite congruent sides
③ $\overline{AD}$ bisects $\angle BAC$	Given
④ $\angle BAD \cong \angle CAD$	An angle bisector creates 2 $\cong$ angles
⑤ $\overline{AD} \cong \overline{AD}$	Reflexive Property
⑥ $\triangle BAD \cong \triangle CAD$	SAS $\cong$ SAS
⑦ $\overline{BD} \cong \overline{DC}$	CPTC

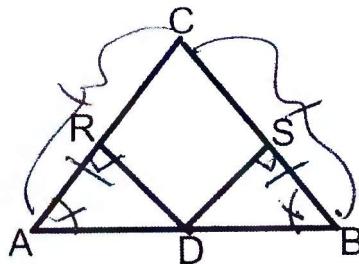
3. Given:  $\angle B \cong \angle S$ ,  $\overline{AB} \parallel \overline{ST}$ ,  $\overline{AR} \cong \overline{TC}$  Statements

Prove:  $\overline{BC} \cong \overline{SR}$



<u>Reasons</u>	
① $\angle B \cong \angle S$	Given
② $\overline{AB} \parallel \overline{ST}$	Given
③ $\angle BAC \cong \angle STR$	Parallel lines cut by a transversal create congruent alternate interior angles
④ $\overline{AR} \cong \overline{TC}$	Given
⑤ $\overline{RC} \cong \overline{RC}$	Reflexive Property
⑥ $\overline{AC} \cong \overline{ST}$ or $\overline{AR} + \overline{RC} = \overline{TC} + \overline{RC}$	Addition Property
⑦ $\triangle ABC \cong \triangle STR$	AAS $\cong$ AAS
	(CPCTC)

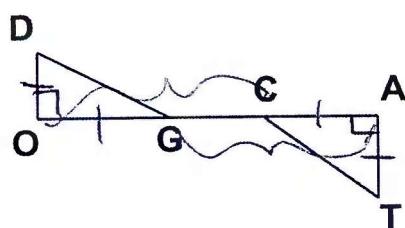
4. Given: In  $\triangle ABC$ ,  $\overline{CA} \cong \overline{CB}$ ,  $\overline{AR} \cong \overline{BS}$ ,  $\overline{DR} \perp \overline{AC}$ , statements Reasons  
 and  $\overline{DS} \perp \overline{BC}$   
 Prove:  $\overline{DR} \cong \overline{DS}$



- ①  $\overline{CA} \cong \overline{CB}$
- ②  $\angle A \cong \angle B$
- ③  $\overline{AR} \cong \overline{BS}$
- ④  $\overline{DR} \perp \overline{AC}$ ,  $\overline{DS} \perp \overline{BC}$
- ⑤  $\angle DRA \cong \angle BSD$
- ⑥  $\triangle ADR \cong \triangle ABS$
- ⑦  $\overline{DR} \cong \overline{DS}$

- ① Given
- ② In a triangle, congruent angles are opposite congruent sides
- ③ Given
- ④ Given
- ⑤ Perpendicular lines form congruent right angles.
- ⑥ ASA  $\cong$  ASA
- ⑦ CPCTC

5. Given:  $\overline{DO} \perp \overline{OA}$ ,  $\overline{TA} \perp \overline{OA}$ ,  $\overline{DO} \cong \overline{TA}$ ,  $\overline{OC} \cong \overline{AG}$  statements Reasons  
 Prove:  $\overline{DG} \cong \overline{TC}$



- ①  $\overline{DO} \perp \overline{OA}$ ,  $\overline{TA} \perp \overline{OA}$
- ②  $\angle DOG \cong \angle TAC$
- ③  $\overline{DO} \cong \overline{TA}$
- ④  $\overline{OC} \cong \overline{AG}$
- ⑤  $\overline{GC} \cong \overline{GC}$
- ⑥  $\overline{OG} \cong \overline{GA}$   
or  
 $\overline{OC} - \overline{GC} = \overline{AG} - \overline{GC}$
- ⑦  $\triangle DOG \cong \triangle TAC$
- ⑧  $\overline{DG} \cong \overline{TC}$

- ① Given
- ② Perpendicular lines form congruent right angles
- ③ Given
- ④ Given
- ⑤ Reflexive property
- ⑥ Subtraction property
- ⑦ SAS  $\cong$  SAS
- ⑧ CPCTC

6. Given:  $\overline{MN} \cong \overline{NT}$ ,  $\angle ROS \cong \angle RSO$ ,  $\angle ORM \cong \angle SRT$

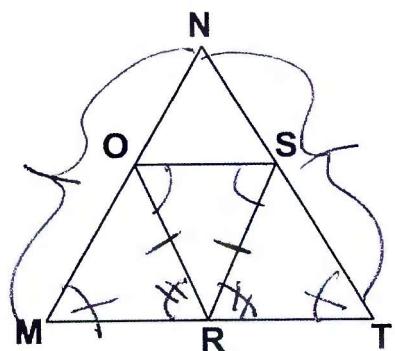
Prove:  $\triangle MOR \cong \triangle TSR$

Statements

Reasons

- ①  $\overline{MN} \cong \overline{NT}$
- ②  $\angle LMN \cong \angle LNT$
- ③  $\angle ROS \cong \angle RSO$
- ④  $\overline{OR} \cong \overline{SR}$
- ⑤  $\angle ORM \cong \angle SRT$
- ⑥  $\triangle MOR \cong \triangle TSR$

- ① Given
- ② In a triangle, congruent angles are opposite congruent sides
- ③ Given
- ④ In a triangle, congruent angles are opposite congruent sides
- ⑤ Given
- ⑥ AAS  $\cong$  AAS



A parallelogram has parallel lines cut by a transversal which forms congruent alternate interior angles

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Date \_\_\_\_\_  
Geometry

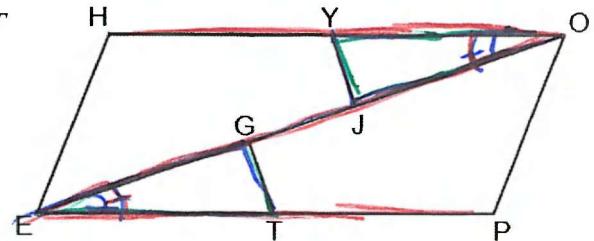
### Proving Alternate Interior Angles Given Parallelogram Mini Proofs (PR1)

1. Given  $HOP$  is a parallelogram, prove  $\triangle JOY \cong \triangle GET$

Statement | Reasons

(1)  $HOP$  is a P-gm  
(2)  $\angle 1 \cong \angle 2$

(1) Given  
(2) A parallelogram has parallel lines cut by a transversal which forms congruent alternate interior angles.

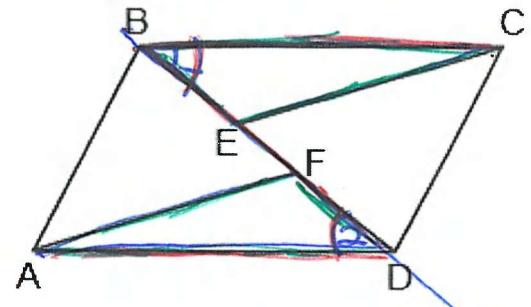


2. Given  $ABCD$  is a parallelogram, prove  $\triangle BCE \cong \triangle DAF$

Statements | Reasons

(1)  $ABCD$  is a P-gm  
(2)  $\angle 1 \cong \angle 2$

(1) Given  
(2) A parallelogram has parallel lines cut by a transversal which forms congruent alternate interior angles

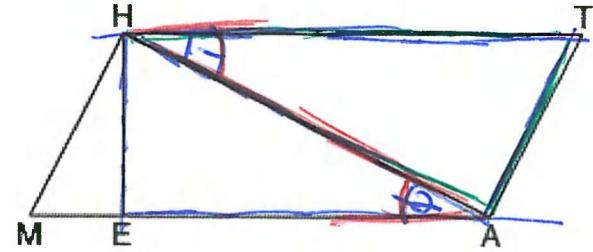


3. Given  $MATH$  is a parallelogram, prove  $\triangle HAT \sim \triangle AEH$

Statements | Reasons

(1)  $MATH$  is a P-gm  
(2)  $\angle 1 \cong \angle 2$

(1) Given  
(2) A parallelogram has parallel lines cut by a transversal which forms congruent alternate interior angles

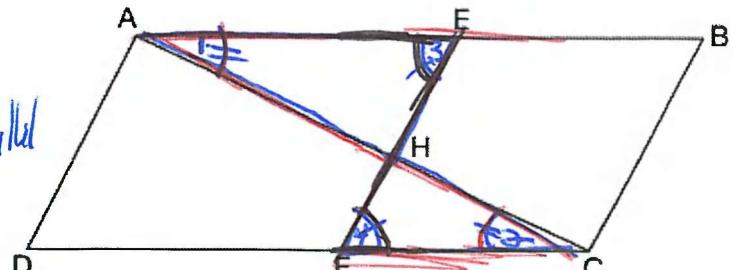


4. Given  $ABCD$  is a parallelogram, prove  $\triangle AEH \sim \triangle CFH$

Statements | Reasons

(1)  $ABCD$  is a P-gm  
(2)  $\angle 1 \cong \angle 2, \angle 3 \cong \angle 4$

(1) Given  
(2) A parallelogram has parallel lines cut by a transversal which forms congruent alternate interior angles



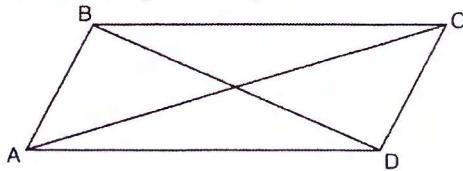
## Euclidean Parallelogram Proofs/Parallelogram Properties

To prove parallelograms: Always prove parallelogram first. You will probably have to use congruent triangles with CPCTC to get at least one of the properties.

1. Quadrilateral  $ABCD$  with diagonals  $\overline{AC}$  and  $\overline{BD}$  is shown in the diagram below.

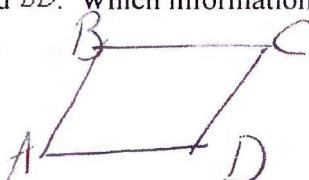
Which information is *not* enough to prove  $ABCD$  is a parallelogram?

- 1)  $\overline{AB} \cong \overline{CD}$  and  $\overline{AB} \parallel \overline{DC}$
- 2)  $\overline{AB} \cong \overline{CD}$  and  $\overline{BC} \cong \overline{DA}$
- 3)  $\overline{AB} \cong \overline{CD}$  and  $\overline{BC} \parallel \overline{AD}$
- 4)  $\overline{AB} \parallel \overline{DC}$  and  $\overline{BC} \parallel \overline{AD}$



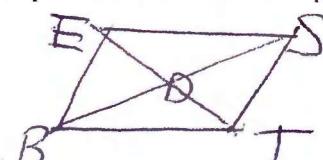
2. Quadrilateral  $ABCD$  has diagonals  $\overline{AC}$  and  $\overline{BD}$ . Which information is *not* sufficient to prove  $ABCD$  is a parallelogram?

- 1)  $\overline{AC}$  and  $\overline{BD}$  bisect each other.
- 2)  $\overline{AB} \cong \overline{CD}$  and  $\overline{BC} \cong \overline{AD}$
- 3)  $\overline{AB} \cong \overline{CD}$  and  $\overline{AB} \parallel \overline{CD}$
- 4)  $\overline{AB} \cong \overline{CD}$  and  $\overline{BC} \parallel \overline{AD}$

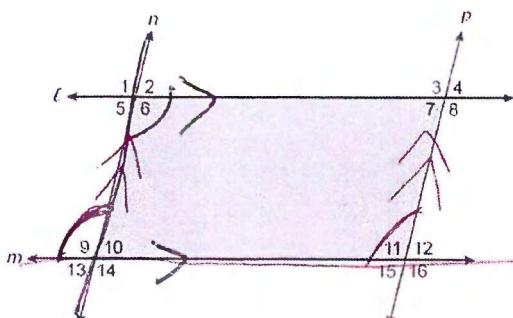


- 3) Quadrilateral  $BEST$  has diagonals that intersect at point  $D$ . Which statement would *not* be sufficient to prove quadrilateral  $BEST$  is a parallelogram?

- 1)  $\overline{BD} \cong \overline{SD}$  and  $\overline{ED} \cong \overline{TD}$
- 2)  $\overline{BE} \cong \overline{ST}$  and  $\overline{ES} \cong \overline{TB}$
- 3)  $\overline{ES} \cong \overline{TB}$  and  $\overline{BE} \parallel \overline{TS}$
- 4)  $\overline{ES} \parallel \overline{BT}$  and  $\overline{BE} \parallel \overline{TS}$



4. In the diagram below, lines  $\ell$  and  $m$  intersect lines  $n$  and  $p$  to create the shaded quadrilateral as shown.



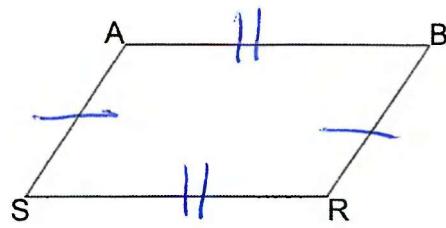
Which congruence statement would be sufficient to prove the quadrilateral is a parallelogram?

- 1)  $\angle 1 \cong \angle 6$  and  $\angle 9 \cong \angle 14$
- 2)  $\angle 5 \cong \angle 10$  and  $\angle 6 \cong \angle 9$
- 3)  $\angle 5 \cong \angle 7$  and  $\angle 10 \cong \angle 15$
- 4)  $\angle 6 \cong \angle 9$  and  $\angle 9 \cong \angle 11$

5. Given:  $\overline{SA} \cong \overline{BR}$ ,  $\overline{AB} \cong \overline{SR}$

Prove: SABR is a parallelogram

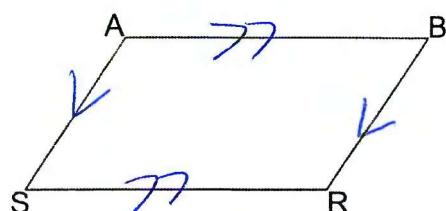
Statements	Reasons
(1) $\overline{SA} \cong \overline{BR}$	(1) Given
(2) $\overline{AB} \cong \overline{SR}$	(2) A parallelogram has two pairs of opposite sides congruent
(3) SABR is a parallelogram	



6. Given:  $\overline{SA} \parallel \overline{BR}$ ,  $\overline{AB} \parallel \overline{SR}$

Prove: SABR is a parallelogram

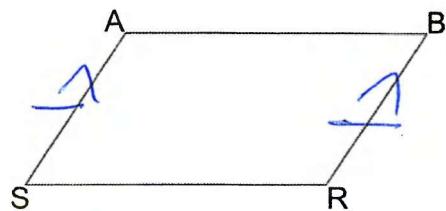
Statements	Reasons
(1) $\overline{SA} \parallel \overline{BR}$ , $\overline{AB} \parallel \overline{SR}$	(1) Given
(2) SABR is a parallelogram	(2) A parallelogram has two pairs of opposite sides parallel



7. Given:  $\overline{SA} \cong \overline{BR}$ ,  $\overline{SA} \parallel \overline{BR}$

Prove: SABR is a parallelogram

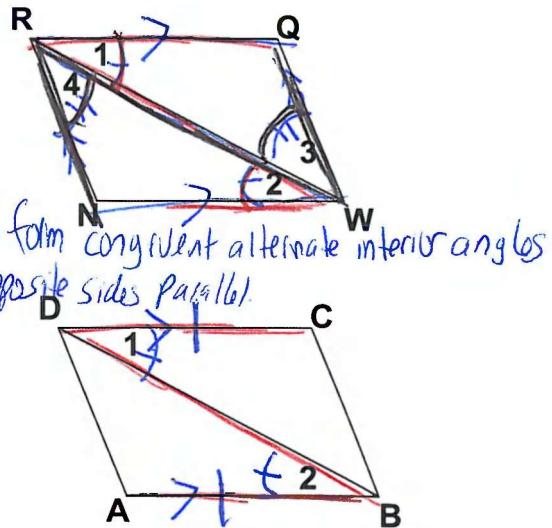
Statements	Reasons
(1) $\overline{SA} \cong \overline{BR}$ , $\overline{SA} \parallel \overline{BR}$	(1) Given
(2) SABR is a parallelogram	(2) A parallelogram has one pair of opposite sides congruent and parallel



8. Given:  $\angle 1 \cong \angle 2$ ,  $\angle 3 \cong \angle 4$

Prove: NRQW is a parallelogram

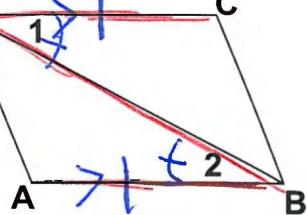
Statements	Reasons
(1) $\angle 1 \cong \angle 2$ , $\angle 3 \cong \angle 4$	(1) Given
(2) $\overline{RQ} \parallel \overline{NW}$ , $\overline{RN} \parallel \overline{QW}$	(2) Parallel lines cut by a transversal form congruent alternate interior angles
(3) NRQW is a parallelogram	(3) A parallelogram has two pairs of opposite sides parallel.



9. Given:  $\overline{AB} \cong \overline{CD}$ ,  $\angle 1 \cong \angle 2$

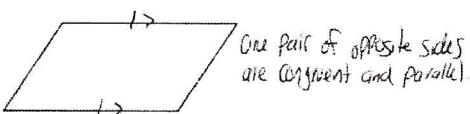
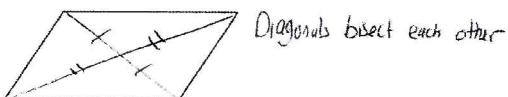
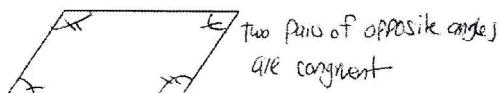
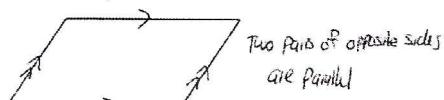
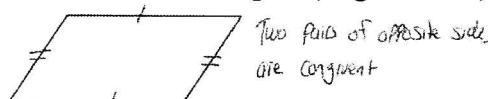
Prove: ABCD is a parallelogram

Statements	Reasons
(1) $\overline{AB} \cong \overline{CD}$	(1) Given
(2) $\angle 1 \cong \angle 2$	(2) Given
(3) $\overline{DC} \parallel \overline{AB}$	(3) Parallel lines cut by a transversal form congruent alternate interior angles
(4) ABCD is a parallelogram	(4) A parallelogram has 1 pair of opposite sides congruent and parallel



## PART IV PARALLELOGRAM PROOFS!

1) Prove the parallelogram (Pages 36-37). You may need to prove sides are parallel using alternate interior angles (Pages 20-21)



2) Use the parallelogram to prove corresponding parts of triangles are congruent (Pages 14-16). **Expect to prove alternate interior angles are congruent! (Page 35).** Opposite sides are congruent is also very common.

If proving sides/angles:	If proving multiplication
3) <b>Expect to use addition/subtraction property (Pages 29-31).</b> 4) State the triangles are congruent 5) State the sides/angles with reason CPCTC	3) Expect to use perpendicular lines form congruent right angles. 4) <b>Work backwards to similar triangles (Pages 12-13)</b>

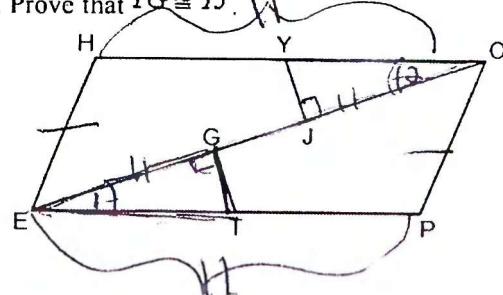
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Mr. Schlansky

Date \_\_\_\_\_  
Geometry

## Parallelogram Proofs Part IV

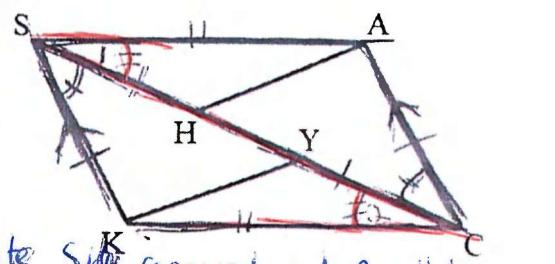
1. In quadrilateral  $HOPE$  below,  $\overline{EH} \cong \overline{OP}$ ,  $\overline{EP} \cong \overline{OH}$ ,  $\overline{EJ} \cong \overline{OG}$ , and  $\overline{TG}$  and  $\overline{YJ}$  are perpendicular to diagonal  $\overline{EO}$  at points  $G$  and  $J$ , respectively. Prove that  $\overline{TG} \cong \overline{YJ}$ .

Statements	Reasons
① $\overline{EH} \cong \overline{OP}$ , $\overline{EP} \cong \overline{OH}$	① Given
② $HOPE$ is a p-gram	② A parallelogram has two pairs of opposite sides congruent
③ $\angle I \cong \angle J$	③ A parallelogram has parallel lines cut by a transversal which create congruent alternate interior angles
④ $\overline{EJ} \cong \overline{OG}$	④ Given
⑤ $\overline{GJ} \cong \overline{GJ}$	⑤ Reflexive property
⑥ $\overline{EG} \cong \overline{OJ}$ or $EJ - GJ = OG - GJ$	⑥ Subtraction property
⑦ $TG \perp EO$ , $YJ \perp EO$	⑦ Given
	⑧ $\angle TGE \cong \angle YJO$
	⑨ $\angle TGE \cong \angle YJO$
	⑩ $\overline{TG} \cong \overline{YJ}$
	⑪ Perpendicular lines form congruent right angles
	⑫ $\overline{ASA} \cong \overline{ASA}$
	⑬ CPCTC



2. In quadrilateral  $SACK$ ,  $\angle KSY \cong \angle ACH$ ,  $\overline{SK} \cong \overline{AC}$ ,  $\overline{SY} \cong \overline{CH}$ . Prove  $\angle SAH \cong \angle CKY$

Statements	Reasons
① $\angle KSY \cong \angle ACH$	① Given
② $\overline{SK} \cong \overline{AC}$	② Parallel lines cut by a transversal create congruent alternate interior angles
③ $\overline{SK} \cong \overline{AC}$	③ Given
④ $SACK$ is a p-gram	④ A parallelogram has 1 pair of opposite sides congruent and parallel
⑤ $\angle L \cong \angle L$	⑤ A p-gram has parallel lines cut by a transversal which creates $\cong$ alternate interior angles
⑥ $\overline{SY} \cong \overline{CH}$	⑥ Given
⑦ $\overline{HY} \cong \overline{HY}$	⑦ Reflexive property
⑧ $\overline{HS} \cong \overline{YL}$ or $SY - HY = CH - HY$	⑧ Subtraction property
⑨ $\overline{SA} \cong \overline{KC}$	⑨ A parallelogram has opposite sides congruent
⑩ $\triangle ASH \cong \triangle CKY$	⑩ SAS $\cong$ SAS
⑪ $\angle SAH \cong \angle CKY$	⑪ CPCTC



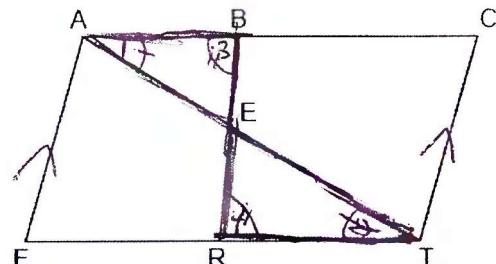
3. In the diagram below of quadrilateral  $FACT$ ,  $\overline{BR}$  intersects diagonal  $\overline{AT}$  at  $E$ ,  $\overline{AF} \parallel \overline{CT}$ , and  $\overline{AF} \cong \overline{CT}$ . Prove  $(AB)(TE) = (AE)(TR)$

statements

- ①  $\overline{AF} \parallel \overline{CT}, \overline{AF} \cong \overline{CT}$
- ②  $FACT$  is a parallelogram
- ③  $\angle 1 \cong \angle 2, \angle 3 \cong \angle 4$

reasons

- ① Given
- ② A parallelogram has 1 pair of opposite sides  $\cong$  and  $\parallel$
- ③ A parallelogram has parallel lines cut by a transversal creating congruent alternate interior angles



\*You could have chose vertical angles

$$\begin{aligned} &④ \triangle ABE \sim \triangle ATR \\ &\text{⑤ } \frac{AB}{TR} = \frac{AE}{TE} \\ &\text{⑥ } (AB)(TE) = (AE)(TR) \end{aligned}$$

$$\text{⑦ } AA \cong AA$$

$$\text{⑧ } \text{C.S.S.T.I.P}$$

$$\text{⑨ } \text{Cross products are equal}$$

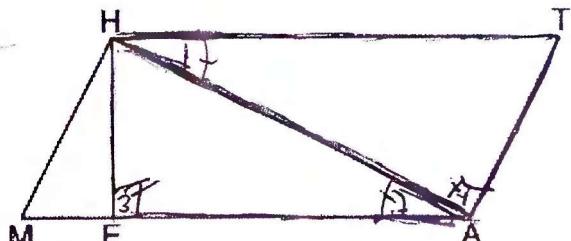
4. Given: Quadrilateral  $MATH$ ,  $\overline{HM} \cong \overline{AT}$ ,  $\overline{HT} \cong \overline{AM}$ ,  $\overline{HE} \perp \overline{MEA}$ , and  $\overline{HA} \perp \overline{AT}$ .  
Prove:  $TA \cdot HA = HE \cdot TH$

statements

- ①  $\overline{HM} \cong \overline{AT}, \overline{HT} \cong \overline{AM}$
- ②  $MATH$  is a parallelogram
- ③  $\angle 1 \cong \angle 2$
- ④  $\overline{HE} \perp \overline{MEA}, \overline{HA} \perp \overline{AT}$
- ⑤  $\angle 3 \cong \angle 4$

reasons

- ① Given
- ② A parallelogram has 2 pairs of opposite sides congruent
- ③ A parallelogram has parallel lines cut by a transversal that create congruent alternate interior angles
- ④ Given
- ⑤ Perpendicular lines form congruent right angles



$$\text{⑥ } \triangle AHT \cong \triangle EAH$$

$$\text{⑦ } AA \cong AA$$

$$\text{⑧ } \text{C.S.S.T.I.P}$$

$$\text{⑨ } \text{Cross products are equal}$$

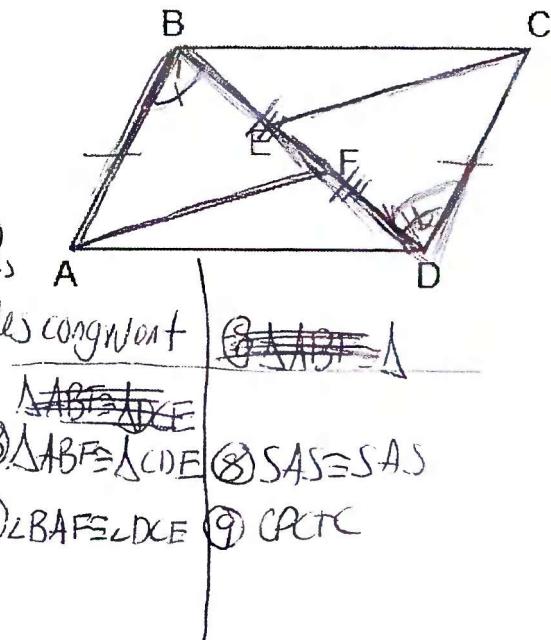
$$\begin{aligned} &\text{⑩ } \frac{TA}{TH} = \frac{HE}{HA} \\ &\text{⑪ } TA \cdot HA = HE \cdot TH \end{aligned}$$

5. In the diagram of quadrilateral  $ABCD$  below,  $\overline{AB} \cong \overline{CD}$ , and  $\overline{AB} \parallel \overline{CD}$ . Segments  $CE$  and  $AF$  are drawn to diagonal  $\overline{BD}$  such that  $\overline{BE} \cong \overline{DF}$ . Prove:  $\angle BAF \cong \angle DCE$ .

Statements

Reasons

- |                                                                                   |                                                                                                       |
|-----------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------|
| (1) $\overline{AB} \cong \overline{CD}$ , $\overline{AB} \parallel \overline{CD}$ | (1) given                                                                                             |
| (2) $ABCD$ is a parallelogram                                                     | (2) A $\square$ has 1 pair of opposite sides congruent and parallel                                   |
| (3) $\angle 1 \cong \angle 2$                                                     | (3) Parallelogram has parallel lines cut by a transversal which create congruent alt. int. $\angle$ s |
| (4) $\overline{AB} \cong \overline{CD}$                                           | (4) A parallelogram has 2 pairs of opposite sides congruent                                           |
| (5) $\overline{BE} \cong \overline{DF}$                                           | (5) given                                                                                             |
| (6) $\overline{EF} \cong \overline{EF}$                                           | (6) reflexive property                                                                                |
| (7) $\overline{BF} \cong \overline{ED}$<br>or<br>$BE + EF = DF + EF$              | (7) addition property                                                                                 |
| (8) $\triangle ABE \cong \triangle CDE$                                           | (8) <del><math>\triangle ABE \cong \triangle CDE</math></del><br>(8) SAS $\cong$ SAS                  |



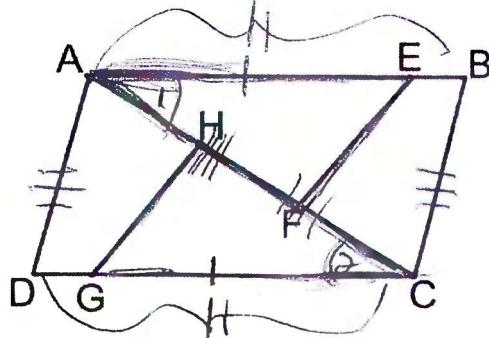
- (9)  $\angle BAF \cong \angle DCE$  (9) CPCTC

6. Given:  $\overline{AE} \cong \overline{CG}$ ,  $\overline{BE} \cong \overline{DG}$ ,  $\overline{AH} \cong \overline{CF}$ ,  $\overline{AD} \cong \overline{CB}$   
Prove:  $\overline{EF} \cong \overline{GH}$

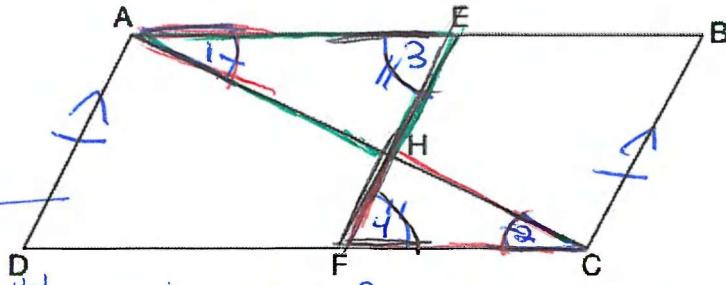
Statements

Reasons

- |                                                                      |                                                                                                    |
|----------------------------------------------------------------------|----------------------------------------------------------------------------------------------------|
| (1) $\overline{AE} \cong \overline{CG}$                              | (1) given                                                                                          |
| (2) $\overline{BE} \cong \overline{DG}$                              | (2) given                                                                                          |
| (3) $\overline{AB} \cong \overline{DC}$<br>or<br>$AE + BE = CG + DG$ | (3) addition property                                                                              |
| (4) $\overline{AD} \cong \overline{CB}$                              | (4) given                                                                                          |
| (5) $ABCD$ is a parallelogram                                        | (5) A parallelogram has 2 pairs of opposite sides congruent                                        |
| (6) $\angle 1 \cong \angle 2$                                        | (6) A parallelogram has parallel lines cut by a transversal that form $\cong$ alt. interior angles |
| (7) $\overline{AH} \cong \overline{CF}$                              | (7) given                                                                                          |
| (8) $\overline{HF} \cong \overline{HF}$                              | (8) reflexive property                                                                             |
| (9) $\overline{AF} \cong \overline{AC}$<br>or<br>$AH + HF = CF + HF$ | (9) addition property                                                                              |
| (10) $\triangle AEF \cong \triangle GCH$                             | (10) SAS $\cong$ SAS                                                                               |
| (11) $\overline{EF} \cong \overline{GH}$                             | (11) CPCTC                                                                                         |



7. Given: Quadrilateral  $ABCD$ ,  $\overline{AC}$  and  $\overline{EF}$  intersect at  $H$ ,  $\overline{AD} \parallel \overline{BC}$  and  $\overline{AD} \cong \overline{BC}$ . Prove:  
 $(EH)(CH) = (FH)(AH)$



statements

- ①  $\overline{AD} \parallel \overline{BC}, \overline{AD} \cong \overline{BC}$
- ②  $ABCD$  is a parallelogram
- ③  $\angle 1 \cong \angle 2, \angle 3 \cong \angle 4$

Reasons

- ④ Given
- ⑤ A Parallelogram has 1 pair of opposite sides  $\cong$  and  $\parallel$
- ⑥ A parallelogram has parallel lines cut by a transversal which creates  $\cong$  alternate interior angles

$$⑦ \triangle EAH \sim \triangle FCH$$

$$⑧ \frac{EH}{AH} = \frac{FH}{CH}$$

$$⑨ (EH)(CH) = (FH)(AH)$$

$$⑩ AA \cong AA$$

CSSTIP

Cross products are equal

8. Given:  $\overline{AB} \cong \overline{DC}, \overline{AD} \cong \overline{BC}, \overline{AF} \cong \overline{GC}, \overline{BH} \cong \overline{DE}$

Prove:  $\overline{EF} \cong \overline{GH}$

statements

- ①  $\overline{AB} \cong \overline{DC}, \overline{AD} \cong \overline{BC}$
- ②  $ABCD$  is a parallelogram

$$③ \angle 1 \cong \angle 2$$

$$④ \overline{AF} \cong \overline{GC}$$

$$⑤ \overline{FG} \cong \overline{FG}$$

$$⑥ \overline{CF} \cong \overline{GA}$$

⑦

$$\overline{AF} - \overline{FG} = \overline{GC} - \overline{FG}$$

$$⑧ \overline{BH} \cong \overline{DE}$$

$$⑨ \overline{HA} \cong \overline{CE}$$

or

$$\overline{AB} - \overline{BH} = \overline{DC} - \overline{DE}$$

$$⑩ \triangle ECF \cong \triangle HAG$$

$$⑪ \overline{EF} \cong \overline{GH}$$

Reasons

② Given

③ A p-gram has 2 pairs of opposite sides  $\cong$

④ A p-gram has parallel lines cut by a transversal which form  $\cong$  alternate interior angles.

⑤ given

⑥ reflexive property

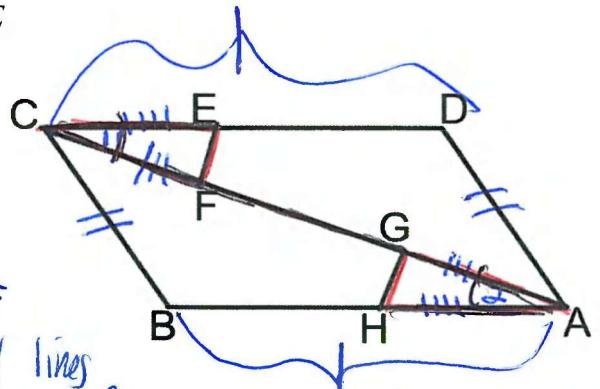
⑦ subtraction property

⑧ given

⑨ subtraction property

⑩ SAS  $\cong$  SAS

⑪ CPCTC



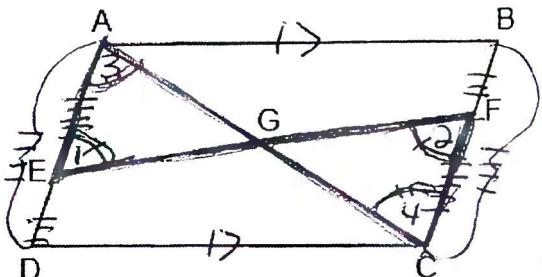
9. Given: Quadrilateral  $ABCD$ ,  $\overline{AB} \cong \overline{CD}$ ,  $\overline{AB} \parallel \overline{CD}$ , diagonal  $\overline{AC}$  intersects  $\overline{EF}$  at  $G$ , and  $\overline{DE} \cong \overline{BF}$ . Prove:  $G$  is the midpoint of  $\overline{EF}$ .

Statements

- (1)  $\overline{AB} \cong \overline{CD}$ ,  $\overline{AB} \parallel \overline{CD}$
- (2)  $ABCD$  is a parallelogram
- (3)  $\angle 1 \cong \angle 2$ ,  $\angle 3 \cong \angle 4$
- (4)  $\overline{DE} \cong \overline{BF}$
- (5)  $\overline{AD} \cong \overline{BC}$
- (6)  $\overline{AE} \cong \overline{FC}$   
or  
 $AD - DE = BC - BF$
- (7)  $\triangle AEG \cong \triangle CFG$
- (8)  $\overline{EG} \cong \overline{GF}$
- (9)  $G$  is the midpoint of  $\overline{EF}$

Reasons

- (1) given
- (2) A parallelogram has 1 pair of opposite sides  $\cong$  and  $\parallel$
- (3) A p-gm has parallel lines cut by a transversal which create congruent alternate interior angles
- (4) given
- (5) A p-gm has opposite sides congruent
- (6) subtraction property
- (7) ASA  $\cong$  ASA
- (8) CPCF
- (9) A midpoint creates 2 congruent segments



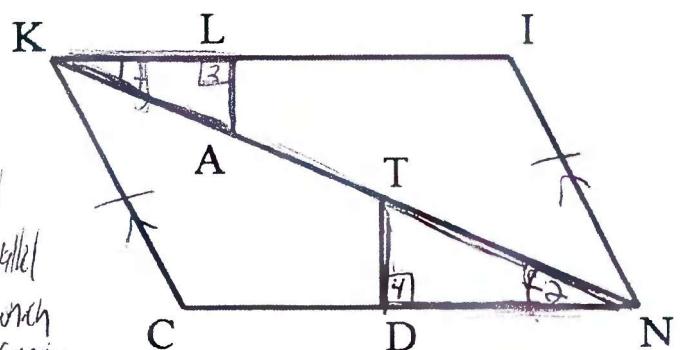
10. Given:  $\overline{KC} \parallel \overline{IN}$ ,  $\overline{KC} \cong \overline{IN}$ ,  $\overline{AL} \perp \overline{KI}$ ,  $\overline{TD} \perp \overline{CN}$ . Prove  $\overline{KL} \cdot \overline{NT} = \overline{DN} \cdot \overline{KA}$

Statement

- (1)  $\overline{KC} \parallel \overline{IN}$ ,  $\overline{KC} \cong \overline{IN}$
- (2)  $KINC$  is a p-gm  
~~not a pair of opposite sides~~
- (3)  $\angle 1 \cong \angle 2$
- (4)  $\overline{AL} \perp \overline{KI}$ ,  $\overline{TD} \perp \overline{CN}$
- (5)  $\angle 3 \cong \angle 4$

Reasons

- (1) given
- (2) A p-gm has 1 pair of opposite sides congruent and parallel
- (3) A parallelogram has parallel lines cut by a transversal which form congruent alternate interior angles
- (4) given
- (5) Perpendicular lines form congruent right angles



(6)  $\triangle KLA \sim \triangle NDT$

$$\begin{aligned} (7) \quad \frac{KL}{KA} &= \frac{DN}{NT} \\ (8) \quad KL \cdot NT &= DN \cdot KA \end{aligned}$$

(6) AA  $\cong$  AA

(7) LSSTIP

(8) cross products are equal



## Proving Rectangles/Rhombuses/Squares MC and Mini Proofs

To prove a rectangle:

- 1) Prove it is a parallelogram
- 2) Prove one of the two rectangle pictures

To prove a rhombus:

- 1) Prove it is a parallelogram
- 2) Prove one of the three rhombus pictures

To prove a square:

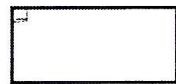
- 1) Prove it is a parallelogram
- 2) Prove one of the two rectangle pictures AND one of the two rhombus pictures.

To prove a rectangle is a square:

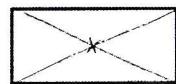
Prove one of the three rhombus pictures

To prove a rhombus is a square:

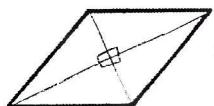
Prove one of the two rectangle pictures



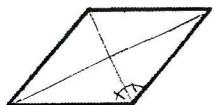
A right angle  
(consecutive sides perpendicular)



Congruent diagonals



diagonals are perpendicular  
to each other



diagonals bisect the  
angles



Consecutive sides  
are congruent

1. A parallelogram must be a rhombus when its

- 1) Diagonals are congruent.
- 2) Opposite sides are parallel.
- ③ 3) Diagonals are perpendicular.
- 4) Opposite angles are congruent.

2. A parallelogram must be a rectangle when its

- 1) diagonals are perpendicular
- ② 2) diagonals are congruent
- 3) opposite sides are parallel
- 4) opposite sides are congruent

3. A rectangle must be a square when its

- 1) angles are right angles
- 2) diagonals are congruent
- ③ 3) consecutive sides are congruent
- 4) opposite sides are parallel

4. A rhombus must be a square when

- 1) its consecutive sides are congruent
- ④ 2) it has a right angle
- 3) its opposite angles are congruent
- 4) its diagonals are perpendicular to each other

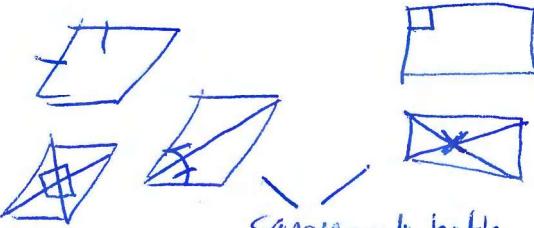
5. A parallelogram must be a rhombus when its

- ① 1) diagonals bisect its angles
- 2) opposite angles are congruent
- 3) angles are right angles
- 4) opposite sides are parallel

6. A rhombus must be a square when its

- 1) diagonals bisect its angles
- 2) opposite angles are congruent
- ③ 3) diagonals are congruent
- 4) opposite sides are parallel

Name Schlansky  
Mr. Schlansky



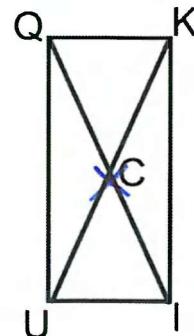
Date \_\_\_\_\_  
Geometry

## Proving Rectangles/Rhombuses/Squares Mini Proofs

7. Given:  $QUIK$  is a parallelogram,  $\angle QUI \cong \angle KIU$

Prove:  $QUIK$  is a rectangle

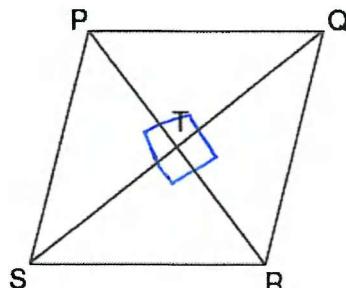
Statements	Reasons
(1) $QUIK$ is a parallelogram	(1) Given
(2) $\overline{QI} \cong \overline{KU}$	(2) Given
(3) $QUIK$ is a rectangle	(3) A rectangle is a parallelogram with congruent diagonals



8. Given:  $PQRS$  is a parallelogram,  $\overline{PR} \perp \overline{SQ}$ .

Prove:  $PQRS$  is a rhombus

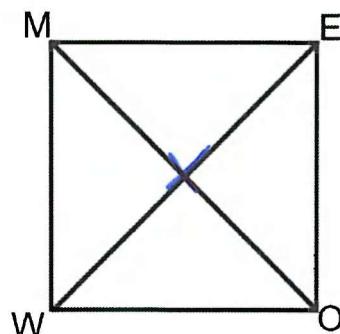
Statements	Reasons
(1) $PQRS$ is a parallelogram	(1) Given
(2) $\overline{PR} \perp \overline{SQ}$	(2) Given
(3) $PQRS$ is a rhombus	(3) A rhombus is a parallelogram with perpendicular diagonals



9. Given:  $MEOW$  is a rhombus,  $\overline{MO} \cong \overline{WE}$

Prove:  $MEOW$  is a square

Statements	Reasons
(1) $MEOW$ is a rhombus	(1) Given
(2) $\overline{MO} \cong \overline{WE}$	(2) <del>Given</del> Given
(3) $MEOW$ is a square	(3) A square is a rhombus with congruent diagonals

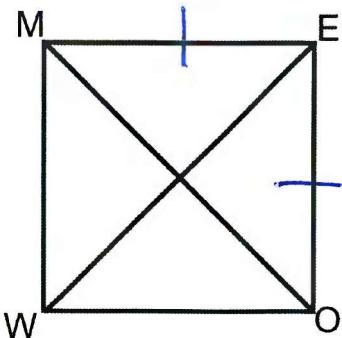


has rectangle, needs rhombus

10. Given:  $MEOW$  is a rectangle,  $\overline{ME} \cong \overline{EO}$

Prove:  $MEOW$  is a square

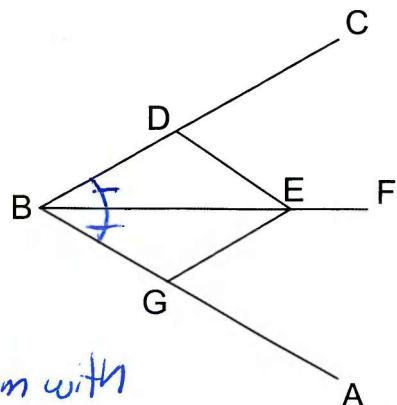
<u>Statements</u>	<u>Reasons</u>
(1) $MEOW$ is a rectangle	(1) given
(2) $\overline{ME} \cong \overline{EO}$	(2) given
(3) $MEOW$ is a square	(3) A square is a rectangle with congruent consecutive sides



11. Given:  $DEGB$  is a parallelogram,  $\overline{BF}$  bisects  $\angle CBA$

Prove:  $DEGB$  is a rhombus

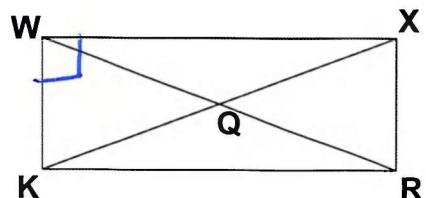
<u>Statements</u>	<u>Reasons</u>
(1) $DEGB$ is a parallelogram	(1) given
(2) $\overline{BF}$ bisects $\angle CBA$	(2) given
(3) $DEGB$ is a rhombus	(3) A rhombus is a parallelogram with diagonals that bisect its angles



12. Given:  $WXRK$  is a parallelogram,  $\overline{KW} \perp \overline{WX}$

Prove:  $WXRK$  is a rectangle

<u>Statements</u>	<u>Reasons</u>
(1) $WXRK$ is a parallelogram	(1) given
(2) $\overline{KW} \perp \overline{WX}$	(2) given
(3) $\angle KWX$ is a right angle	(3) Perpendicular lines form right angles
(4) $WXRK$ is a rectangle	(4) A rectangle is a parallelogram with a right angle



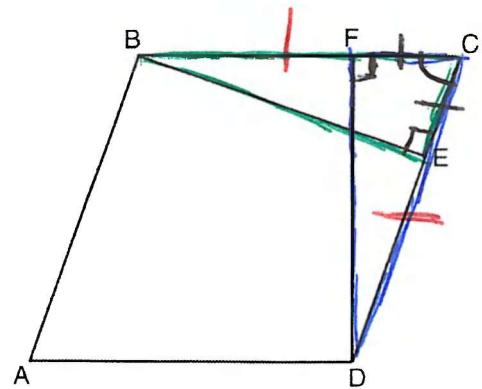
Name Schlansky  
Mr. Schlansky

Date \_\_\_\_\_  
Geometry

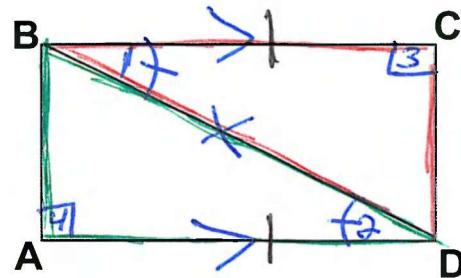
## Proving All Parallelograms Part IV

1. In the diagram of parallelogram  $ABCD$  below,  $\overline{BE} \perp \overline{CD}$ ,  $\overline{DF} \perp \overline{BC}$ ,  $\overline{CE} \cong \overline{CF}$ .  
Prove  $ABCD$  is a rhombus.

Statements	Reasons
(1) $ABCD$ is a p-gram	(1) Given
(2) $BE \perp CD$ , $DF \perp BC$	(2) Given
(3) $\angle DFC \cong \angle BEC$	(3) Perpendicular lines form congruent right angles
(4) $CE \cong CF$	(4) Given (5) Reflexive Property
(5) $\angle C \cong \angle C$	(6) $\angle BCE \cong \angle DCF$
(7) $\triangle BCE \cong \triangle DCF$	(7) ASA $\cong$ ASA
(8) $\overline{BC} \cong \overline{CD}$	(8) CPCTC
(9) $ABCD$ is a rhombus	(9) A rhombus is a parallelogram with consecutive sides $\cong$



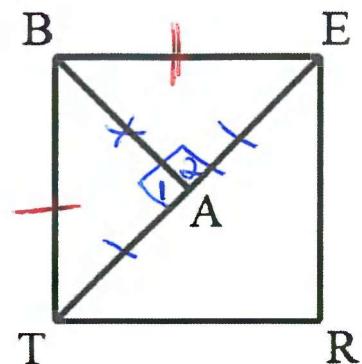
2. Given:  $\overline{BC} \parallel \overline{AD}$ ,  $\overline{BA} \perp \overline{AD}$ ,  $\overline{BC} \perp \overline{CD}$   
Prove:  $ABCD$  is a rectangle



Statements	Reasons
(1) $\overline{BC} \parallel \overline{AD}$	(1) Given
(2) $\angle 1 \cong \angle 2$	(2) Parallel lines cut by a transversal creates $\cong$ alternate interior angles
(3) $\overline{BA} \perp \overline{AD}$ , $\overline{BC} \perp \overline{CD}$	(3) Given (4) Perpendicular lines create congruent right angles
(5) $\angle 3 \cong \angle 4$	(5) Reflexive property
(6) $\overline{BD} \cong \overline{BD}$	(6) AAS $\cong$ AAS
(7) $\triangle BAD \cong \triangle DCB$	(7) CPCTC
(8) $\overline{BC} \cong \overline{AD}$	(8) A parallelogram has 1 pair of opposite sides $\cong$ and $\parallel$
(9) $ABCD$ is a p-gram	(9) Perpendicular lines form right angles
(10) $\angle 4$ is a right angle	(10) A rectangle is a parallelogram with a right angle.
(11) $ABCD$ is a rectangle	

have rectangle and rhombus property

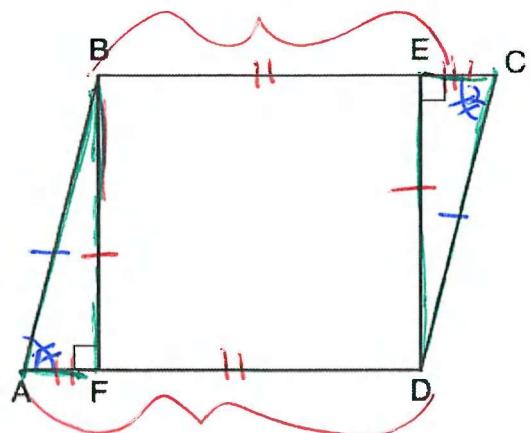
3. Given:  $BERT$  is a rectangle,  $\overline{BA}$  is the perpendicular bisector of  $\overline{TE}$ .  
Prove  $BERT$  is a square.



Statements	Reasons
① $BERT$ is a rectangle	① Given
② $\overline{BA}$ is the perpendicular bisector of $\overline{TE}$	② Given
③ $\overline{TA} \cong \overline{AE}$	③ A line bisector creates two congruent segments
④ $\angle 1 \cong \angle 2$	④ Perpendicular lines form congruent right angles
⑤ $\overline{BA} \cong \overline{BA}$	⑤ Reflexive property
⑥ $\triangle BAT \cong \triangle BAE$	⑥ SAS $\cong$ SAS
⑦ $\overline{BE} \cong \overline{BT}$	⑦ CPCTC
⑧ $BERT$ is a square	⑧ A square is a rectangle with consecutive sides $\cong$

→ need  $BEDF$  is parallelogram first

4. Given: Parallelogram  $ABCD$ ,  $\overline{BF} \perp \overline{AFD}$ , and  $\overline{DE} \perp \overline{BEC}$   
Prove:  $BEDF$  is a rectangle



Statements	Reasons
① $ABCD$ is a parallelogram	① Given
② $\angle 1 \cong \angle 2$	② A parallelogram has opposite angles congruent
③ $\overline{AB} \cong \overline{DC}$	③ A parallelogram has opposite sides congruent.
④ $\overline{BF} \perp \overline{AFD}$ , $\overline{DE} \perp \overline{BEC}$	④ Given
⑤ $\angle BFA \cong \angle DEC$	⑤ Perpendicular lines form congruent right angles
⑥ $\triangle BFA \cong \triangle DEC$	⑥ AAS $\cong$ AAS
⑦ $\overline{BF} \cong \overline{ED}$	⑦ CPCTC
⑧ $\overline{BC} \cong \overline{AD}$	⑧ A parallelogram has opposite sides congruent
⑨ $\overline{EC} \cong \overline{AF}$	⑨ CPCTC
⑩ $\overline{BE} \cong \overline{FD}$	⑩ Subtraction property
⑪ $BC - EC = AD - AF$	⑪ A parallelogram has 2 pairs of opposite sides $\cong$
⑫ $BEDF$ is a parallelogram	⑫ Perpendicular lines form right angles
⑬ $\angle BFA$ is a right angle	⑬ A rectangle is a parallelogram with a right angle
⑭ $BEDF$ is a rectangle	