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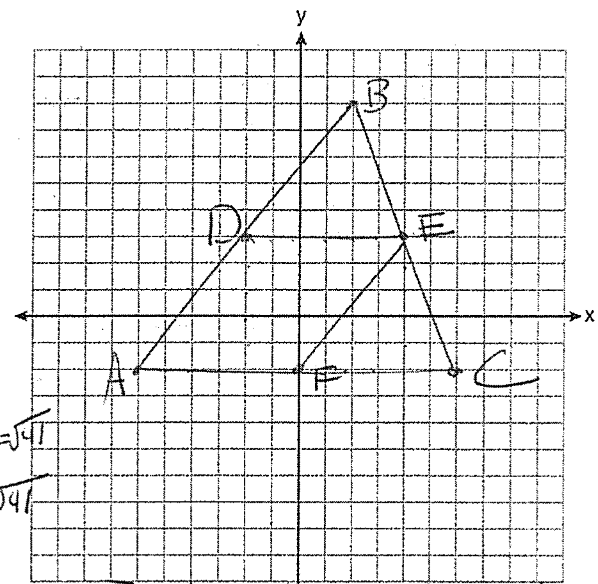
Date ______________
Geometry

Coordinate Geometry Proofs Applications

1. Given: $\triangle ABC$ with vertices $A(-6,-2)$, $B(2,8)$, and $C(6,-2)$. $\overline{AB}$ has midpoint D, $\overline{BC}$ has midpoint E, and $\overline{AC}$ has midpoint F.
Prove: *ADEF* is a parallelogram
ADEF is *not* a rhombus
[The use of the grid is optional.]



D
midpoint $\overline{AB}$
$\frac{-6+2}{2}, \frac{-2+8}{2}$
$\frac{-4}{2}, \frac{6}{2}$
$(-2,3)$

E
midpoint $\overline{BC}$
$\frac{2+6}{2}, \frac{8+-2}{2}$
$\frac{8}{2}, \frac{6}{2}$
$(4,3)$

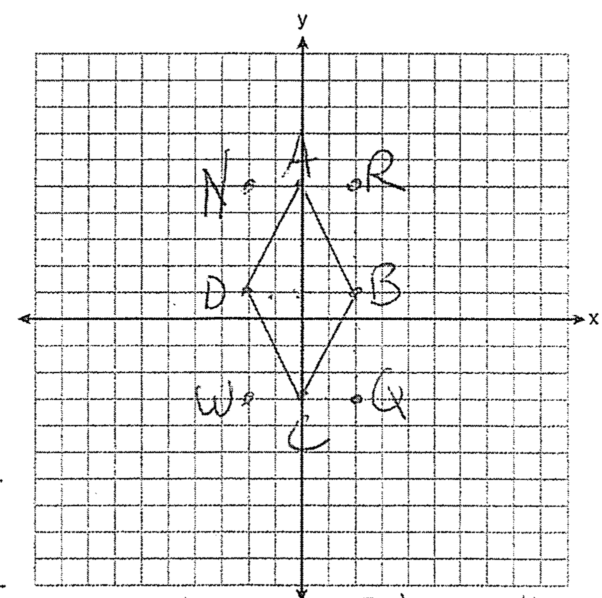
F
midpoint $\overline{AC}$
$\frac{-6+6}{2}, \frac{-2+-2}{2}$
$\frac{0}{2}, \frac{-4}{2}$
$(0,-2)$

1) ADEF is a parallelogram because it has 2 pairs of opposite sides congruent. It is not a rhombus because not all sides are congruent

2) $d\,\overline{AD} = \sqrt{4^2+5^2} = \sqrt{16+25} = \sqrt{41}$
$d\,\overline{FE} = \sqrt{4^2+5^2} = \sqrt{16+25} = \sqrt{41}$
$d\,\overline{DE} = 6$
$d\,\overline{AF} = 6$

3) $\overline{AD} \cong \overline{FE}$ and $\overline{DE} \cong \overline{AF}$ because they have the same distance
$\overline{AD} \not\cong \overline{DE}$ because they don't have the same distance.

2. The vertices of rectangle NRQW are N(-2,5), R(2,5), Q(2,-3), and W(-2,-3). If A is the midpoint $\overline{NR}$, B is the midpoint of $\overline{RQ}$, C is the midpoint of $\overline{QW}$, and D is the midpoint of $\overline{WN}$, prove that ABCD is a ~~parallelogram but *not* a~~ rhombus.



A
midpoint $\overline{NR}$
$\frac{-2+2}{2}, \frac{5+5}{2}$
$\frac{0}{2}, \frac{10}{2}$
$(0,5)$

B
midpoint $\overline{RQ}$
$\frac{2+2}{2}, \frac{5+-3}{2}$
$\frac{4}{2}, \frac{2}{2}$
$(2,1)$

C
midpoint $\overline{QW}$
$\frac{2+-2}{2}, \frac{-3+-3}{2}$
$\frac{0}{2}, \frac{-6}{2}$
$(0,-3)$

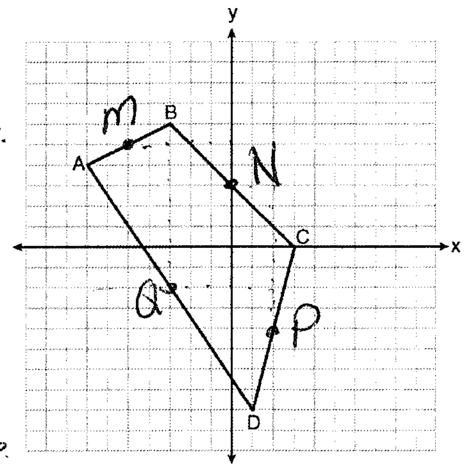
D
midpoint $\overline{WN}$
$\frac{-2+-2}{2}, \frac{5+-3}{2}$
$\frac{-4}{2}, \frac{2}{2}$
$(-2,1)$

1) ABCD is a rhombus because all sides are congruent

2) $d\,\overline{DA} = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20}$
$d\,\overline{AB} = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20}$
$d\,\overline{BC} = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20}$
$d\,\overline{CD} = \sqrt{2^2+4^2} = \sqrt{4+16} = \sqrt{20}$

3) $\overline{DA} \cong \overline{AB} \cong \overline{BC} \cong \overline{CD}$ because they have the same distance

3. Quadrilateral *ABCD* with vertices $A(-7,4)$, $B(-3,6)$, $C(3,0)$, and $D(1,-8)$ is graphed on the set of axes below. Quadrilateral *MNPQ* is formed by joining *M*, *N*, *P*, and *Q*, the midpoints of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, and $\overline{AD}$, respectively. Prove that quadrilateral *MNPQ* is a parallelogram. Prove that quadrilateral *MNPQ* is *not* a rhombus.



1) MNPQ is a parallelogram because it has 2 pairs of opposite sides congruent. It is not a rhombus because not all sides are congruent.

2) $d\,\overline{MN} = \sqrt{5^2+2^2} = \sqrt{25+4} = \sqrt{29}$

$d\,\overline{NP} = \sqrt{2^2+7^2} = \sqrt{4+49} = \sqrt{53}$

$d\,\overline{PQ} = \sqrt{5^2+2^2} = \sqrt{25+4} = \sqrt{29}$

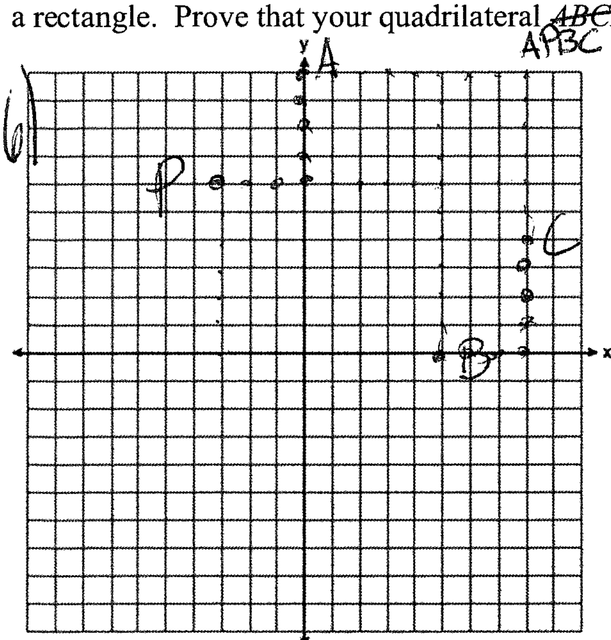
$d\,\overline{QM} = \sqrt{2^2+7^2} = \sqrt{4+49} = \sqrt{53}$

3) $\overline{MN} \cong \overline{PQ}$, $\overline{NP} \cong \overline{QM}$ because they have the same distance.

$\overline{MN} \not\cong \overline{NP}$ because they don't have the same distance.

4. In the coordinate plane, the vertices of Triangle *ABC* are A(0,10) B(5,0) and C(8,4). Prove that Triangle *ABC* is a right triangle. State the coordinates of point *P* such that quadrilateral *ABCP* is a rectangle. Prove that your quadrilateral ~~*ABCP*~~ APBC is a rectangle.

P(-3,6)



1) ABC is a right triangle because its sides fit into Pythagorean Theorem.

2) $d\,\overline{AB} = \sqrt{5^2+10^2} = \sqrt{25+100} = \sqrt{125}$

$d\,\overline{BC} = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25}$

$d\,\overline{CA} = \sqrt{8^2+6^2} = \sqrt{64+36} = \sqrt{100}$

3) $a^2+b^2=c^2$

$\cancel{\sqrt{125}^2 + \sqrt{25}^2}$

$\sqrt{25}^2 + \sqrt{100}^2 \stackrel{?}{=} \sqrt{125}^2$

$25+100 = 125$

$125 = 125$ ✓

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1) APBC is a rectangle because it has 2 pairs of opposite sides congruent and diagonals congruent.

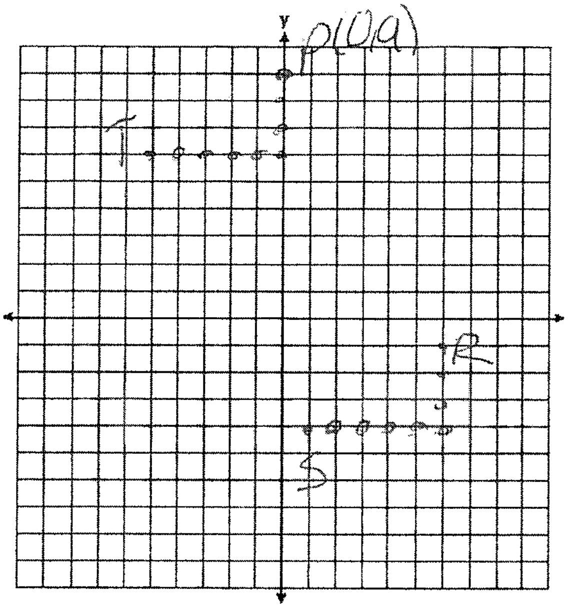
2) $d\,\overline{PA} = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25}$

$d\,\overline{PB} = \sqrt{8^2+6^2} = \sqrt{64+36} = \sqrt{100}$

$d\,\overline{PC} = \sqrt{11^2+2^2} = \sqrt{121+4} = \sqrt{125}$

3) $\overline{AC} \cong \overline{PB}$, $\overline{AP} \cong \overline{CB}$, $\overline{AB} \cong \overline{PC}$ because they have the same distance.

5 8. In the coordinate plane, the vertices of ΔRST are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$. Prove that ΔRST is a right triangle. State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle. Prove that your quadrilateral $RSTP$ is a rectangle. [The use of the set of axes below is optional.]



1) ΔRST is a right triangle because the sides fit into Pythagorean Theorem.

2) $d\overline{ST} = \sqrt{6^2+10^2} = \sqrt{36+100} = \sqrt{136}$

$d\overline{SR} = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$

$d\overline{RT} = \sqrt{11^2+7^2} = \sqrt{121+49} = \sqrt{170}$

3) $a^2+b^2=c^2$

$\sqrt{136}^2+\sqrt{34}^2=\sqrt{170}^2$

$170=170$

1) RSTP is a rectangle because it has 2 pairs of opposite sides congruent and diagonals are congruent.

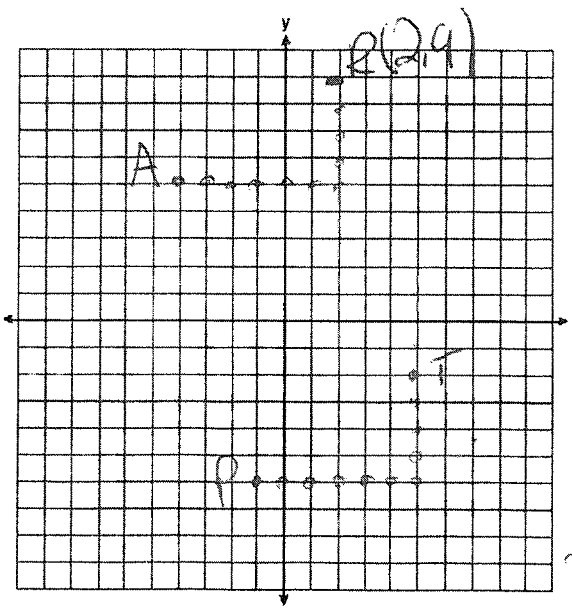
2) $d\overline{TP} = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$

$d\overline{PR} = \sqrt{6^2+10^2} = \sqrt{36+100} = \sqrt{136}$

$d\overline{PS} = \sqrt{1^2+13^2} = \sqrt{1+169} = \sqrt{170}$

3) $\overline{TP}\cong\overline{SR}$, $\overline{TS}\cong\overline{PR}$, $\overline{TR}\cong\overline{PS}$ because they have the same distance.

6 9. In the coordinate plane, the vertices of triangle PAT are $P(-1,-6)$, $A(-4,5)$, and $T(5,-2)$. Prove that $\triangle PAT$ is an isosceles triangle. [The use of the set of axes below is optional.] State the coordinates of R so that quadrilateral $PART$ is a parallelogram. Prove that quadrilateral $PART$ is a parallelogram.



1) ΔPAT is isosceles because two sides are congruent.

2) $d\overline{AP} = \sqrt{3^2+11^2} = \sqrt{9+121} = \sqrt{130}$

$d\overline{AT} = \sqrt{9^2+7^2} = \sqrt{81+49} = \sqrt{130}$

3) $\overline{AP}\cong\overline{AT}$ because they have the same distance.

1) PART is a parallelogram because two pairs of opposite sides are congruent.

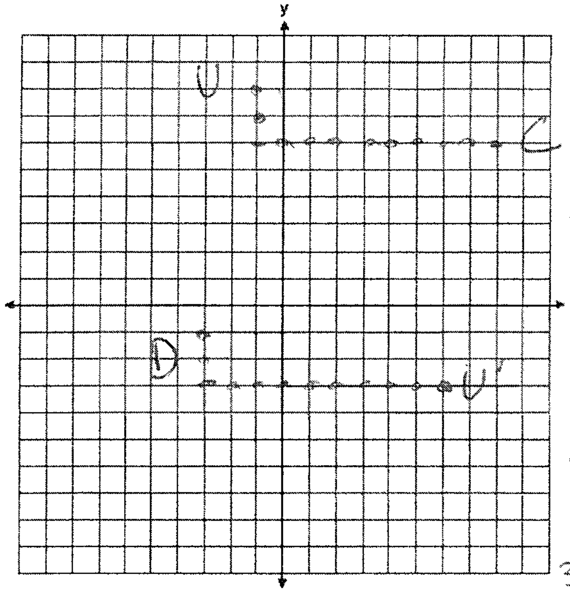
2) $d\overline{AR} = \sqrt{6^2+4^2} = \sqrt{36+16} = \sqrt{52}$

$d\overline{TR} = \sqrt{3^2+11^2} = \sqrt{9+121} = \sqrt{130}$

$d\overline{PT} = \sqrt{6^2+4^2} = \sqrt{36+16} = \sqrt{52}$

3) $\overline{AR}\cong\overline{PT}$, $\overline{AP}\cong\overline{RT}$ because they have the same distance.

7 ~~10~~. Given: Triangle DUC with coordinates $D(-3,-1)$, $U(-1,8)$, and $C(8,6)$
Prove: $\triangle DUC$ is a right triangle
Point U is reflected over $\overline{DC}$ to locate its image point, U', forming quadrilateral $DUCU'$.
Prove quadrilateral $DUCU'$ is a square.
[The use of the set of axes below is optional.]



1) $\triangle DUC$ is a right triangle because its sides fit into Pythagolean theolem.

2) $d\overline{DU} = \sqrt{2^2+9^2} = \sqrt{4+81} = \sqrt{85}$

$d\overline{UC} = \sqrt{9^2+2^2} = \sqrt{81+4} = \sqrt{85}$

$d\overline{CD} = \sqrt{11^2+7^2} = \sqrt{121+49} = \sqrt{170}$

3) $a^2+b^2=c^2$

$\sqrt{85}^2+\sqrt{85}^2=\sqrt{170}^2$

$170=170$

1) $DUCU'$ is a square because all sides are congruent and diagonals are congruent

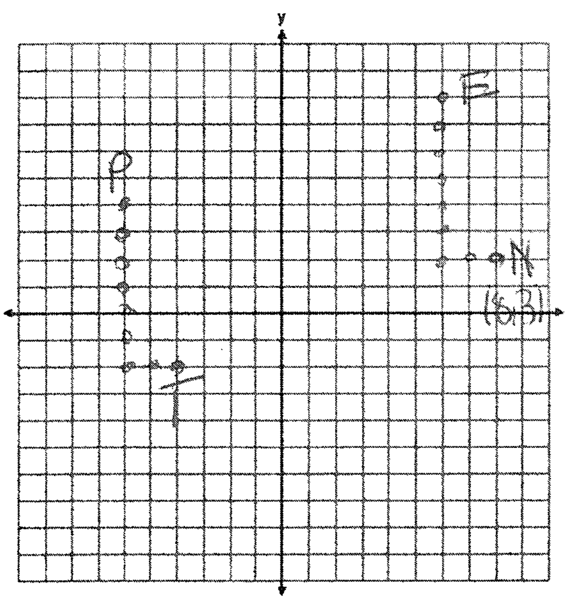
2) $d\overline{DU'} = \sqrt{9^2+2^2} = \sqrt{81+4} = \sqrt{85}$

$d\overline{UU'} = \sqrt{7^2+11^2} = \sqrt{49+121} = \sqrt{170}$

$d\overline{CU'} = \sqrt{2^2+9^2} = \sqrt{4+81} = \sqrt{85}$

3) $\overline{DU} \cong \overline{UC} \cong \overline{CU'} \cong \overline{U'D}$ and $\overline{UU'} \cong \overline{DC}$ because they have the same distance.

8 ~~11~~. Triangle PET has vertices with coordinates $P(-6,4)$, $E(6,8)$, and $T(-4,-2)$. Prove $\triangle PET$ is a right triangle. State the coordinates of N, the image of P, after a 180° rotation centered at $(1,3)$. Prove $PENT$ is a rectangle. [The use of the set of axes below is optional.]



1) $\triangle PET$ is a right triangle because its sides fit into Pythagolean theolem.

2) $d\overline{PE} = \sqrt{12^2+4^2} = \sqrt{144+16} = \sqrt{160}$

$d\overline{ET} = \sqrt{10^2+10^2} = \sqrt{100+100} = \sqrt{200}$

$d\overline{TP} = \sqrt{2^2+6^2} = \sqrt{4+36} = \sqrt{40}$

3) $a^2+b^2=c^2$

$\sqrt{40}^2+\sqrt{160}^2=\sqrt{200}^2$

$40+160=200$

$200=200$

1) $PENT$ is a rectangle because it has two pairs of opposite sides congruent and diagonals congruent.

2) $d\overline{TN} = \sqrt{12^2+4^2} = \sqrt{144+16} = \sqrt{160}$

$d\overline{EN} = \sqrt{2^2+6^2} = \sqrt{4+36} = \sqrt{40}$

$d\overline{PN} = \sqrt{14^2+2^2} = \sqrt{196+4} = \sqrt{200}$

3) $\overline{PE} \cong \overline{TN}$, $\overline{PT} \cong \overline{EN}$, $\overline{PN} \cong \overline{TE}$ because they have the same distance

You can just find the 4th point the same way as the previous page by counting!