

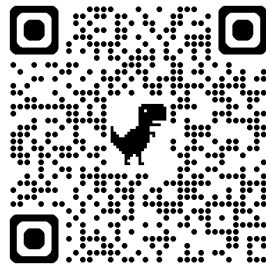
**Name:**

# **Common Core Algebra II**

## **Unit 4/5**

### **Functions Exponents and Logarithms**

**Mr. Schlansky**



**Lesson 1: I can evaluate exponents using the exponents rules.**

**Multiplying: Add exponents**

$$x^2 \bullet x^3 = x^{2+3} = x^5$$

**Dividing: Subtract exponents:**

$$\frac{x^8}{x^5} = x^{8-5} = x^3$$

**When raising a power to a power, multiply exponents:**

$$(x^2)^3 = x^{2 \cdot 3} = x^6$$

Anything to the zero power is equal to 1

$$x^0 = 1$$

**Negative exponents are fractions!**

$$x^{-2} = \frac{1}{x^2}$$

**If exponent is outside parenthesis, everything gets it**

$$\left(\frac{xy}{z}\right)^3 = \frac{x^3 y^3}{z^3}$$

Radicals are fractional exponents (Fractional exponent =  $\frac{\text{power}}{\text{root}}$ )

Get rid of parenthesis

Negative exponents are fractions (Move whatever is being raised to the negative power)

Clean it up (Multiply/Divide/Put back in radical)

**Lesson 2: I can evaluate negative exponents by making a fraction and moving what's being raised to the negative power.**

Same notes as Lesson 1.

**Lesson 3: I can evaluate fractional exponents by understanding radicals are fractional exponents and putting  $\frac{\text{power}}{\text{root}}$ .**

Same notes as Lesson 1.

**Lesson 4: I can evaluate radicals by understanding radicals are fractional exponents and putting  $\frac{\text{power}}{\text{root}}$ .**

If given a radical, re-write as a fractional exponent. Put everything inside the radical into parentheses and the fractional exponent on the outside of the parentheses.

Follow notes from lesson 1 from there.

**Lesson 5: I can prepare for my exponents quiz by practicing and following my procedure.**

Radicals are fractional exponents

Get rid of parenthesis

Negative exponents are fractions

Clean it up MDE

(Use notes from Lesson 1)

**Lesson 6: I can solve constant exponential equations by raising both sides to the reciprocal power.**

- 1) Isolate the base
- 2) Raise both sides to the reciprocal power
- 3) Solve Equation

\*If the root is even, don't forget  $\pm$

**Lesson 7: I can evaluate logarithms by raising the base to a power to get the solution.**

Evaluating Logs: The base to what power equals the answer.  $\log_8 64 \rightarrow 8^x = 64$

**Lesson 8: I can convert between exponential and logarithmic form using the base of the log is the base of the exponent**

The base of the log is the base of the exponent!

$$\log_8 64 \rightarrow 8^x = 64$$

**Lesson 9: I can expand logarithms using the product, quotient, and power rules.**

**Product Rule:**  $\log a \bullet b = \log a + \log b$

**Quotient Rule:**  $\log \frac{a}{b} = \log a - \log b$

**Power Rule:**  $\log a^p = p \log a$

Apply product and quotient rules first

Apply power rule last

\*If given radical, re-write as a fractional exponent first

**Lesson 10: I can solve variable exponential equations by taking the log of both sides.**

**Variable Exponential Equations**

Isolate the base

Take the log of both sides

Bring the exponent in front of the log

Solve the equation

\*If the base is  $e$ , use  $\ln$  instead of  $\log$  (you don't have to but this is what is generally done in upper level courses)

**Lesson 11: I can solve variable exponential equations multiple choice problems by storing each potential answer, typing in the left hand side, typing in the right hand side, and seeing if they match.**

Same strategy as any multiple choice equation!

Store the expression given in each answer

Type the left hand side into the calculator  
Type the right hand side into the calculator  
If the left hand side matches the right hand side, that is the answer!

**Lesson 12: I can solve word problems by solving exponential equations and taking the log of both sides.**

Same notes as Lesson 8.

Read carefully to identifying what variable you are substituting in for.

If you're doubling, multiply initial value by 2, if tripling multiplying initial value by 3.

**Lesson 13: I can solve Newton's Law of Heating/Cooling Problems by carefully substituting in for each variable, taking the problem one sentence at a time, and taking log of both sides if finding  $k$  or  $t$ .**

The formula will be given to you. Write out what each variable represents and carefully substitute in. There may be multiple questions within the problem so make sure you read only one sentence at a time.

If solving for  $T$ , type the entire right hand side in.

If solving for  $k$  or  $t$ , solve the exponential equation by ISOLATING and taking the log/ln of both sides.

\*Once you find  $k$ , you will need to use that  $k$  value to answer the next question. THE VALUES YOU USED IN THE FIRST QUESTION DO NOT APPLY TO THE SECOND QUESTION.

**Lesson 14: I can transform functions using the translation and reflection rules.**

**Translations (+ or -)**

If adding to  $f(x)$ , the graph moves up or down

If adding to  $x$ , the graph moves left or right (the opposite direction in which you would think)

$y = f(x) + a$  moves UP  $a$  units

$y = f(x) - a$  moves DOWN  $a$  units

$y = f(x + a)$  moves LEFT  $a$  units

$y = f(x - a)$  moves RIGHT  $a$  units

**Reflection (-)**

Reflect over the axis that you are *not* negating.

$y = -f(x)$ , reflection over the  $x$  - axis (negate the  $y$ , reflect over the  $x$ )

$y = f(-x)$ , reflection over the  $y$  - axis (negate the  $x$ , reflect over the  $y$ )

**Lesson 15: I can dilate functions using the dilation rules.**

If multiplying a function, a dilation is being performed.

$y = af(x)$ , Vertical Dilation

If  $|a| > 1$ , vertical stretch by a scale factor of  $a$

If  $|a| < 1$ , vertical shrink/compression by a scale factor of  $a$

$y = f(ax)$ , Horizontal Dilation

If  $|a| > 1$ , horizontal shrink/compression by a scale factor of  $\frac{1}{a}$

If  $|a| < 1$ , horizontal stretch by a scale factor of  $\frac{1}{a}$

\*Horizontal transformations are always the “opposite” of what you would expect.

**Lesson 16: I can transform points using the translation and dilation rules.**

Same notes as Lessons 14 and 15.

If a point moves up, add to the y value.

If a point moves down, subtract from the y value.

If a point moves right, add to the x value

If a point moves left, subtract from the x value

If a vertical dilation is performed, multiply the y value by the scale factor

If a horizontal dilation is performed, multiply the x value by the scale factor (the scale factor is the reciprocal of what you are multiplying by).

**Lesson 17: I can find the inverse of a function graphically by switching the x and y coordinates.**

INVERSE MEANS SWITCH X AND Y!

- 1) Pull “nice” points from the original function and write them down into a table.
- 2) Switch x and y to obtain coordinates of the inverse function.
- 3) Plot those new points.

**Lesson 18: I can find the inverse of a function algebraically by switching x and y and solving for y.**

INVERSE MEANS SWITCH X AND Y!

- 1) Switch x and y
- 2) Solve for y

**Lesson 19: I can solve multiple choice inverse problems using my graphing calculator.**

**If multiple choice:**

Type the original equation into  $y =$  and pull a few nice points from the table

Switch x and y in your table

Type the four choices into  $Y =$  and see which has the switched points in its table

**Lesson 20: I can solve systems of equations graphically using my graphing calculator to find the point(s) of intersection.**

**Systems of Equations Graphically Using TI-84+ (  $f(x) = g(x)$  )**

- 1) Type equations into  $Y_1$  and  $Y_2$
- 2) Zoom 6 (Standard) is your standard window. Adjust window OR try Zoom 0(Fit) if you don't see what you want to see.

- 3) 2<sup>nd</sup> Trace (Calc), 5 (Intersect)
- 4) Place cursor over point of intersection, hit enter, enter, enter. Repeat the process for any other points of intersection.

\*The solutions to the system of equations are the x values of the intersections.

**Lesson 21: I can solve systems of inequalities graphically using my graphing calculator to find the points(s) of intersection and listing the appropriate interval.**

Same notes as lesson 20 to find the intersection points.

Write the interval between the appropriate point(s) of intersection. Intervals are always x values from left to right.

< or > gets a parenthesis (),  $\leq$  or  $\geq$  get a bracket []

**Lesson 22: I can graph exponential and logarithmic graphs by typing the equation into y =, plotting points from the table, and finding the asymptote using the horizontal shift or table tricks.**

**To graph an exponential function:**

- 1) Type equation into y =, plot nice points from the table
- 2) Graph horizontal asymptote (The asymptote is the vertical shift OR the repeated value in the table. DO NOT plot the repeated value points, that value is the asymptote)
- 3) Draw a curve that gets closer and closer to the asymptote but never touches it

**Domain:**  $(-\infty, \infty)$

**Range:**  $(HA, \infty)$  or  $(-\infty, HA)$

**Asymptote:** y = vertical shift/repeated value in table

**End Behavior:**

One direction will be the asymptote value

The other will be  $\pm\infty$

**To graph a logarithmic function:**

- 1) Type equation into y =, plot nice points from the table
- 2) Graph vertical asymptote (The asymptote is the horizontal shift OR the last error in the table before nice values start)
- 3) Draw a curve that gets closer and closer to the asymptote but never touches it

**Domain:**  $(VA, \infty)$  or  $(-\infty, VA)$

**Range:**  $(-\infty, \infty)$

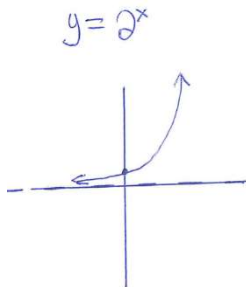
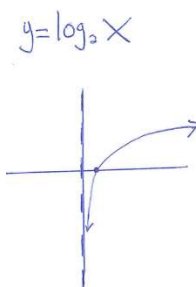
**Asymptote:** x = horizontal shift/last error in table

**End Behavior:**

As x approaches the asymptote, y will approach  $\pm\infty$

As x approaches  $\pm\infty$ , y will approach  $\pm\infty$

**Lesson 23: I can answer exponential and logarithmic graph multiple choice problems by understanding their shape and that they are inverses of each other.**

Exponential	Logarithmic
	
Horizontal Asymptote at $y = 0$	Vertical Asymptote at $x = 0$
Passes through $(0, 1)$	Passes through $(1, 0)$
Domain is all real numbers	Domain is all positive real numbers
Range is all positive real numbers	Range is all real numbers
<b>Exponents and logarithms are inverses of each other!!!!!!!!!!!!</b>	

**Lesson 24: I can graph exponential equations (Part IV) by typing the equation into y = and plotting points, finding the intersection using second trace, and finding t by solving exponential equations using logs.**

1) Type equation(s) into y = and plot points. They will tell you what to graph between either in the problem or on the given graph.

\*Create an appropriate scale that will fit all of the points. Each box must be worth the same value on each axis.

2) Plot the points carefully.

3) For  $f(x) = g(x)$ , 2<sup>nd</sup> Trace: Intersect and state the **x value**.

4) If given a value, substitute it into the appropriate values and solve the equation by taking log of both sides or Y1 Y2 Intersect strategy.

**Lesson 25: I can graph square root and cube root functions by typing the equation into y = and plotting the points.**

1) Type equation into y =

2) Plot points

Cube root functions have a domain and range of all real numbers

Square root functions begin/end at the horizontal shift

**Lesson 26: I can find the average rate of change of a function using  $\frac{y_2 - y_1}{x_2 - x_1}$ .**

**Average rate of change:**  $\frac{y_2 - y_1}{x_2 - x_1}$

ALWAYS CREATE A TABLE

If given a table: circle the values in the table and do  $\frac{\text{bottom} - \text{top}}{\text{bottom} - \text{top}}$

If given a graph: create a table and pull the y values from the graph. Then circle the values in the table and do  $\frac{\text{bottom} - \text{top}}{\text{bottom} - \text{top}}$ .

If given an equation: create a table by typing into Y= and going to 2<sup>nd</sup> Graph (Table). Then circle the values in the table and do  $\frac{\text{bottom} - \text{top}}{\text{bottom} - \text{top}}$ .

**Lesson 27: I can find the average rate of change and write a context sentence using  $\frac{y_2 - y_1}{x_2 - x_1}$**

and the given script.

**Average rate of change:**  $\frac{y_2 - y_1}{x_2 - x_1}$

“On average, from x to x, the y topic is increasing/decreasing by AROC y units per x unit”

**Lesson 28: I can determine which intervals have the fastest and slowest average rate of change by looking at the slopes (graph) or calculating the AROC for each interval (table).**

**Average rate of change:**  $\frac{y_2 - y_1}{x_2 - x_1}$

**Graphs:** The steeper the slope, the greater the average rate of change. The flatter the slope, the slower the average rate of change.

**Tables:** Calculate the average rate of change for each interval.

**Lesson 29: I can determine if functions are even, odd, or neither by determining if it is symmetric to the y axis or the origin.**

**IF GIVEN AN EQUATION, TYPE INTO Y=!!!**

**Even Functions:** Symmetric to the y-axis

**Odd Functions:** Symmetry to the origin (Turn it upside down and see the exact same thing)

**Lesson 30: I can prepare for my exponents/logarithms test by practicing!**



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## ***Exponents Rules***

**Express each of the following in simplest form**

1.  $(4x^3)(2x^5)$

2.  $(3x^4)(7x^8)$

3.  $(-2x^2y^3)(3x^5y)$

4.  $\frac{12x^8}{4x^3}$

5.  $\frac{24x^9}{3x^2}$

6.  $\frac{48x^8y^9}{4xy^8}$

7.  $(x^2)^3$

8.  $(x^4)^6$

9.  $(y^3)^6$

10.  $x^0$

11.  $(4x)^0$

12.  $(12x^3y^2)^0$

13.  $7^2$

14.  $2^4$

15.  $5^3$

$$16. (3x^4y)^2$$

$$17. \left(\frac{2x^2}{y^3}\right)^3$$

$$18. \left(\frac{x^2y}{2mn^3}\right)^4$$

$$19. (x^4y^3)^{\frac{1}{2}}$$

$$20. \left(\frac{x^2}{y^6z^9}\right)^{\frac{1}{3}}$$

$$21. \left(\frac{x^4y^8}{z^3}\right)^{\frac{3}{2}}$$

$$22. \frac{x^4y^5}{x^2y^8}$$

$$23. \frac{2x^7y^4}{4xy^6}$$

$$24. \frac{15x^8y^7}{10x^{10}y^2}$$

$$25. \frac{5x^6 \bullet 4x^3}{2x^5}$$

$$26. (2x^2y^3)^2(3xy)$$

$$27. \frac{(2x^3)^4}{4x^{15}}$$

$$28. \left(\frac{4x^0y^3}{z}\right)^3\left(\frac{2z}{y}\right)^2$$

$$29. \left(\frac{3x^4}{2z^2}\right)^3\left(\frac{x^2}{z}\right)^4$$

$$30. \left(\frac{x^2y}{z^8}\right)^{\frac{1}{2}}(xz^{10})$$

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## *Negative Exponents*

**Reduce each of the following and express with positive exponents**

1.  $\frac{14x^{-2}y^3}{-8x^{-5}y^5}$

2.  $\frac{x^2y^{-3}}{x^{-3}y^{-2}}$

3.  $(3y)^2 (3zy^4)^{-2}$

4.  $\frac{(x^2y)^{-2}}{x^2y^{-3}}$

5. Which expression is equivalent to  $x^{-1} \cdot y^2$ ?

1)  $xy^2$                       3)  $\frac{x}{y^2}$

2)  $\frac{y^2}{x}$                         4)  $xy^{-2}$

6. Which expression is equivalent to  $\frac{x^{-1}y^4}{3x^{-5}y^{-1}}$ ?

1)  $\frac{x^4y^5}{3}$                       3)  $3x^4y^5$

2)  $\frac{x^5y^4}{3}$                       4)  $\frac{y^4}{3x^5}$

7. The expression  $\frac{a^2b^{-3}}{a^{-4}b^2}$  is equivalent to

1)  $\frac{a^6}{b^5}$                       3)  $\frac{a^2}{b}$   
2)  $\frac{b^5}{a^6}$                       4)  $a^{-2}b^{-1}$

**Simplify the following expressions**

8.  $\frac{2x^{-2}y^{-2}}{4y^{-5}}$

9.  $(5^{-2}a^3b^{-4})^{-1}$

10.  $\frac{(3x^{-2}y^2)^2}{9x^{-3}y^{-3}}$

11.  $\frac{3x^{-4}y^5}{(2x^3y^{-7})^{-2}}$

12.  $\frac{(4x^{-2})^{-2}}{(2x^2)(2y)^{-3}}$

13.  $\frac{(2x^{-3})^{-3}}{16(x^2y^{-1})^{-2}}$

14.  $\frac{(2xy^2)^{-2}}{(8x^{-2}y)^{-1}(2y^2)^{-2}}$

15.  $\frac{(3x^2y^{-2})^2}{(2x^2y^{-1})^2(3x^{-5})}$

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## ***Fractional Exponents***

**Rewrite the following as radicals**

1.  $x^{\frac{2}{3}}$

2.  $x^{\frac{3}{4}}$

3.  $x^{\frac{5}{6}}$

4.  $x^{\frac{1}{3}}$

5.  $x^{\frac{3}{2}}$

6.  $x^{\frac{1}{2}}$

7.  $x^{\frac{4}{5}}$

8.  $x^{\frac{1}{7}}$

9.  $x^{\frac{5}{2}}$

**Rewrite the following using fractional exponents**

10.  $\sqrt[3]{x^4}$

11.  $\sqrt[5]{x^3}$

12.  $\sqrt[4]{x^7}$

13.  $\sqrt{x^3}$

14.  $\sqrt[6]{x^5}$

15.  $\sqrt{x}$

16.  $\sqrt[8]{x^3}$

17.  $\sqrt[5]{x^3}$

18.  $\sqrt[3]{x}$

**Evaluate each of the following:**

19.  $25^{\frac{1}{2}}$

20.  $8^{\frac{1}{3}}$

21.  $100^{\frac{1}{2}}$

22.  $4^{\frac{3}{2}}$

23.  $27^{\frac{2}{3}}$

24.  $125^{\frac{5}{3}}$

25.  $8^{\frac{5}{3}}$

26.  $81^{\frac{3}{4}}$

27.  $16^{\frac{3}{2}}$

28.  $16^{\frac{5}{4}}$

29.  $36^{\frac{3}{2}}$

30.  $32^{\frac{2}{5}}$

31. Explain what a rational exponent, such as  $\frac{5}{2}$  means. Use this explanation to evaluate  $9^{\frac{5}{2}}$ .

32. Explain how  $125^{\frac{4}{3}}$  can be evaluated using properties of rational exponents to result in an integer answer.

33. Explain how  $(-8)^{\frac{4}{3}}$  can be evaluated using properties of rational exponents to result in an integer answer.

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## *Given Radicals*

**Express the following without using radicals:**

1.  $\sqrt{x^{-2}y^5}$

2.  $\sqrt[3]{27x^6y^{-8}}$

3.  $\sqrt{25x^3y^4}$

4.  $\sqrt[3]{64x^{-5}y^{-8}}$

5. The expression  $\sqrt[4]{16x^2y^7}$  is equivalent to

1)  $2x^{\frac{1}{2}}y^{\frac{7}{4}}$

2)  $2x^8y^{28}$

3)  $4x^{\frac{1}{2}}y^{\frac{7}{4}}$

4)  $4x^8y^{28}$

6. The expression  $\sqrt[4]{81x^2y^5}$  is equivalent to

1)  $3x^{\frac{1}{2}}y^{\frac{5}{4}}$

2)  $3x^{\frac{1}{2}}y^{\frac{4}{5}}$

3)  $9xy^{\frac{5}{2}}$

4)  $9xy^{\frac{2}{5}}$

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## ***Fractional Exponents Regents Practice***

**For Multiple Choice, Use Multiple Choice Strategy!!!!!!**

1. Given  $y > 0$ , the expression  $\sqrt{3x^2y} \cdot \sqrt[3]{27x^3y^2}$  is equivalent to

- |                 |  |
|-----------------|--|
| 1) $81x^5y^3$   | 3) $3^{\frac{5}{2}}x^2y^{\frac{5}{3}}$ |
| 2) $3^{15}x^2y$ | 4) $3^{\frac{3}{2}}x^2y^{\frac{7}{6}}$ |

2. The expression  $\left(\frac{m^2}{\frac{1}{m^{\frac{1}{3}}}}\right)^{-\frac{1}{2}}$  is equivalent to

- |                              |                            |
|------------------------------|----------------------------|
| 1) $-\sqrt[6]{m^5}$          | 3) $-m^5\sqrt{m}$          |
| 2) $\frac{1}{\sqrt[6]{m^5}}$ | 4) $\frac{1}{m^5\sqrt{m}}$ |

3. Which equation is equivalent to  $P = 210x^{\frac{4}{3}}y^{\frac{7}{3}}$

- |                              |                                  |
|------------------------------|----------------------------------|
| 1) $P = \sqrt[3]{210x^4y^7}$ | 3) $P = 210xy^2\sqrt[3]{xy}$     |
| 2) $P = 70xy^2\sqrt[3]{xy}$  | 4) $P = 210xy^2\sqrt[3]{x^3y^5}$ |

4. For  $x \geq 0$ , which equation is *false*?

- |  |  |
|--|--|
| 1) $(x^{\frac{3}{2}})^2 = \sqrt[4]{x^3}$ | 3) $(x^{\frac{3}{2}})^{\frac{1}{2}} = \sqrt[4]{x^3}$ |
| 2) $(x^3)^{\frac{1}{4}} = \sqrt[4]{x^3}$ | 4) $(x^{\frac{2}{3}})^2 = \sqrt[3]{x^4}$             |

5. For  $x \neq 0$ , which expressions are equivalent to one divided by the sixth root of  $x$ ?

I.  $\frac{\sqrt[6]{x}}{\sqrt[3]{x}}$     II.  $\frac{x^{\frac{1}{6}}}{x^{\frac{1}{3}}}$     III.  $x^{-\frac{1}{6}}$

- |                    |                     |
|--------------------|---------------------|
| 1) I and II, only  | 3) II and III, only |
| 2) I and III, only | 4) I, II, and III   |



Express the following in simplest form, with a rational exponent.

6.  $a^5\sqrt{a^4}$

7.  $2xy^2\sqrt[3]{x^2y}$

8.  $\frac{\sqrt[3]{x^2} \cdot \sqrt{x^5}}{\sqrt[6]{x}}$

9.  $\frac{x\sqrt{x^3}}{\sqrt[3]{x^5}}$

Express the following in simplest radical form:

10.  $\frac{x^{\frac{1}{5}}}{x^{\frac{1}{2}}}$

11.  $\left(\frac{1}{x^{-2}}\right)^{-\frac{3}{4}}$

12.  $\frac{2x^{\frac{3}{2}}}{\left(16x^4\right)^{\frac{1}{4}}}$

13.  $\frac{(x^2y^4)^{\frac{1}{3}}}{xy}$

Determine the value of  $n$  in each of the following equations:

$$14. \frac{\sqrt[3]{x^8}}{(x^4)^{\frac{1}{3}}} = x^n \qquad 15. \left( \frac{1}{\sqrt[3]{y^2}} \right) y^4 = y^n \qquad 16. \left( \frac{y^{\frac{17}{8}}}{y^{\frac{5}{4}}} \right)^4 = y^n$$

17. Kenzie believes that for  $x \geq 0$ , the expression  $\left(\sqrt[7]{x^2}\right)\left(\sqrt[5]{x^3}\right)$  is equivalent to  $\sqrt[35]{x^6}$ . Is she correct? Justify your response algebraically.

18. Justify why  $\frac{\sqrt[3]{x^2y^5}}{\sqrt[4]{x^3y^4}}$  is equivalent to  $x^{\frac{-1}{12}}y^{\frac{2}{3}}$  using properties of rational exponents, where  $x \neq 0$  and  $y \neq 0$ .

19. For  $n$  and  $p > 0$ , is the expression  $\left(p^2n^{\frac{1}{2}}\right)^8\sqrt{p^5n^4}$  equivalent to  $p^{18}n^6\sqrt{p}$ ? Justify your answer.

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## ***Constant Exponential Equations***

**Solve the following equations for x. Round values to the nearest tenth if necessary.**

1.  $x^{\frac{1}{2}} = 7$

2.  $x^{\frac{2}{3}} = 4$

3.  $2x^3 + 1 = 55$

4.  $3x^{\frac{4}{3}} - 5 = 43$

5.  $3x^{\frac{2}{5}} - 11 = 289$

6.  $x^{\frac{1}{5}} - 6 = -8$

7.  $x^{\frac{4}{3}} - 11 = 5$

8.  $4x^{\frac{3}{4}} - 2 = 254$

9.  $3x^{\frac{2}{5}} + 1 = 76$

10.  $4x^{\frac{2}{3}} - 5 = 95$

11.  $-2x^{\frac{5}{4}} - 7 = -71$

12.  $2x^{\frac{3}{2}} - 5 = 76$

13. Given the equal terms  $\sqrt[3]{x^5}$  and  $y^{\frac{5}{6}}$ , determine and state  $y$ , in terms of  $x$ .

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## *Evaluating Logarithms*

**Evaluate each of the following logarithms:**

1.  $\log_4 4$

2.  $\log_{15} 1$

3.  $\log_2 16$

4.  $\log_3 27$

5.  $\log_6 36$

6.  $\log_4 64$

7.  $\log_2 8$

8.  $\log_5 125$

9.  $\log 100000$

10.  $\log_7 \frac{1}{49}$

11.  $\log_2 \frac{1}{8}$

12.  $\log 0.1$

13.  $\log_6 216$

14.  $\log_{11} \frac{1}{121}$

15.  $\ln e$

16.  $\log_{\frac{1}{2}} \frac{1}{64}$

17.  $\log_9 27$

18.  $\log_4 32$

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## *Converting Logarithms*

Write each of the following in exponential form:

1.  $\log_4 256 = 4$

2.  $\log_x k = 5$

3.  $\log_{12} \frac{1}{144} = -2$

4.  $\log_2 \frac{1}{q} = p$

5.  $\log 100 = 2$

6.  $\ln 50 = x$

7.  $\log_{x+2}(x^2 - 5x + 4) = 4$

8.  $\log_3(x^2 + 4) = x + 7$

9.  $\log(x^3 - 7x) = 4$

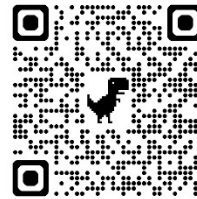
10.  $\log_{x-9} 2 = x + 1$

11.  $\log_2\left(\frac{x^2 - 4x + 1}{x - 7}\right) = x + 2$

12.  $\log_x\left(\frac{x^2 + x - 3}{x + 1}\right) = 6$

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## ***Logarithm Rules***

**Express as multiple logs**

1.  $\log xy$

2.  $\ln \frac{7}{x}$

3.  $\ln x^3$

4.  $\log \frac{x^4 y^2}{z}$

5.  $\ln x^3 y^2$

6.  $\log \frac{a^2 b}{c^4}$

$$7. \log \frac{a^5 b^3}{c^6}$$

$$8. \ln \frac{x^2 y^6}{c^3}$$

$$9. \ln \frac{\sqrt{x}}{y^3}$$

$$10. \log \frac{\sqrt[4]{x^3} y^2}{\sqrt[3]{z}}$$

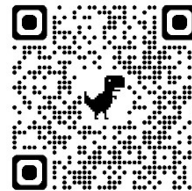
$$11. \log \frac{x^5 y}{\sqrt[3]{z}}$$

$$12. \log \frac{m^3 \sqrt{n}}{k^2}$$



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## *Variable Exponential Equations*

Solve each of the following and round to the *nearest hundredth*.

1.  $3^{2x} = 182$

2.  $e^{2n} = 245$

3.  $3(5)^{2x} = 60$

4.  $20e^{4x} = 120$

5.  $250(1.04)^{4x} = 500$

6.  $48e^{12x} = 60$

7.  $1.2(4)^{2x} = 20$

8.  $400(.987)^{2.5x} = 300$

9.  $2(3)^{2x} + 8 = 18$

10.  $4(2)^{3x} + 3 = 15$

11.  $8 + 2e^{-5x} = 14$

12.  $12 + 2(5)^{8x} = 2000$

13.  $500e^{\frac{x}{2}} = 200$

14.  $2000(2)^{\frac{x}{4.2}} = 1500$

15.  $1.2(3)^{\frac{x}{4.1}} + 15 = 195$

16.  $18 - 4(6)^{\frac{x}{3}} = 16$

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## *Exponential Equations Multiple Choice*

1. Which is the solution to:  $2(3)^{4x} + 1 = 11$  ?

- |                              |                              |
|------------------------------|------------------------------|
| 1) $\frac{\log 5}{4 \log 3}$ | 3) $\frac{\log 3}{4 \log 5}$ |
| 2) $\frac{4 \log 5}{\log 3}$ | 4) $\frac{4 \log 3}{\log 5}$ |

2. Which is the solution to:  $256 + 4(2)^{6x} = 2700$  ?

- |                              |                               |
|------------------------------|-------------------------------|
| 1) $\frac{\ln 4}{6 \ln 2}$   | 3) $\frac{\ln 611}{6 \ln 2}$  |
| 2) $\frac{6 \ln 423}{\ln 4}$ | 4) $\frac{6 \ln 2444}{\ln 4}$ |

3. Which is the solution to:  $1 - 2(5)^{2x} = -5$  ?

- |                            |                            |
|----------------------------|----------------------------|
| 1) $\frac{\ln 6}{2 \ln 3}$ | 3) $\frac{2 \ln 4}{\ln 3}$ |
| 2) $\frac{2 \ln 5}{\ln 1}$ | 4) $\frac{\ln 3}{2 \ln 5}$ |

4. Which is the solution to:  $5(3)^{2x} = 30$  ?

- |                              |                              |
|------------------------------|------------------------------|
| 1) $\frac{\log 6}{3 \log 2}$ | 3) $\frac{2 \log 6}{\log 3}$ |
| 2) $\frac{\log 6}{2 \log 3}$ | 4) $\frac{2 \log 3}{\log 6}$ |

5. The solution to the equation  $5e^{x+2} = 7$  is

- 1)  $-2 + \ln\left(\frac{7}{5}\right)$       3)  $\frac{-3}{5}$   
2)  $\left(\frac{\ln 7}{\ln 5}\right) - 2$       4)  $-2 + \ln(2)$

6. What is the solution of  $2(3^{x+4}) = 56$ ?

- 1)  $x = \log_3(28) - 4$       3)  $x = \log(25) - 4$   
2)  $x = -1$       4)  $x = \frac{\log(56)}{\log(6)} - 4$

7. The solution to the equation  $6(2^{x+4}) = 36$  is

- 1)  $-1$       3)  $\ln(3) - 4$   
2)  $\frac{\ln 36}{\ln 12} - 4$       4)  $\frac{\ln 6}{\ln 2} - 4$

8. Which expression is *not* a solution to the equation  $2^t = \sqrt{10}$ ?

- 1)  $\frac{1}{2} \log_2 10$       3)  $\log_4 10$   
2)  $\log_2 \sqrt{10}$       4)  $\log_{10} 4$

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## ***Exponential Equations Word Problems***

1. A population of wolves in a county is represented by the equation  $P(t) = 80(0.98)^t$ , where  $t$  is the number of years since 1998. After how many years will the population of wolves be 60 rounded to the *nearest year*?

2. Juliette deposits \$3000 into a bank account where the balance of the account  $b(t)$  after  $t$  years can be represented by  $b(t) = 3000e^{.042t}$ . To the nearest tenth of a year, how long will it take for Juliette's money to double?

3. 200 grams of a radioactive substance decays according to the formula  $a(t) = 200(.094)^{2t}$  where  $a(t)$  is the amount of the radioactive substance remaining after  $t$  years. To the nearest hundredth of a year, how long will it take until there are 150 grams remaining?

4. After an oven is turned on, its temperature,  $T$ , is represented by the equation  $T = 400 - 350(3.2)^{-0.1m}$ , where  $m$  represents the number of minutes after the oven is turned on and  $T$  represents the temperature of the oven, in degrees Fahrenheit.

How many minutes does it take for the oven's temperature to reach  $300^{\circ}\text{F}$ ? Round your answer to the *nearest minute*.

5. Drew's parents invested \$1,500 in an account such that the value of the investment doubles every seven years. The value of the investment,  $V$ , is determined by the equation  $V = 1500(2)^{\frac{t}{7}}$ , where  $t$  represents the number of years since the money was deposited. How many years, to the *nearest tenth of a year*, will it take the value of the investment to triple?

6. The number of houses in Central Village, New York, grows every year according to the function  $H(t) = 540(1.039)^{1.02t}$ , where  $H$  represents the number of houses, and  $t$  represents the number of years since January 1995. A civil engineering firm has suggested that a new, larger well must be built by the village to supply its water when the number of houses exceeds 1,000. During which year will this first happen?

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## ***Newton's Law of Heating and Cooling***

1. The Fahrenheit temperature of a heated object can be modeled by the function below.

$$F(t) = F_s + (F_0 - F_s)e^{-kt}$$

$F(t)$  = the temperature of the object after  $t$  minutes

$t$  = time in minutes

$F_s$  = the surrounding temperature

$F_0$  = the initial temperature of the object

$k$  = a constant

Hot chocolate at a temperature of  $200^\circ\text{F}$  is poured into a container. The room temperature is kept at a constant  $68^\circ\text{F}$  and  $k = 0.05$ .

After how much time, to the *nearest minute*, will the temperature of the hot chocolate be  $150^\circ\text{F}$ ?

After how much time, to the *nearest tenth of a minute*, will the temperature of the hot chocolate be  $120^\circ\text{F}$ ?

2. The Fahrenheit temperature,  $F(t)$ , of a heated object at time  $t$ , in minutes, can be modeled by the function below.  $F_s$  is the surrounding temperature,  $F_0$  is the initial temperature of the object, and  $k$  is a constant.

$$F(t) = F_s + (F_0 - F_s)e^{-kt}$$

Coffee at a temperature of  $195^\circ\text{F}$  is poured into a container. The room temperature is kept at a constant  $68^\circ\text{F}$  and  $k = 0.05$ . Coffee is safe to drink when its temperature is, at most,  $120^\circ\text{F}$ . To the *nearest minute*, how long will it take until the coffee is safe to drink?

3. After sitting out of the refrigerator for a while, a turkey at room temperature ( $68^{\circ}\text{F}$ ) is placed into an oven at 8 a.m., when the oven temperature is  $325^{\circ}\text{F}$ . Newton's Law of Heating explains that the temperature of the turkey will increase proportionally to the difference between the temperature of the turkey and the temperature of the oven, as given by the formula below:

$$T = T_a + (T_0 - T_a)e^{-kt}$$

$T_a$  = the temperature surrounding the object

$T_0$  = the initial temperature of the object

$t$  = the time in hours

$T$  = the temperature of the object after  $t$  hours

$k$  = decay constant

The turkey reaches the temperature of approximately  $100^{\circ}\text{F}$  after 2 hours. Find the value of  $k$ , to the *nearest thousandth*. Determine the Fahrenheit temperature of the turkey, to the *nearest degree*, at 3 p.m.

4. Empanadas are taken out of an oven when they reached a temperature of  $168^{\circ}\text{F}$  and put on the kitchen table at room temperature ( $68^{\circ}\text{F}$ ). After 8 minutes, the temperature of the empanadas is  $125^{\circ}\text{F}$ . The temperature of a cooled object can be given by the formula below:

$$T = T_a + (T_0 - T_a)e^{-kt}$$

$T$  = the temperature of the object after  $t$  minutes

$t$  = time in minutes

$T_a$  = the surrounding temperature

$T_0$  = the initial temperature of the object

$k$  = decay constant

Find the value of  $k$ , rounded to the *nearest thousandth*. Using your value of  $k$ , to the *nearest minute*, how long will it take for the empanadas to reach  $100^{\circ}\text{F}$ ?



5. Megan is performing an experiment in a lab where the air temperature is a constant 73°F and the liquid is 237°F. One and a half hours later, the temperature of the liquid is 112°F. Newton's law of cooling states  $T(t) = T_a + (T_0 - T_a)e^{-kt}$  where:

$T(t)$ : temperature, °F, of the liquid at  $t$  hours

$T_a$ : air temperature

$T_0$ : initial temperature of the liquid

$k$ : constant

Determine the value of  $k$ , to the *nearest thousandth*, for this liquid. Determine the temperature of the liquid using your value for  $k$ , to the *nearest degree*, after two and a half hours. Megan needs the temperature of the liquid to be 80°F to perform the next step in her experiment. Use your value for  $k$  to determine, to the *nearest tenth of an hour*, how much time she must wait since she first began the experiment.

6. Objects cool at different rates based on the formula below.

$$T = (T_0 - T_R)e^{-rt} + T_R$$

$T_0$ : initial temperature

$T_R$ : room temperature

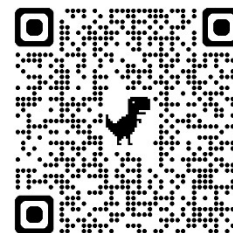
$r$ : rate of cooling of the object

$t$ : time in minutes that the object cools to a temperature,  $T$

Mark makes T-shirts using a hot press to transfer designs to the shirts. He removes a shirt from a press that heats the shirt to 400°F. The rate of cooling for the shirt is 0.0735 and the room temperature is 75°F. Find the temperature of the shirt, to the *nearest degree*, after five minutes. At the same time, Mark's friend Jeanine removes a hoodie from a press that heats the hoodie to 450°F. After eight minutes, the hoodie measured 270°F. The room temperature is still 75°F. Determine the rate of cooling of the hoodie, to the *nearest ten thousandth*. The T-shirt and hoodie were removed at the same time. Determine when the temperature will be the same, to the *nearest minute*.

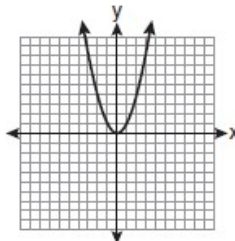
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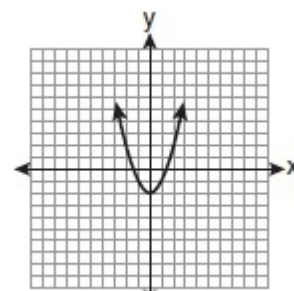
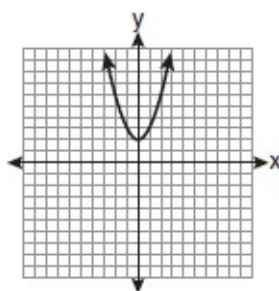
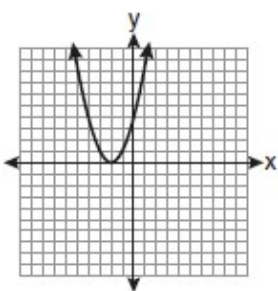
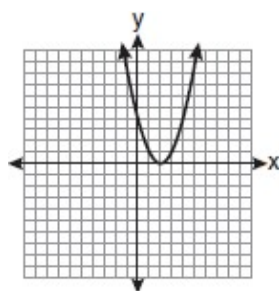
## ***Translations and Reflections Rules***

1. The graph below represents  $f(x)$ .

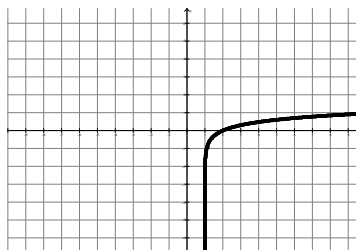


Match the following equations with their graphs:

- a)  $f(x+2)$
- b)  $f(x)+2$
- c)  $f(x-2)$
- d)  $f(x)-2$

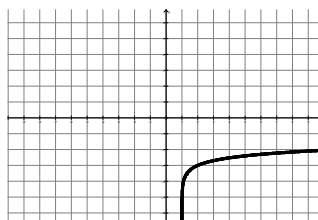
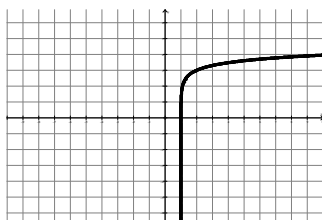
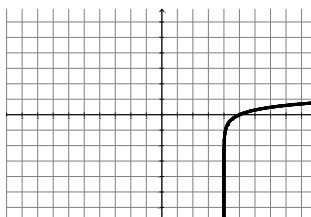
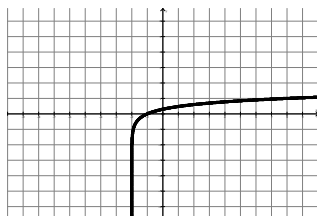


2. The graph below represents  $f(x)$ .



Match the following equations with their graphs:

- a)  $f(x+2)$
- b)  $f(x)+2$
- c)  $f(x-2)$
- d)  $f(x)-2$



3. Which transformation of  $y = 2^x$  results in the function  $y = 2^x - 2$ ?

- 1) Up two units                      3) Right two units
- 2) Down two units                4) Left 2 Units

4. Which transformation of  $y = 2^x$  results in the function  $y = 2^{x-2}$ ?

- 1) Up two units                      3) Right two units
- 2) Down two units                4) Left 2 Units

5. The function  $f(x) = \sqrt{x}$ . Which function represents a shift of the graph left 3 units?

- 1)  $f(x-3) = \sqrt{x-3}$                       3)  $f(x)+3 = \sqrt{x}+3$
- 2)  $f(x+3) = \sqrt{x+3}$                       4)  $f(x)-3 = \sqrt{x}-3$

6. Joey's math class is studying the basic quadratic function,  $f(x) = x^2$ . Each student is supposed to make two new functions by adding or subtracting a constant to the function. Joey chooses the functions  $g(x) = x^2 - 5$  and  $h(x) = x^2 + 2$ . What transformations would map  $f(x)$  to  $g(x)$  and  $f(x)$  to  $h(x)$ ?

- 1) shift left 5, shift right 2                      3) shift up 5, shift down 2
- 2) shift right 5, shift left 2                      4) shift down 5, shift up 2

7. If  $g(x) = f(x-4)+2$ , how is the graph of  $f(x)$  translated to form the graph of  $g(x)$ ?

8. If  $h(x) = f(x+1)-3$ , how is the graph of  $f(x)$  translated to form the graph of  $g(x)$ ?

9. How is the parent function transformed to create  $f(x) = |x+3|-2$ ?

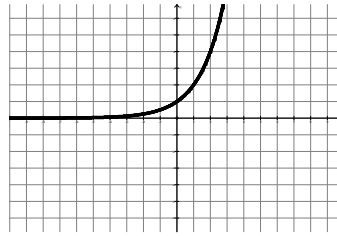
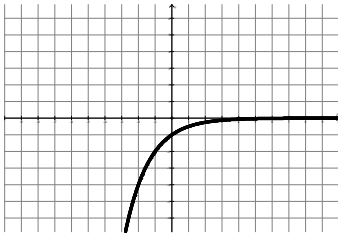
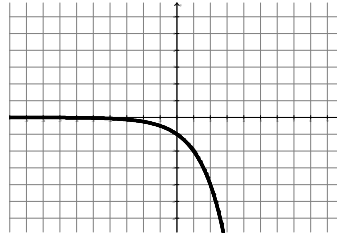
10. How is the parent function transformed to create  $f(x) = (x-4)^2 + 3$ ?

11. The graph to the right represents  $f(x)$ .

Match the following with their graphs:

a) Which graph represents  $f(-x)$

b) Which graph represents  $-f(x)$

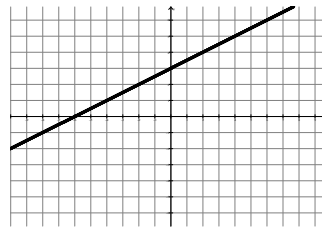
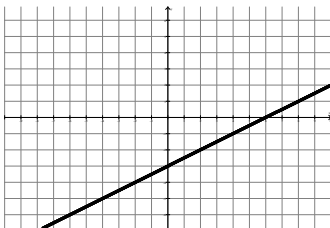
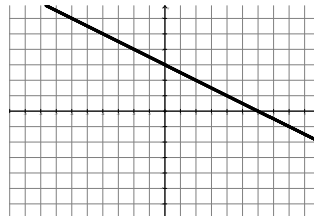


12. The graph to the right represents  $g(x)$ .

Match the following with their graphs:

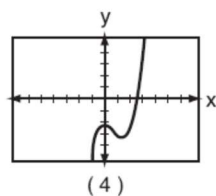
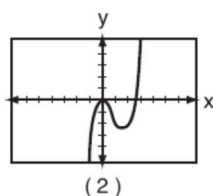
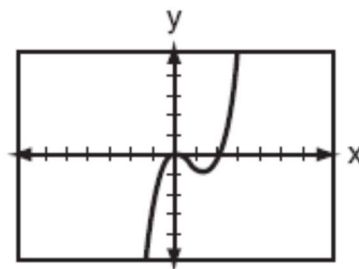
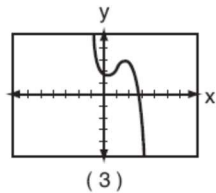
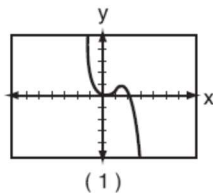
a) Which graph represents  $g(-x)$

b) Which graph represents  $-g(x)$

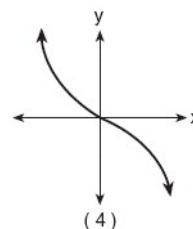
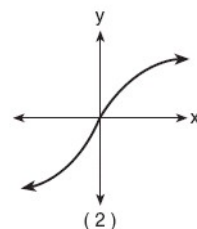
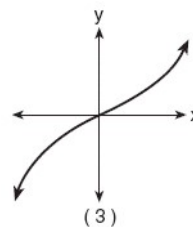
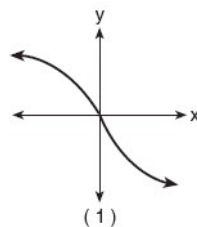
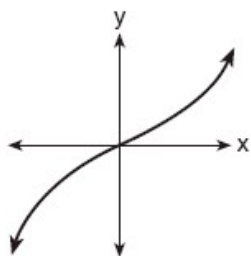


13. The accompanying graph represents the equation  $y = f(x)$ .

Which graph represents  $g(x)$ , if  $g(x) = -f(x)$ ?



14. The graph below represents  $f(x)$ .



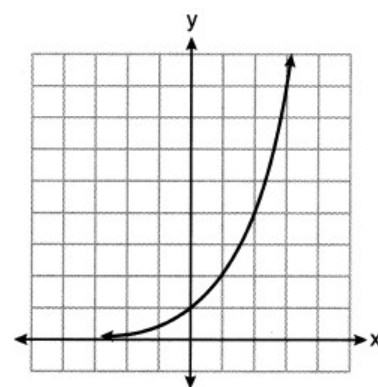
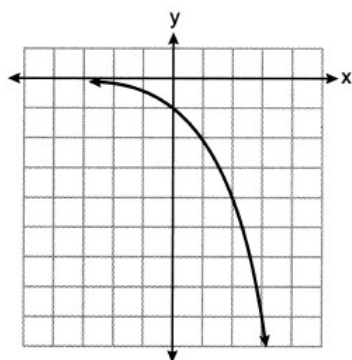
Which graph best represents  $f(-x)$ ?

15. Consider the function  $y = h(x)$ , defined by the graph to the right.

Which equation could be used to represent the graph shown below?

- 1)  $y = h(x) - 2$
- 2)  $y = h(x - 2)$

- 3)  $y = -h(x)$
- 4)  $y = h(-x)$



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## ***Dilating Functions***

**State the transformations that were performed on  $f(x)$  to produce  $g(x)$**

1.  $g(x) = 2f(x)$

2.  $g(x) = f(2x)$

3.  $g(x) = \frac{1}{3}f(x)$

4.  $g(x) = f\left(\frac{1}{3}x\right)$

5.  $g(x) = f(4x)$

6.  $g(x) = \frac{2}{5}f(x)$

7.  $g(x) = f\left(\frac{1}{5}x\right)$

8.  $g(x) = 7f(x)$

9.  $g(x) = f(3x)$

10.  $g(x) = 2f(x-1)-3$

11.  $g(x) = f(2(x+1))-4$

$$12. g(x) = -f\left(\frac{1}{3}(x-5)\right) + 1$$

$$13. g(x) = -\frac{1}{2}f(x+5)$$

$$14. g(x) = 3f(-2(x+1)) - 5$$

$$15. g(x) = -f(4(x+3)) + 1$$

**State the transformations that were applied to the parent function**

$$16. g(x) = -2f(x+1) - 4$$

$$17. f(x) = -\frac{1}{4}f(x-2) + 1$$

$$18. g(x) = \frac{2}{3}f(-(x+1)) - 2$$

$$19. f(x) = -2f(x-1) + 4$$

$$20. f(x) = -f(3(x-1)) + 6$$

$$21. f(x) = -2f(x-1)$$

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## ***Transforming Points***

1. If (2,4) is included in  $f(x)$ , what point must be included in  $g(x)$  if  $g(x) = f(x) + 3$ .
2. If (2,4) is included in  $f(x)$ , what point must be included in  $g(x)$  if  $g(x) = f(x + 3)$ .
3. If (5,1) is included in  $f(x)$ , what point must be included in  $g(x)$  if  $g(x) = f(x - 5)$ .
4. If (4,7) is included in  $f(x)$ , what point must be included in  $g(x)$  if  $g(x) = f(x) - 2$ .
5. If (-3,4) is included in  $f(x)$ , what point must be included in  $g(x)$  if  $g(x) = f(x - 4)$ .
6. If (-3,-2) is included in  $f(x)$ , what point must be included in  $g(x)$  if  $g(x) = f(x) + 4$ .
7. If (3,5) is included in  $f(x)$ , what point must be included in  $g(x)$  if  $g(x) = f(x + 4) - 7$ .
8. If (4,-6) is included in  $f(x)$ , what point must be included in  $g(x)$  if  $g(x) = f(x - 1) + 3$ .



9. If  $(-2,4)$  is included in  $f(x)$ , what point must be included in  $g(x)$  if  $g(x) = 2f(x)$ .
10. If  $(-2,4)$  is included in  $f(x)$ , what point must be included in  $g(x)$  if  $g(x) = f(2x)$ .
11. If  $(4,-8)$  is included in  $f(x)$ , what point must be included in  $g(x)$  if  $g(x) = \frac{1}{2}f(x)$ .
12. If  $(4,-8)$  is included in  $f(x)$ , what point must be included in  $g(x)$  if  $g(x) = f\left(\frac{1}{2}x\right)$ .
13. If  $(-3,2)$  is included in  $f(x)$ , what point must be included in  $g(x)$  if  $g(x) = f\left(\frac{1}{3}x\right)$ .
14. If  $(2,-1)$  is included in  $f(x)$ , what point must be included in  $g(x)$  if  $g(x) = 4f(x)$ .
15. If  $(-8,1)$  is included in  $f(x)$ , what point must be included in  $g(x)$  if  $g(x) = f(4x)$ .
16. If  $(-3,-5)$  is included in  $f(x)$ , what point must be included in  $g(x)$  if  $g(x) = 2f(x)$ .

17. The function  $f(x)$  is given by the following table of values. Which table of values would represent  $g(x)$  if  $g(x) = f(x) + 5$ ?

$x$	$f(x)$
1	2
2	4
3	8

- 1) 

$x$	$g(x)$
5	2
6	4
7	8

 2) 

$x$	$g(x)$
1	7
2	9
3	13

 3) 

$x$	$g(x)$
1	-3
2	-1
3	3

 4) 

$x$	$g(x)$
-4	2
-3	4
-2	8

18. The function  $f(x)$  is given by the following table of values. Which table of values would represent  $g(x)$  if  $g(x) = f(x + 5)$ ?

$x$	$f(x)$
1	2
2	4
3	8

- 1) 

$x$	$g(x)$
5	2
6	4
7	8

 2) 

$x$	$g(x)$
1	7
2	9
3	13

 3) 

$x$	$g(x)$
1	-3
2	-1
3	3

 4) 

$x$	$g(x)$
-4	2
-3	4
-2	8

19. The function  $f(x)$  is given by the following table of values. Which table of values would represent  $g(x)$  if  $g(x) = f(2x)$ ?

$x$	$f(x)$
2	18
4	10
8	2

- 1) 

$x$	$g(x)$
2	36
4	20
8	4

 2) 

$x$	$g(x)$
1	18
2	10
4	2

 3) 

$x$	$g(x)$
2	9
4	5
8	1

 4) 

$x$	$g(x)$
4	18
8	10
16	2

20. The function  $f(x)$  is given by the following table of values. Which table of values would represent  $g(x)$  if  $g(x) = 2f(x)$ ?

$x$	$f(x)$
2	18
4	10
8	2

- 1) 

$x$	$g(x)$
2	36
4	20
8	4

 2) 

$x$	$g(x)$
1	18
2	10
4	2

 3) 

$x$	$g(x)$
2	9
4	5
8	1

 4) 

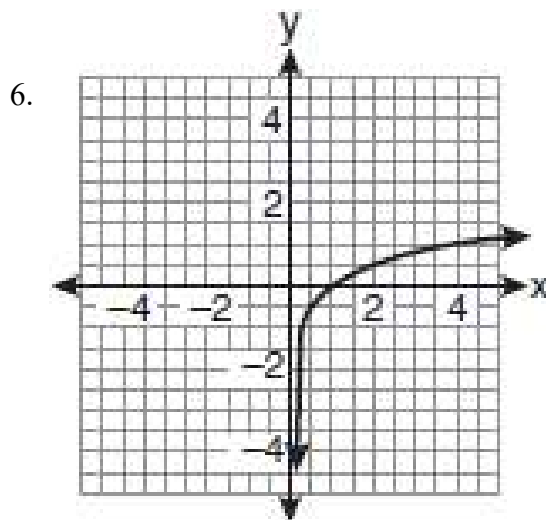
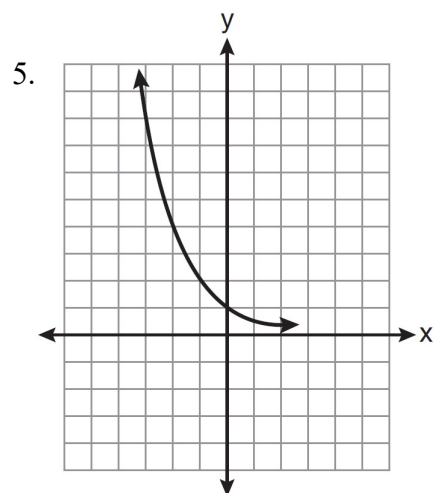
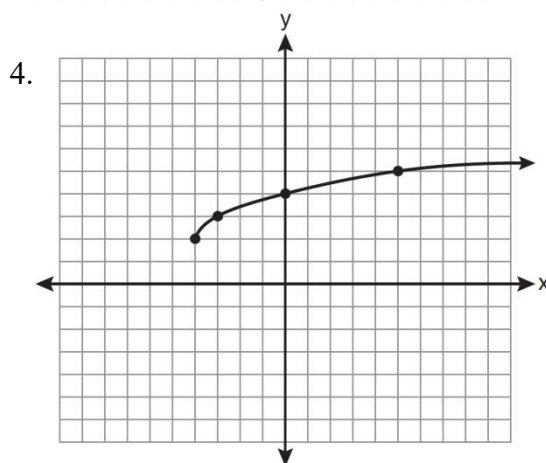
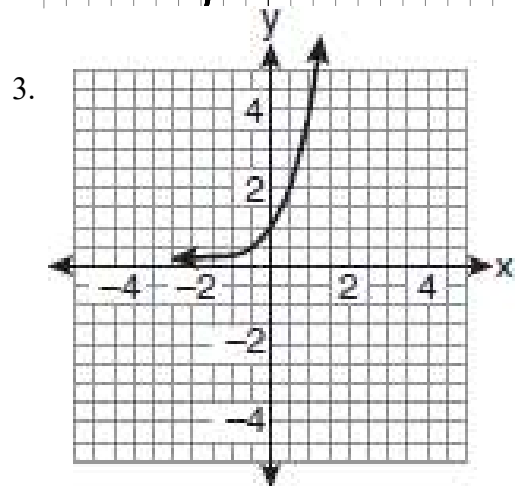
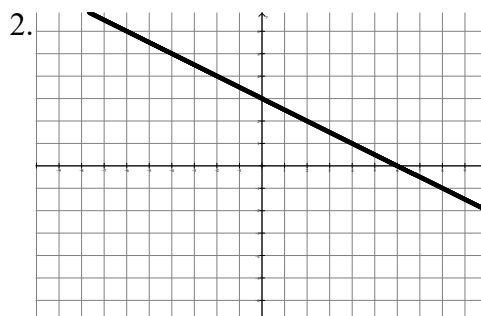
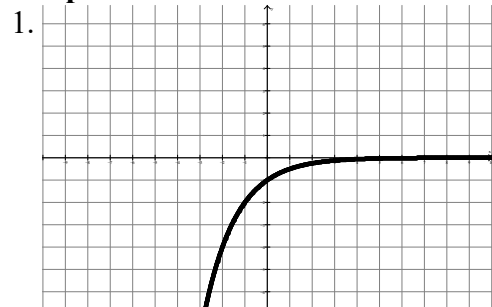
$x$	$g(x)$
4	18
8	10
16	2

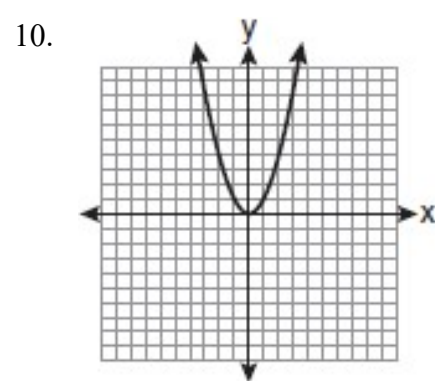
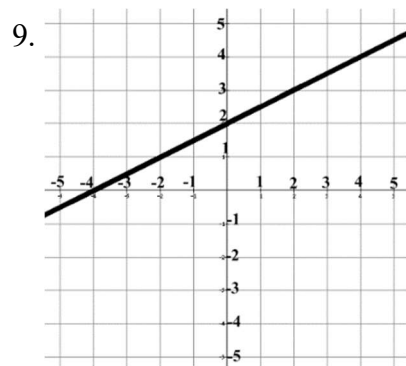
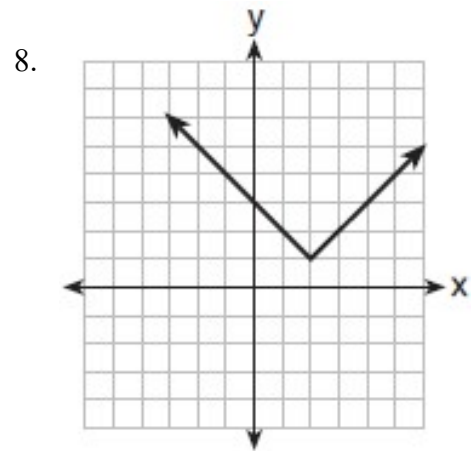
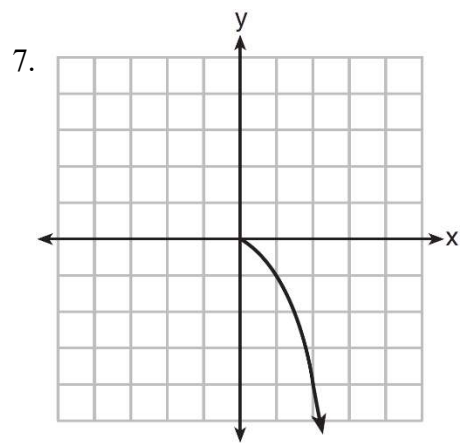
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Pre Calculus

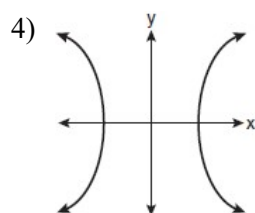
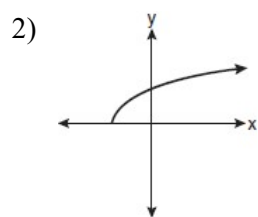
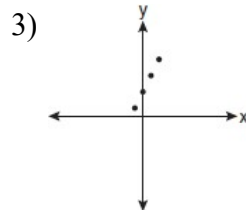
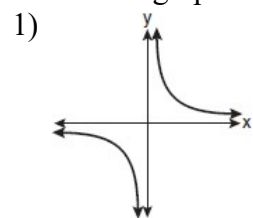
## ***Inverse of a Function Graphically***

**Graph the inverse of the functions below on the same axes**





11. Which graph has an inverse that is itself?



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Algebra II



## ***Finding the Inverse of a Function Algebraically***

1. What is the inverse of the function  $y = 2x - 3$ ?

2. Find the inverse of  $y = 3x + 2$ .

3. If  $f(x) = 5x - 7$ , find  $f^{-1}(x)$

4. What is  $g^{-1}(x)$  if  $g(x) = 3x + 6$

5. What is the inverse of  $y = \frac{1}{2}x + 2$ ?

6. If  $f(x) = x^2$ , find  $f^{-1}(x)$

7. What is  $h^{-1}(x)$  if  $h(x) = x^2 + 2$

8. Given  $f(x) = \frac{2}{3}x + 6$ , write the equation of  $f^{-1}(x)$ .

9. What is the inverse of the function  $y = 4x + 5$ ?

10. What is the inverse of  $f(x) = -6(x - 2)$ ?

## ***Finding the Inverse of a Function Multiple Choice***

1. What is the inverse of the function  $y = 2x - 3$ ?

- 1)  $y = \frac{x+3}{2}$                       3)  $y = -2x + 3$   
2)  $y = \frac{x}{2} + 3$                       4)  $y = \frac{1}{2x-3}$

2. If a function is defined by the equation  $y = 3x + 2$ , which equation defines the inverse of this function?

- 1)  $x = \frac{1}{3}y + \frac{1}{2}$                       3)  $y = \frac{1}{3}x - \frac{2}{3}$   
2)  $y = \frac{1}{3}x + \frac{1}{2}$                       4)  $y = -3x - 2$

3. What is the inverse of  $f(x) = 2x + 6$ ?

- 1)  $f^{-1}(x) = -2(x + 3)$                       3)  $f^{-1}(x) = \frac{x}{2} - 3$   
2)  $f^{-1}(x) = x - 3$                       4)  $f^{-1}(x) = \frac{x}{2} + 3$

4. What is the inverse of the function  $y = 4x + 5$ ?

- 1)  $x = \frac{1}{4}y - \frac{5}{4}$                       3)  $y = 4x - 5$   
2)  $y = \frac{1}{4}x - \frac{5}{4}$                       4)  $y = \frac{1}{4x+5}$

5. What is the inverse of  $f(x) = -6(x - 2)$ ?

- 1)  $f^{-1}(x) = -2 - \frac{x}{6}$                       3)  $f^{-1}(x) = \frac{1}{-6(x-2)}$   
2)  $f^{-1}(x) = 2 - \frac{x}{6}$                       4)  $f^{-1}(x) = 6(x + 2)$



6. Given  $f(x) = \frac{1}{2}x + 8$ , which equation represents the inverse,  $g(x)$ ?

1)  $g(x) = 2x - 8$

3)  $g(x) = -\frac{1}{2}x + 8$

2)  $g(x) = 2x - 16$

4)  $g(x) = -\frac{1}{2}x - 16$

7. If  $f(x) = 12x - 4$ , then the inverse function  $f^{-1}(x)$  is

1)  $f^{-1}(x) = \frac{x+1}{3}$

3)  $f^{-1}(x) = \frac{x+4}{12}$

2)  $f^{-1}(x) = \frac{x}{3} + 1$

4)  $f^{-1}(x) = \frac{x}{12} + 4$

8. The inverse of the function  $f(x) = \frac{x+1}{x-2}$  is

1)  $f^{-1}(x) = \frac{x+1}{x+2}$

3)  $f^{-1}(x) = \frac{x+1}{x-2}$

2)  $f^{-1}(x) = \frac{2x+1}{x-1}$

4)  $f^{-1}(x) = \frac{x-1}{x+1}$

9. What is the inverse of  $f(x) = \frac{x}{x+2}$ , where  $x \neq -2$ ?

1)  $f^{-1}(x) = \frac{2x}{x-1}$

3)  $f^{-1}(x) = \frac{x}{x-2}$

2)  $f^{-1}(x) = \frac{-2x}{x-1}$

4)  $f^{-1}(x) = \frac{-x}{x-2}$

\*. Given the inverse function  $f^{-1}(x) = \frac{2}{3}x + \frac{1}{6}$ , which function represents  $f(x)$ ?

1)  $f(x) = -\frac{2}{3}x + \frac{1}{6}$

3)  $f(x) = \frac{3}{2}x - \frac{1}{4}$

2)  $f(x) = -\frac{3}{2}x + \frac{1}{4}$

4)  $f(x) = \frac{3}{2}x - \frac{1}{6}$

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Algebra II



## *Solving Systems Graphically Using TI*

1. To the *nearest tenth*, the value of  $x$  that satisfies  $2^x = -2x + 11$  is

- 1) 2.5
- 2) 2.6
- 3) 5.8
- 4) 5.9

2. For which values of  $x$ , rounded to the *nearest hundredth*, will  $|x^2 - 9| - 3 = \log_3 x$ ?

- 1) 2.29 and 3.63
- 2) 2.37 and 3.54
- 3) 2.84 and 3.17
- 4) 2.92 and 3.06

3. For which approximate value(s) of  $x$  will  $\log(x + 5) = |x - 1| - 3$ ?

- 1) 5, 1
- 2) -2.41, 0.41
- 3) -2.41, 5
- 4) 5, only

4. Which value, to the *nearest tenth*, is an approximate solution for the equation  $f(x) = g(x)$ , if

$$f(x) = \frac{5}{x-3} \text{ and } g(x) = 2(1.3)^x?$$

- 1) 3.2
- 2) 3.9
- 3) 4.0
- 4) 5.6

5. If  $p(x) = 2\ln(x) - 1$  and  $m(x) = \ln(x + 6)$ , then what is the solution for  $p(x) = m(x)$ ?

- 1) 1.65
- 2) 3.14
- 3) 5.62
- 4) no solution

6. Which value, to the *nearest tenth*, is *not* a solution of  $p(x) = q(x)$  if  $p(x) = x^3 + 3x^2 - 3x - 1$  and  $q(x) = 3x + 8$ ?

- 1) -3.9
- 2) -1.1
- 3) 2.1
- 4) 4.7

7. If  $f(x) = g(x)$   $f(x) = 3|x| - 1$  and  $g(x) = 0.03x^3 - x + 1$ , an approximate solution for the equation  $f(x) = g(x)$  is

- 1) 1.96
- 2) 11.29
- 3) (-0.99, 1.96)
- 4) (11.29, 32.87)

8. Given:  $h(x) = \frac{2}{9}x^3 + \frac{8}{9}x^2 - \frac{16}{13}x + 2$

$$k(x) = -|0.7x| + 5$$

State the solutions to the equation  $h(x) = k(x)$ , rounded to the *nearest hundredth*.

9. If  $f(t) = 325e^{-0.0735t} + 75$  and  $g(t) = 375e^{-0.0817t} + 75$ , for what value of  $t$  does  $f(t) = g(t)$  rounded to the *nearest tenth*?

10. A technology company is comparing two plans for speeding up its technical support time. Plan  $A$  can be modeled by the function  $A(x) = 15.7(0.98)^x$  and plan  $B$  can be modeled by the function  $B(x) = 11(0.99)^x$  where  $x$  is the number of customer service representatives employed by the company and  $A(x)$  and  $B(x)$  represent the average wait time, in minutes, of each customer. To the *nearest integer*, solve the equation  $A(x) = B(x)$ .

11. Website popularity ratings are often determined using models that incorporate the number of visits per week a website receives. One model for ranking websites is  $P(x) = \log(x - 4)$ , where  $x$  is the number of visits per week in thousands and  $P(x)$  is the website's popularity rating.

An alternative rating model is represented by  $R(x) = \frac{1}{2}x - 6$ , where  $x$  is the number of visits per week in thousands. For what number of weekly visits will the two models provide the same rating?

12. The value of a certain small passenger car based on its use in years is modeled by  $V(t) = 28482.698(0.684)^t$ , where  $V(t)$  is the value in dollars and  $t$  is the time in years. Zach had to take out a loan to purchase the small passenger car. The function  $Z(t) = 22151.327(0.778)^t$ , where  $Z(t)$  is measured in dollars, and  $t$  is the time in years, models the unpaid amount of Zach's loan over time. State when  $V(t) = Z(t)$ , to the *nearest hundredth*.

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Algebra II

## ***Solving Systems of Inequalities Graphically with TI***

1. Given  $f(x) = x^2$  and  $g(x) = -\frac{1}{2}x + 5$ , over what interval is  $f(x) < g(x)$ ?
  
  
  
  
  
  
  
  
  
  
2. Given  $f(x) = -|x|$  and  $g(x) = -\sqrt{x+4}$ , over what interval is  $f(x) \geq g(x)$ ?
  
  
  
  
  
  
  
  
  
  
3. Given  $m(x) = \log(x)$  and  $n(x) = (x-5)^2$ , over what interval is  $m(x) \geq n(x)$ ?
  
  
  
  
  
  
  
  
  
  
4. Given  $a(x) = e^x - 9$  and  $b(x) = -|x-3| - 2$ , over what interval is  $a(x) < b(x)$ ?
  
  
  
  
  
  
  
  
  
  
5. If  $f(x) = \frac{1}{2}x^3 + 3x^2 - 4x$  and  $g(x) = 5\log_3(x+10)$ , then which value, rounded to the *nearest tenth*, is a solution to  $f(x) > g(x)$ ?  

1) -7.0	3) -1.1
2) -6.8	4) 2.1
  
  
  
  
  
  
  
  
  
  
6. For which value of  $x$  will  $\log(x+5) \geq |x-1| - 3$ ?  

1) -6	3) 4
2) -4	4) 6

7. For which value of  $x$  will  $\sqrt[3]{x-1} > -\frac{1}{2}|x| + 3$ ?

- |         |        |
|---------|--------|
| 1) -3.1 | 3) 2.7 |
| 2) 1.1  | 4) 3.9 |

8. The function  $r(x) = \frac{1}{12}x$  represents the revenue from Carla's business and  $c(x) = 2\log(x)$  represents her cost for selling  $x$  unites of merchandise. To the *nearest tenth*, over what interval will  $c(x) > r(x)$ ? Explain the meaning of this interval in the context of the problem.

9. The height of a ball thrown in the air can be modeled by  $b(t) = -16t^2 + 32t$  and the height of an eagle can be modeled by  $e(t) = -\frac{1}{2}t + 14$  after  $t$  seconds. To the *nearest hundredth*, over what interval is  $e(t) < b(t)$ ? Explain the meaning of this interval in the context of the problem.

10. The height of object A can be represented by  $A(x) = 2\sqrt[3]{x} + 15$  and the height of object B can be represented by  $B(x) = 20(0.8)^x$  after  $x$  seconds. Over what interval is  $A(x) > B(x)$ ? Explain its meaning in the context of the problem.

11. The value of stock A can be modeled by  $A(t) = 2\sqrt{t+10} + 1$  and the value of stock B can be represented by  $B(t) = t^3 - 3t^2 - 3t + 10$ , where  $t$  represents time in days. Over what positive interval, rounded to the *nearest tenth*, is  $A(x) > B(x)$ ? Explain the meaning of this interval in the context of the problem.

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Algebra II



## *Graphing Exponential and Logarithmic Functions*

For the following equations, graph the equation and the asymptote. State the domain, range, equation of the asymptote, and end behavior.

1.  $y = 2^x - 3$

Domain:

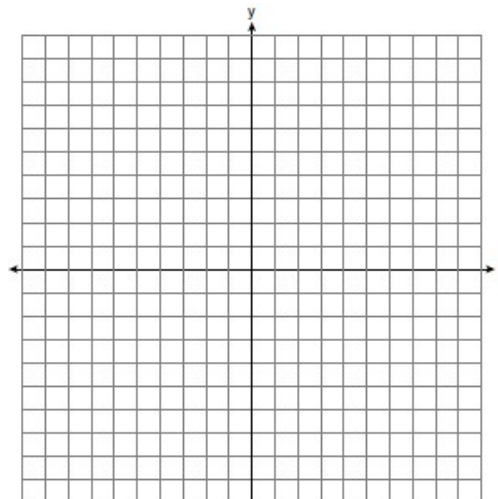
Range:

Asymptote:

End Behavior:

$$x \rightarrow -\infty, f(x) \rightarrow$$

$$x \rightarrow \infty, f(x) \rightarrow$$



2.  $y = \frac{1}{2}^{x-3} + 1$

Domain:

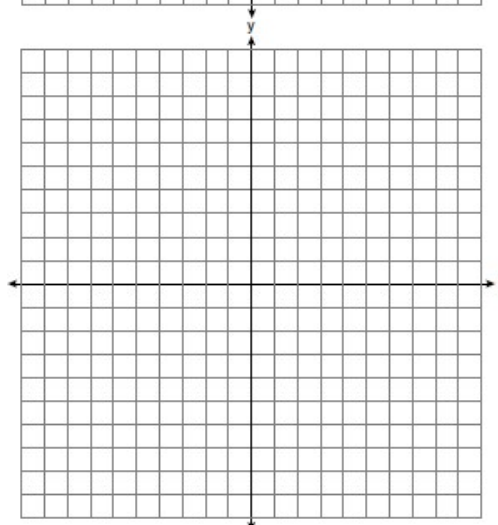
Range:

Asymptote:

End Behavior:

$$x \rightarrow -\infty, f(x) \rightarrow$$

$$x \rightarrow \infty, f(x) \rightarrow$$



3.  $y = -3^{x-2} + 4$

Domain:

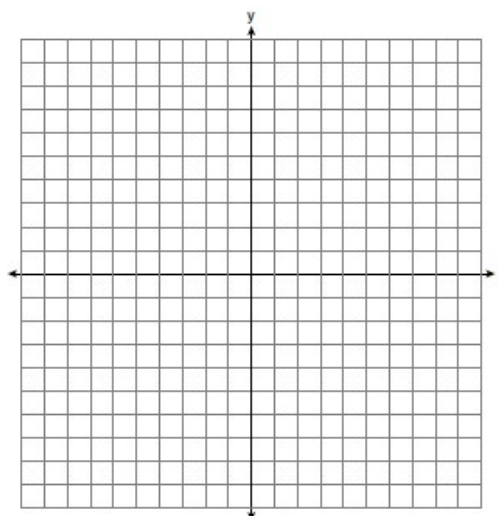
Range:

Asymptote:

End Behavior:

$$x \rightarrow -\infty, f(x) \rightarrow$$

$$x \rightarrow \infty, f(x) \rightarrow$$



4.  $y = 2(3)^{x+1} - 8$

Domain:

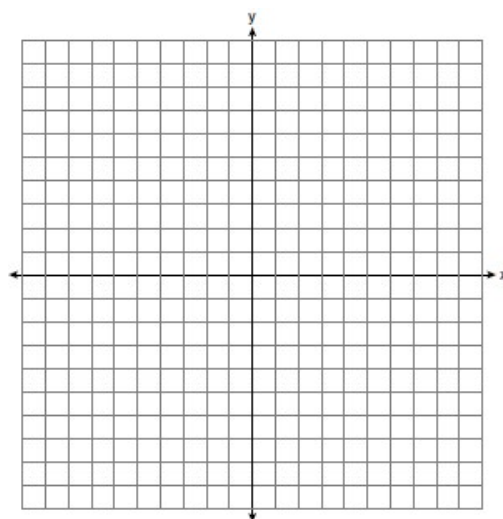
Range:

Asymptote:

End Behavior:

$x \rightarrow -\infty, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$



5.  $y = -2\left(\frac{1}{3}\right)^{x-5} + 9$

Domain:

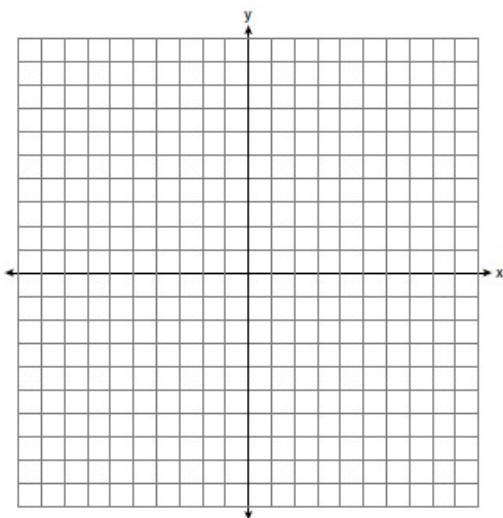
Range:

Asymptote:

End Behavior:

$x \rightarrow -\infty, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$



6.  $y = 3\left(\frac{1}{2}\right)^{x+1} - 7$

Domain:

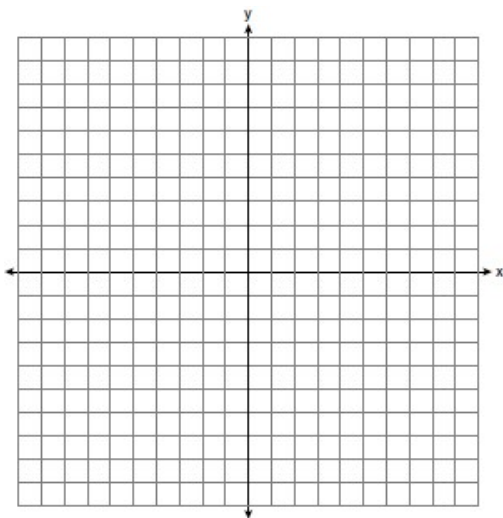
Range:

Asymptote:

End Behavior:

$x \rightarrow -\infty, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$



7.  $y = \log_2(x) + 3$

Domain:

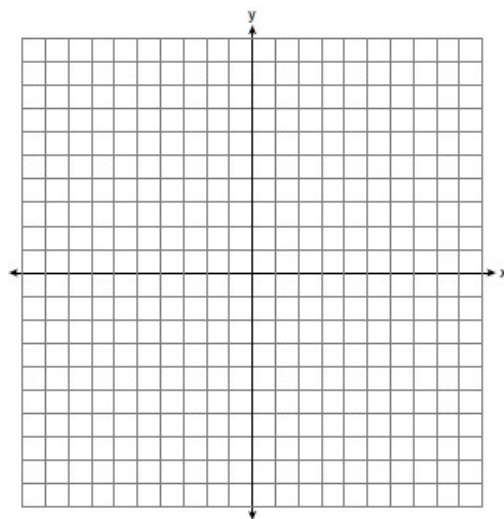
Range:

Asymptote:

End Behavior:

$x \rightarrow 0, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$



8.  $y = \log_3(x+2) - 1$

Domain:

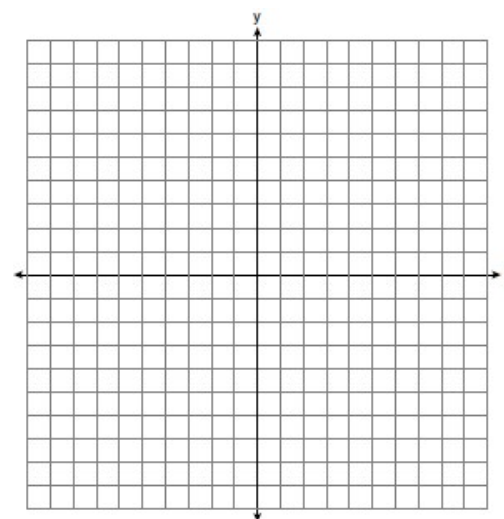
Range:

Asymptote:

End Behavior:

$x \rightarrow -2, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$



9.  $y = -2\log_2(x+6) - 4$

Domain:

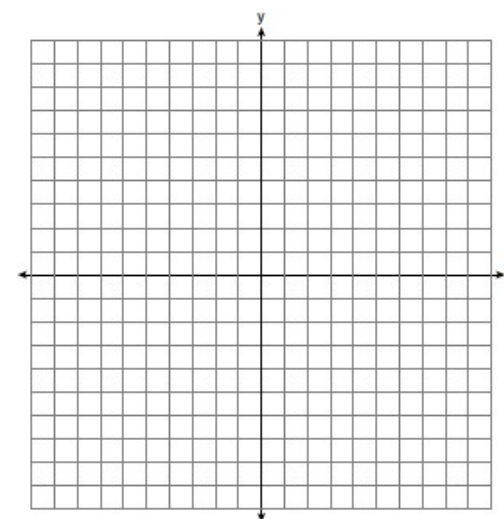
Range:

Asymptote:

End Behavior:

$x \rightarrow -6, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$





10.  $y = 4\log_{\frac{1}{2}}(x-3) + 1$

Domain:

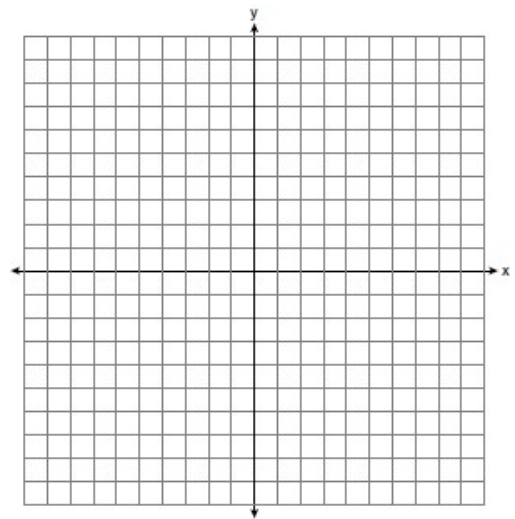
Range:

Asymptote:

End Behavior:

$x \rightarrow 3, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$



11.  $y = 3\log_4(x+1) - 8$

Domain:

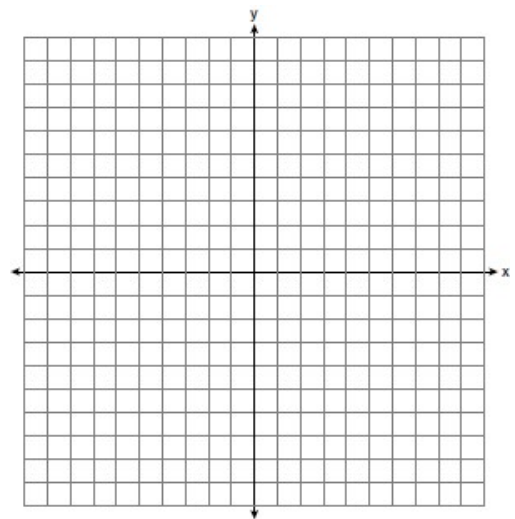
Range:

Asymptote:

End Behavior:

$x \rightarrow -1, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$



12.  $y = -4\log_2(x+9) + 4$

Domain:

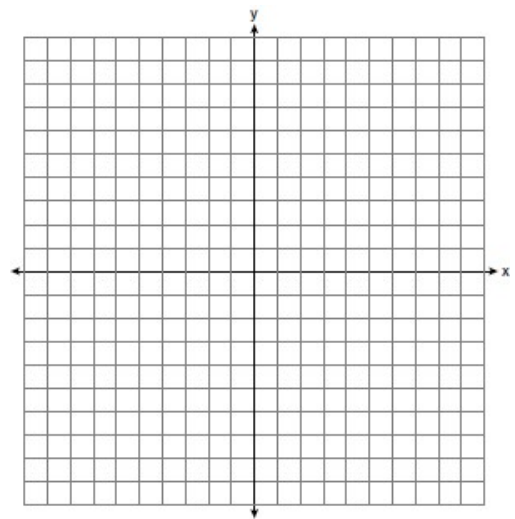
Range:

Asymptote:

End Behavior:

$x \rightarrow -9, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$



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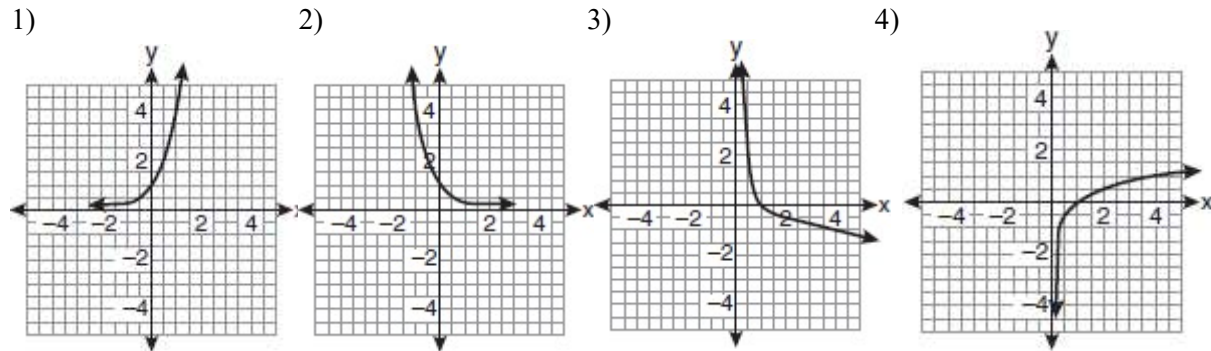
Date \_\_\_\_\_  
Algebra II



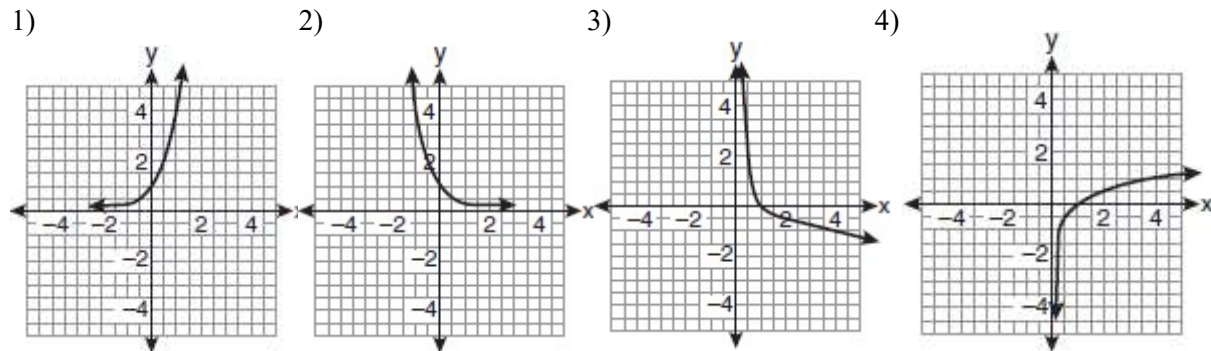
## ***Exponential and Logarithmic Graphs Multiple Choice***

1. Which statement about the graph of  $c(x) = \log_6 x$  is *false*?
  - 1) The asymptote has equation  $y = 0$ .
  - 2) The graph has no  $y$ -intercept.
  - 3) The domain is the set of positive reals.
  - 4) The range is the set of all real numbers.
  
2. Which statement about the graph of the equation  $y = e^x$  is *not* true?
  - 1) It is asymptotic to the  $x$ -axis.
  - 2) The domain is the set of all real numbers.
  - 3) It lies in Quadrants I and II.
  - 4) It passes through the point  $(e, 1)$ .
  
3. Which statement is true about the graph of  $f(x) = \left(\frac{1}{8}\right)^x$ ?
  - 1) The graph is always increasing.
  - 2) The graph is always decreasing.
  - 3) The graph passes through  $(1, 0)$ .
  - 4) The graph has an asymptote,  $x = 0$ .
  
4. Which statement is *true* regarding the equation  $f(x) = \log_7 x$ ?
  - 1) It is always increasing
  - 2) The graph passes through  $(0, 1)$
  - 3) The domain is all real numbers
  - 4) The equation of the asymptote is  $y = 0$
  
5. Given the equation  $f(x) = \pi^x$ , which of the following statements is true?
  - 1) The graph passes through  $(\pi, 1)$
  - 2) The domain is  $[0, \infty)$
  - 3) The graph passes through  $(0, 1)$
  - 4) The range is all real numbers
  
6. Which statement is *false* regarding the equation  $f(x) = \ln x$ ?
  - 1) It passes through  $(1, 0)$
  - 2) It is always decreasing
  - 3) The equation of the asymptote is  $x = 0$
  - 4) Its range is  $(-\infty, \infty)$

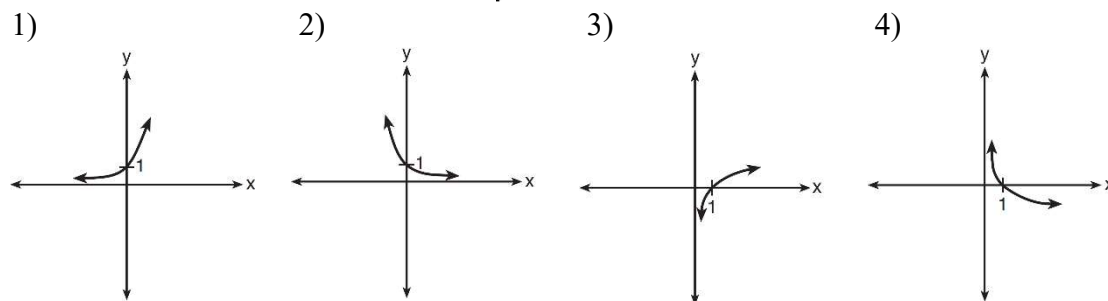
7. If a function is defined by the equation  $f(x) = 4^x$ , which graph represents the inverse of this function?



8. If a function is defined by the equation  $f(x) = \log_4 x$ , which graph represents the inverse of this function?



9. Which sketch shows the inverse of  $y = a^x$ , where  $a > 1$ ?



10. What is the inverse of the function  $y = \log_3 x$ ?

- 1)  $y = x^3$     2)  $y = \log_x 3$     3)  $y = 3^x$     4)  $x = 3^y$

11. If  $f(x) = a^x$  where  $a > 1$ , then the inverse of the function is

- 1)  $f^{-1}(x) = \log_x a$     3)  $f^{-1}(x) = \log_a x$   
 2)  $f^{-1}(x) = a \log x$     4)  $f^{-1}(x) = x \log a$

12. The asymptote of the graph of  $f(x) = 5 \log(x + 4)$  is

- 1)  $y = 6$
- 2)  $x = -4$
- 3)  $x = 4$
- 4)  $y = 5$

13. The asymptote of the graph of  $j(x) = 2e^{x-4} - 1$  is

- 1)  $x = 4$
- 2)  $x = -4$
- 3)  $y = -1$
- 4)  $y = 2$

14. The asymptote of the graph of  $e(x) = \log_3(x - 5) + 1$  is

- 1)  $y = 1$
- 2)  $x = 1$
- 3)  $y = 5$
- 4)  $x = 1$

15. The asymptote of the graph of  $m(x) = -3(2)^{x+1} - 4$  is

- 1)  $x = -1$
- 2)  $x = 3$
- 3)  $y = -4$
- 4)  $y = -3$

16. For the equation  $f(x) = 2^{x-3} + 1$ , as  $x \rightarrow -\infty$

- 1)  $f(x) \rightarrow -\infty$
- 2)  $f(x) \rightarrow 1$
- 3)  $f(x) \rightarrow \infty$
- 4)  $f(x) \rightarrow 3$

17. For the equation  $f(x) = \log_2(x - 4) + 3$ , as  $x \rightarrow 4$

- 1)  $f(x) \rightarrow -\infty$
- 2)  $f(x) \rightarrow 3$
- 3)  $f(x) \rightarrow \infty$
- 4)  $f(x) \rightarrow 4$

18. For the equation  $f(x) = -\log_3(x + 1) - 2$ , as  $x \rightarrow \infty$

- 1)  $f(x) \rightarrow -\infty$
- 2)  $f(x) \rightarrow -1$
- 3)  $f(x) \rightarrow \infty$
- 4)  $f(x) \rightarrow -2$

19. Given  $f(x) = 3^{x-1} + 2$ , as  $x \rightarrow -\infty$

- 1)  $f(x) \rightarrow -1$
- 2)  $f(x) \rightarrow 0$
- 3)  $f(x) \rightarrow 2$
- 4)  $f(x) \rightarrow -\infty$

20. For the equation  $f(x) = 3 \ln(x - 4) + 1$ ,  $f(x) \rightarrow -\infty$  as

- 1)  $x \rightarrow 4$
- 2)  $x \rightarrow 1$
- 3)  $x \rightarrow \infty$
- 4)  $x \rightarrow -\infty$

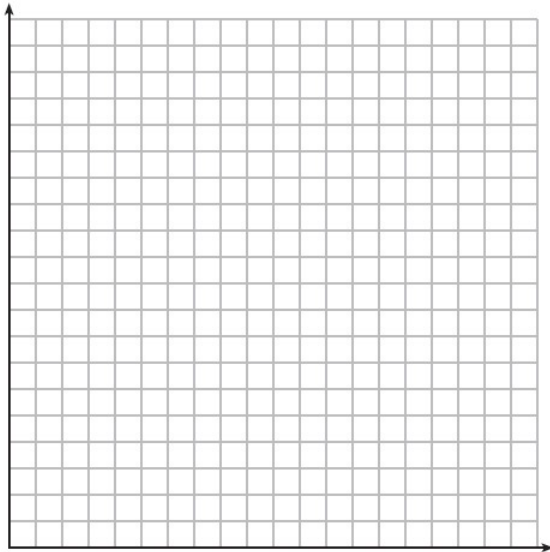
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Algebra II



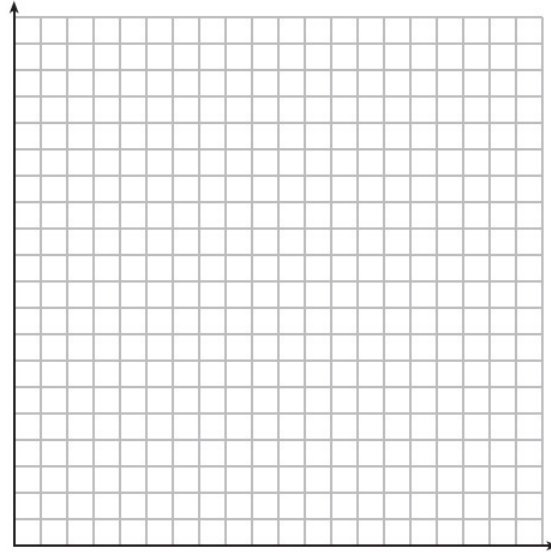
## *Exponential Graphs (Part IV)*

1. The value of Tom's bank account is currently 100000 and is decreasing according to the equation  $V(t) = 100000(.876)^t$ . The amount of money he has paid for his mortgage can be represented by the equation  $M(t) = 20000(1.1304)^t$ . Graph and label  $V(t)$  and  $M(t)$  over the interval  $[0, 10]$ .



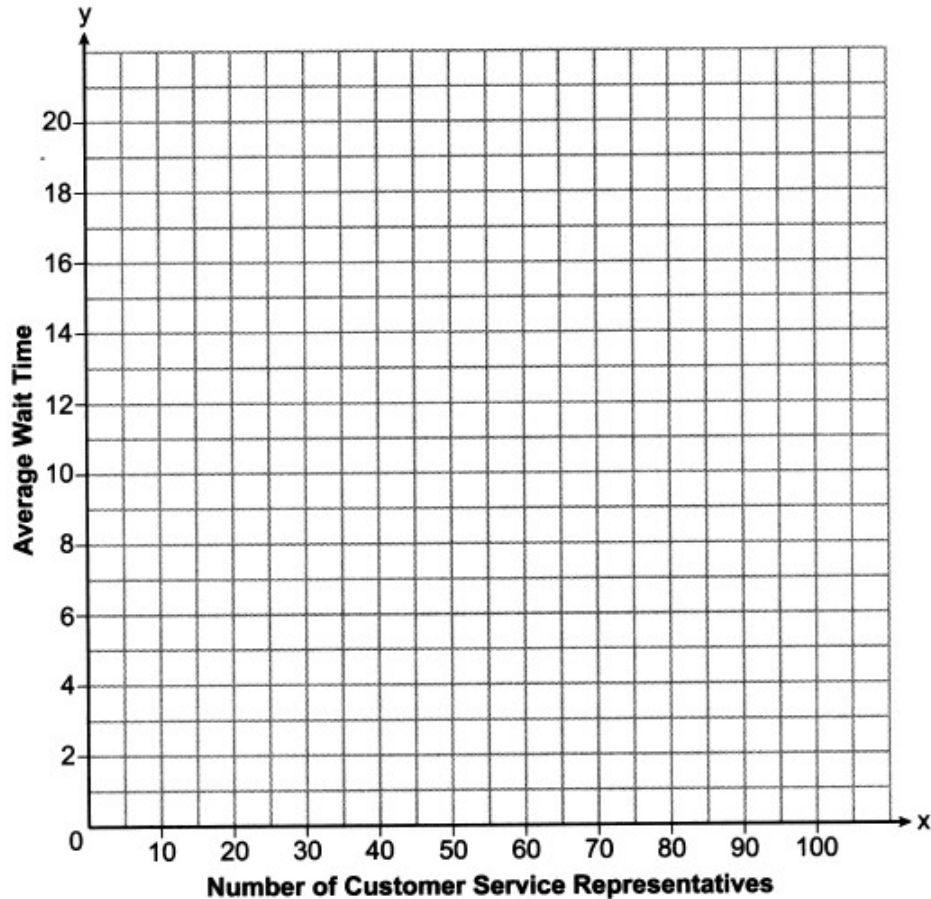
After how many years will the value of Tom's bank account be equal to the amount of money he paid for his mortgage? Round your answer to the *nearest tenth of a year*. Tom will open a new bank account when the value of his account is \$20,000. After how many years, to the *nearest hundredth of a year*, will that happen?

2. The value of a certain small passenger car based on its use in years is modeled by  $V(t) = 28482.698(0.684)^t$ , where  $V(t)$  is the value in dollars and  $t$  is the time in years. Zach had to take out a loan to purchase the small passenger car. The function  $Z(t) = 22151.327(0.778)^t$ , where  $Z(t)$  is measured in dollars, and  $t$  is the time in years, models the unpaid amount of Zach's loan over time. Graph  $V(t)$  and  $Z(t)$  over the interval  $0 \leq t \leq 5$ , on the set of axes below.



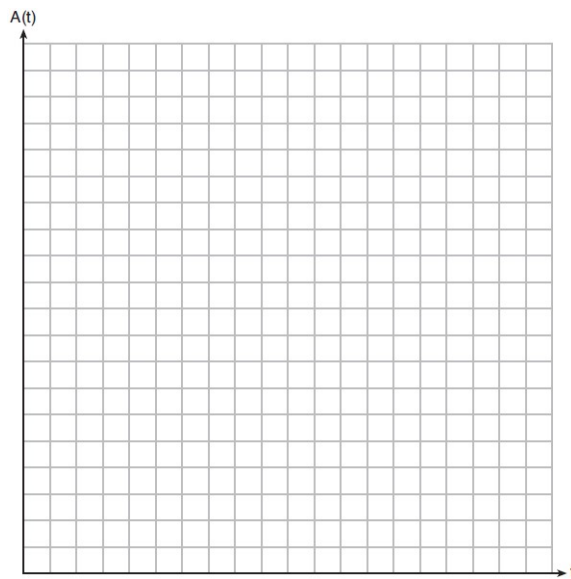
State when  $V(t) = Z(t)$ , to the *nearest hundredth*, and interpret its meaning in the context of the problem. Zach will cancel the collision policy when the value of his car equals \$3000. To the *nearest tenth of a year*, how long will it take Zach to cancel this policy? Justify your answer.

3. A technology company is comparing two plans for speeding up its technical support time. Plan  $A$  can be modeled by the function  $A(x) = 15.7(0.98)^x$  and plan  $B$  can be modeled by the function  $B(x) = 11(0.99)^x$  where  $x$  is the number of customer service representatives employed by the company and  $A(x)$  and  $B(x)$  represent the average wait time, in minutes, of each customer. Graph  $A(x)$  and  $B(x)$  in the interval  $0 \leq x \leq 100$  on the set of axes below.



To the *nearest integer*, solve the equation  $A(x) = B(x)$ . How many Customer Service Representatives would the Company B need in order to the average wait time to be 3 minutes? Round to the *nearest representative*.

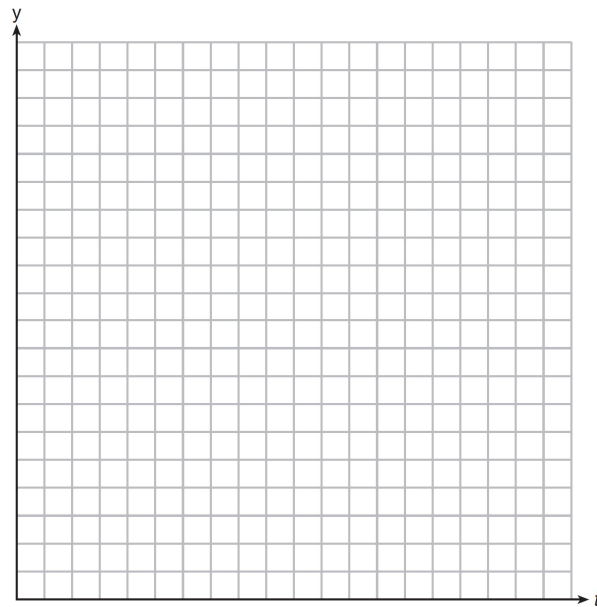
4. Tony is evaluating his retirement savings. The value of his account can be represented by  $A(t) = 318000(1.07)^t$ . Graph  $A(t)$  where  $0 \leq t \leq 20$  on the set of axes below.



Tony's goal is to save \$1,000,000. Determine algebraically, to the *nearest year*, how many years it will take for him to achieve his goal. Explain how your graph of  $A(t)$  confirms your answer.



5. Drugs break down in the human body at different rates and therefore must be prescribed by doctors carefully to prevent complications, such as overdosing. The breakdown of a drug is represented by the function  $N(t) = N_0(e)^{-rt}$ , where  $N(t)$  is the amount left in the body,  $N_0$  is the initial dosage,  $r$  is the decay rate, and  $t$  is time in hours. Patient  $A$ ,  $A(t)$ , is given 800 milligrams of a drug with a decay rate of 0.347. Patient  $B$ ,  $B(t)$ , is given 400 milligrams of another drug with a decay rate of 0.231. Write two functions,  $A(t)$  and  $B(t)$ , to represent the breakdown of the respective drug given to each patient. Graph each function on the set of axes below.



To the *nearest tenth of an hour*,  $t$ , when does the amount of the given drug remaining in patient  $B$  begin to exceed the amount of the given drug remaining in patient  $A$ ? The doctor will allow patient  $A$  to take another dose of the drug once 120 milligrams is left in the body. Determine, to the *nearest tenth of an hour*, how long patient  $A$  will have to wait to take another dose of the drug.

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Algebra II

## *Graphing Square and Cube Root Functions*

For the following equations, graph the equation, state the domain, range, and end behavior.

1.  $f(x) = 2 - \sqrt{x+3}$

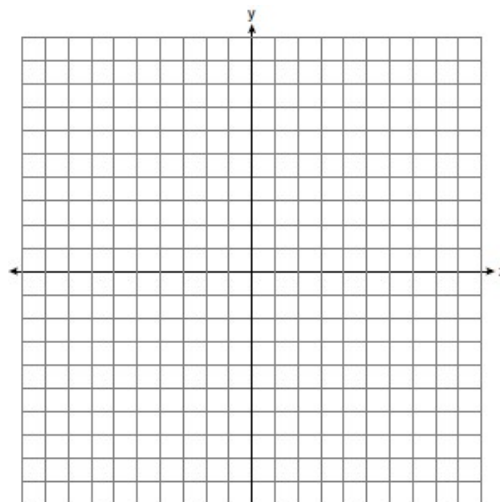
Domain:

Range:

End Behavior:

$$x \rightarrow -3, f(x) \rightarrow$$

$$x \rightarrow \infty, f(x) \rightarrow$$



2.  $f(x) = -\sqrt[3]{2x+8}$

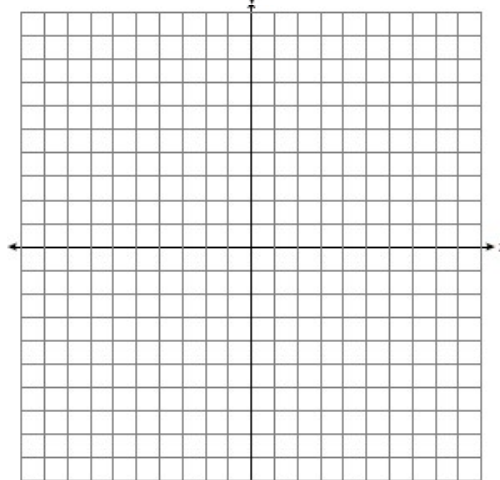
Domain:

Range:

End Behavior:

$$x \rightarrow -\infty, f(x) \rightarrow$$

$$x \rightarrow \infty, f(x) \rightarrow$$



3.  $y = \sqrt[3]{x-2}$

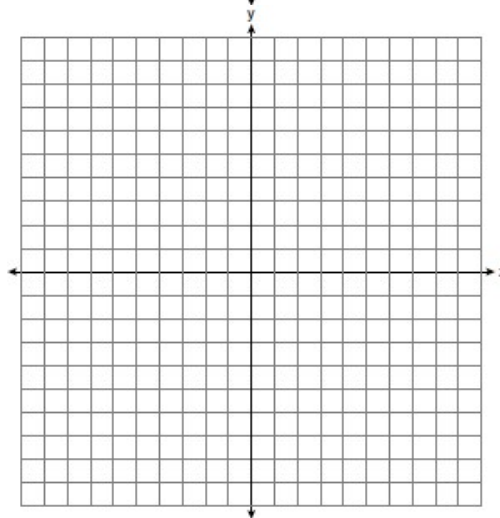
Domain:

Range:

End Behavior:

$$x \rightarrow -\infty, f(x) \rightarrow$$

$$x \rightarrow \infty, f(x) \rightarrow$$



4.  $y = \sqrt{x} - 1$

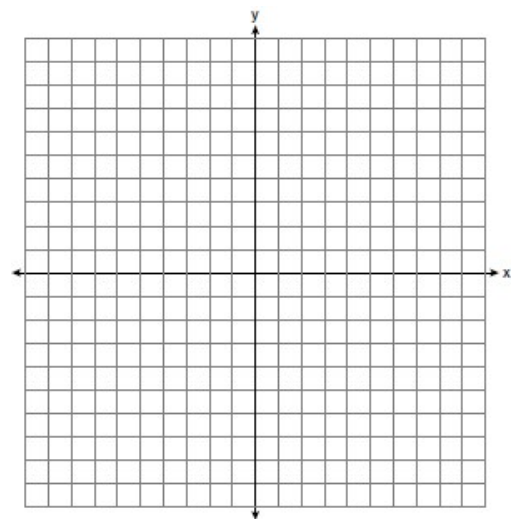
Domain:

Range:

End Behavior:

$x \rightarrow 0, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$



5.  $y = -2\sqrt{x-2}$

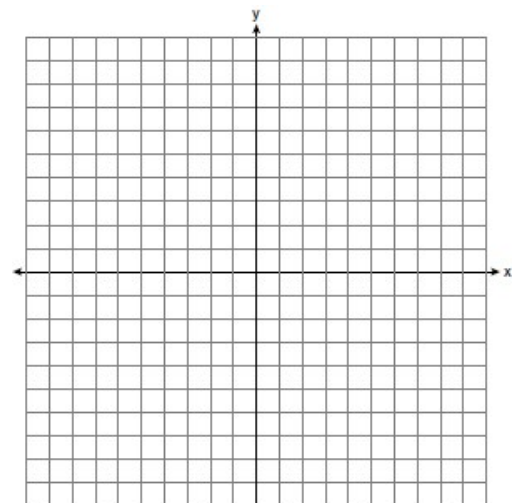
Domain:

Range:

End Behavior:

$x \rightarrow 2, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$



6.  $y = 2 - \sqrt[3]{x+5}$

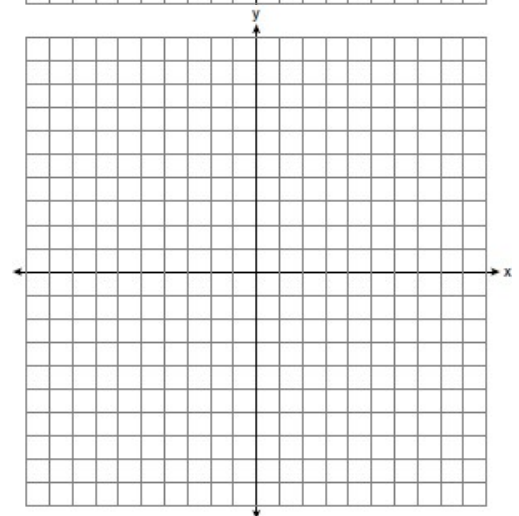
Domain:

Range:

End Behavior:

$x \rightarrow -\infty, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$



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Algebra II



## *Average Rate of Change*

1. The function  $h(x)$  is given in the table below. Which of the following gives its average rate of change over the interval  $2 \leq x \leq 6$ ?

(1)  $-\frac{3}{2}$

(3)  $-\frac{7}{6}$

(2)  $\frac{6}{4}$

(4)  $-1$

$x$	$h(x)$
0	10
2	9
4	6
6	3

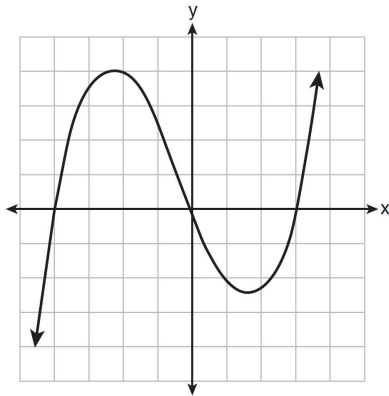
2. The distance needed to stop a car after applying the brakes varies directly with the square of the car's speed. The table below shows stopping distances for various speeds. Determine the average rate of change in braking distance, in ft/mph, between one car traveling at 50 mph and one traveling at 70 mph.

Speed (mph)	10	20	30	40	50	60	70
Distance (ft)	6.25	25	56.25	100	156.25	225	306.25

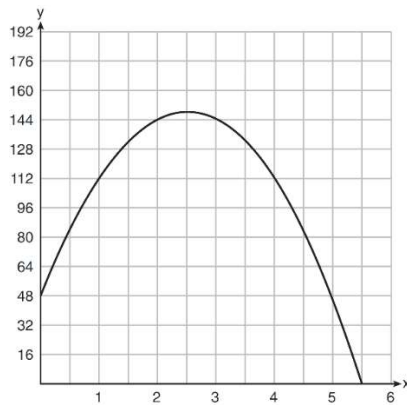
3. What is the average rate of change from 0 to 2?

$x$	$f(x)$
0	1
1	2
2	5
3	7

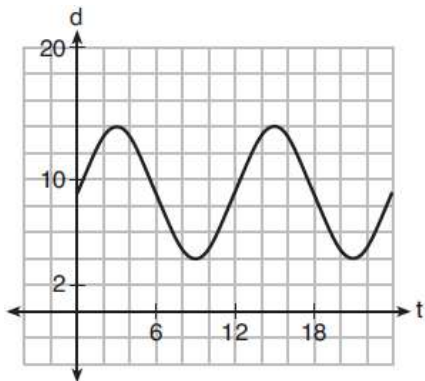
4. The graph of  $p(x)$  is shown below. What is the average rate of change over the interval  $-4 \leq x \leq 1$ ?



5. A ball is thrown into the air from the edge of a 48-foot-high cliff so that it eventually lands on the ground. The graph below shows the height,  $y$ , of the ball from the ground after  $x$  seconds. What is the average rate of change of the ball between 1 and 5 seconds?



6. The depth of the water at a marker 20 feet from the shore in a bay is depicted in the graph below. If the depth,  $d$ , is measured in feet and time,  $t$ , is measured in hours since midnight, what is the average rate of change of the depth of the water between 3AM and 9AM?



7. For the function  $f(x) = 3^x$ , find the average rate of change over the interval -5 to -1 rounded to the nearest thousandth.

8. Find the average rate of change of the function  $f(t) = 2500(0.97)^{4t}$  over the interval  $10 \leq t \leq 15$  rounded to the nearest tenth.

9. An initial investment of \$1000 reaches a value,  $V(t)$ , according to the model  $V(t) = 1000(1.01)^{4t}$ , where  $t$  is the time in years. Determine the average rate of change, to the *nearest dollar per year*, of this investment from year 2 to year 7.

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## *Average Rate of Change with Context*

**“On average, from  $x$  to  $x$ , the  $y$  topic is increasing/decreasing by AROC  $y$  units per  $x$  unit”**

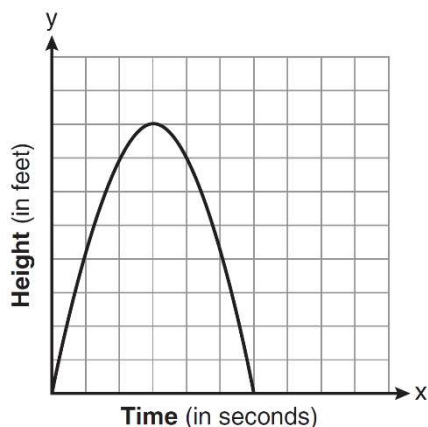
1. A family is traveling from their home to a vacation resort hotel. The table below shows their distance from home as a function of time.

Determine the average rate of change between hour 2 and hour 7. Explain its meaning in the given context.

Time (hrs)	0	2	5	7
Distance (mi)	0	140	375	480

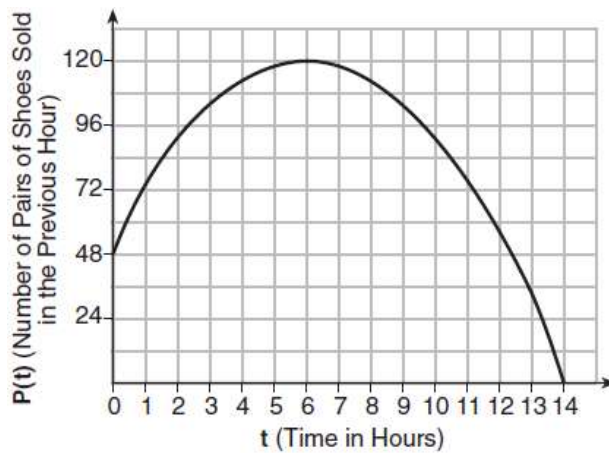
2. The average monthly high temperature in Buffalo, in degrees Fahrenheit, can be modeled by the function  $B(t) = 25.29 \sin(0.4895t - 1.9752) + 55.2877$ , where  $t$  is the month number (January = 1). State, to the *nearest tenth*, the average monthly rate of temperature change between August and November. Explain its meaning in the given context.

3. The graph below represents the parabolic path of a ball kicked by a young child. Find the average rate of change from 3 to 6 seconds. Explain its meaning in the context of the problem.



4. The population,  $P(t)$ , of a town increased according to the function  $P(t) = 12,000(1.03)^t$ , where  $t$  is the number of years since 2000. Find the average rate of change from  $t = 10$  to  $t = 20$  rounding to the nearest integer. Explain its meaning in the context of the problem.

5. A manager wanted to analyze the online shoe sales for his business. He created a graph to model the data, as shown below. Determine the average rate of change between the sixth and fourteenth hours, and explain what it means in the context of the problem.



6. The table below shows the number of hours of daylight on the first day of each month in Rochester, NY. Given the data, what is the average rate of change in hours of daylight per month from January 1st to April 1st? Interpret what this means in the context of the problem.

Month	Hours of Daylight
Jan.	9.4
Feb.	10.6
March	11.9
April	13.9
May	14.7
June	15.4
July	15.1
Aug.	13.9
Sept.	12.5
Oct.	11.1
Nov.	9.7
Dec.	9.0



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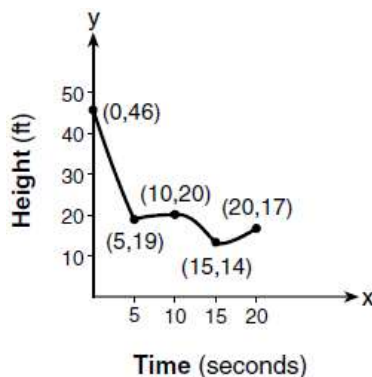


## *Average Rate of Change with Intervals*

1. The graph below models the height of a remote-control helicopter over 20 seconds during flight.

Over which interval does the helicopter have the *fastest* average rate of change?

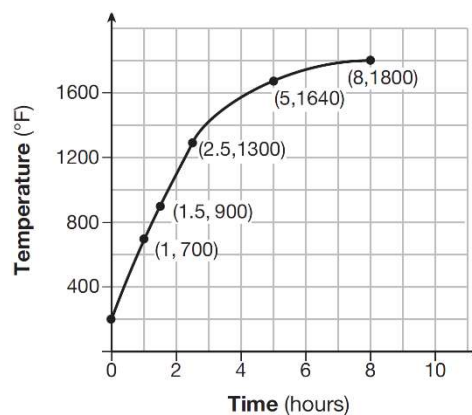
- 1) 0 to 5 seconds
- 2) 5 to 10 seconds
- 3) 10 to 15 seconds
- 4) 15 to 20 seconds



2. Firing a piece of pottery in a kiln takes place at different temperatures for different amounts of time. The graph below shows the temperatures in a kiln while firing a piece of pottery after the kiln is preheated to 200°F.

During which time interval did the temperature in the kiln show the greatest average rate of change?

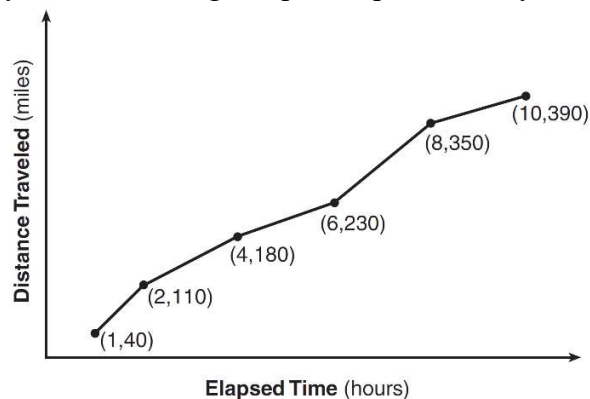
- 1) 0 to 1 hour
- 2) 1 hour to 1.5 hours
- 3) 2.5 hours to 5 hours
- 4) 5 hours to 8 hours



3. The Jamison family kept a log of the distance they traveled during a trip, as represented by the graph below.

During which interval was their average speed the greatest?

- 1) the first hour to the second hour
- 2) the second hour to the fourth hour
- 3) the sixth hour to the eighth hour
- 4) the eighth hour to the tenth hour



4. The table below shows the year and the number of households in a building that had high-speed broadband internet access.

Number of Households	11	16	23	33	42	47
Year	2002	2003	2004	2005	2006	2007

For which interval of time was the average rate of change the *smallest*?

- 1) 2002 - 2004
- 2) 2003 - 2005
- 3) 2004 - 2006
- 4) 2005 - 2007

5. Joelle has a credit card that has a 19.2% annual interest rate compounded monthly. She owes a total balance of  $B$  dollars after  $m$  months. Assuming she makes no payments on her account, the table below illustrates the balance she owes after  $m$  months.

$m$	$B$
0	1000.00
10	1172.00
19	1352.00
36	1770.80
60	2591.90
69	2990.00
72	3135.80
73	3186.00

Over which interval of time is her average rate of change for the balance on her credit card account the greatest?

- 1) month 10 to month 60
- 2) month 19 to month 69
- 3) month 36 to month 72
- 4) month 60 to month 73

6. The function  $N(t) = 100(2.6)^{-0.023t}$  models the number of grams in a sample of cesium-137 that remain after  $t$  years. On which interval is the sample's average rate of decay the fastest?

- 1)  $[1, 10]$
- 2)  $[10, 20]$
- 3)  $[15, 25]$
- 4)  $[1, 30]$

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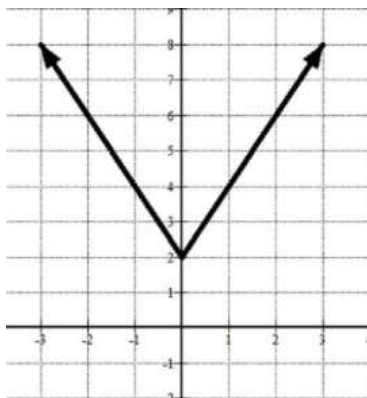
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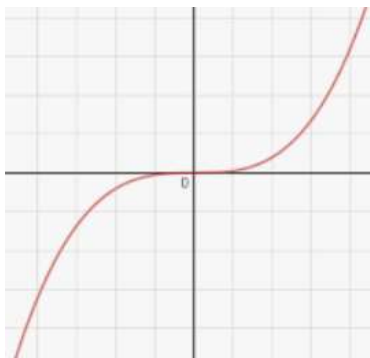
## *Even and Odd Functions*

**Determine graphically whether the following functions are even, odd, or neither**

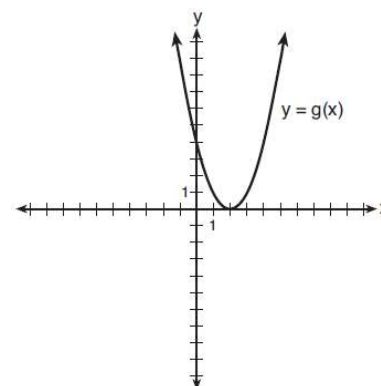
1.



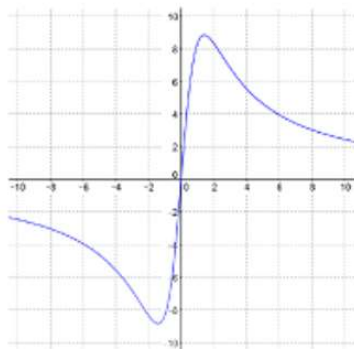
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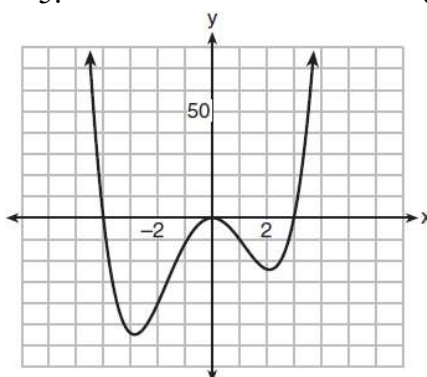
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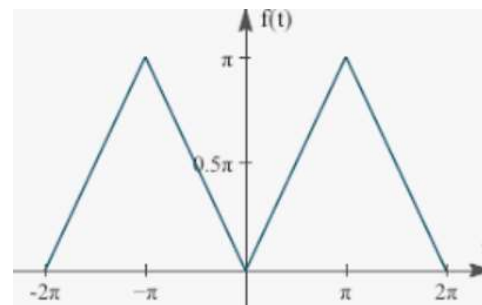
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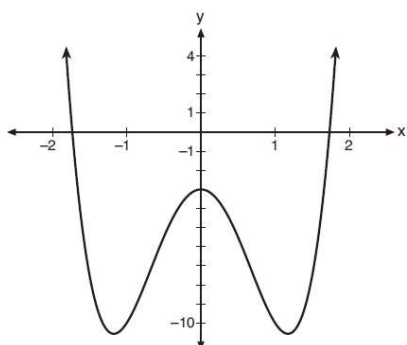
5.



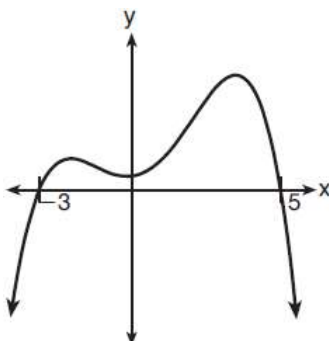
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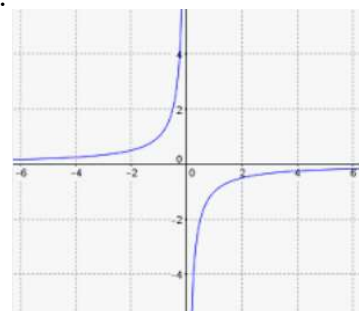
7.



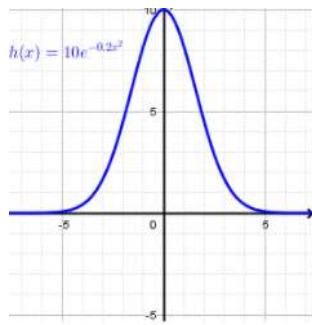
8.



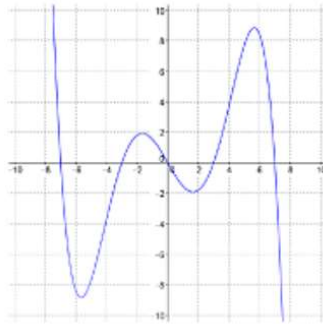
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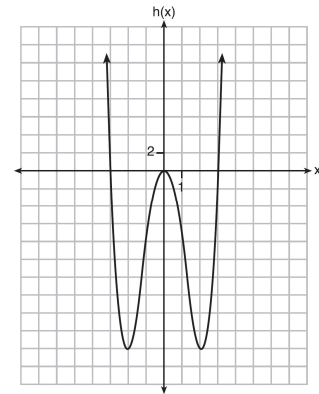
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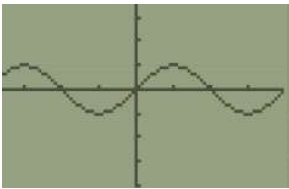
11.



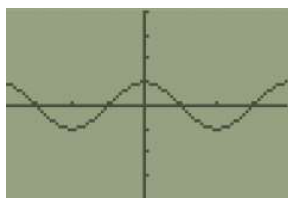
12.



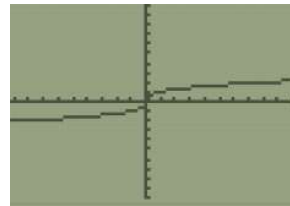
13.



14.



15.



16.  $f(x) = -x^4 + 4$

17.  $f(x) = \frac{1}{2}x^5 - 2x$

18.  $f(x) = 4x^3 - 6$

19.  $f(x) = |x| + 4$

20.  $f(x) = |x + 4|$

21.  $f(x) = \frac{10}{x}$

22.  $f(x) = x^3 + x$

23.  $f(x) = -2x^4 + 8$

24.  $f(x) = 2^{x+1}$

Name \_\_\_\_\_  
Mr. Schlansky

Date \_\_\_\_\_  
Algebra II



## *Exponents/Logarithms/Functions Review Sheet*

1. Which value, to the *nearest tenth*, is an approximate solution for the equation  $f(x) = g(x)$ , if

$$f(x) = \frac{5}{x-3} \text{ and } g(x) = 2(1.3)^x?$$

- |        |        |
|--------|--------|
| 1) 3.2 | 3) 4.0 |
| 2) 3.9 | 4) 5.6 |

2. If  $p(x) = 2\ln(x) - 1$  and  $m(x) = \ln(x + 6)$ , then what is the solution for  $p(x) = m(x)$ ?

- |         |                |
|---------|----------------|
| 1) 1.65 | 3) 5.62        |
| 2) 3.14 | 4) no solution |

3. The function  $f(x) = \sqrt{x}$ . Which function represents a shift of the graph left 3 units and up 2 units?

- |                            |                            |
|----------------------------|----------------------------|
| 1) $g(x) = \sqrt{x-3} - 2$ | 3) $g(x) = \sqrt{x+2} - 3$ |
| 2) $g(x) = \sqrt{x+3} + 2$ | 4) $g(x) = \sqrt{x-2} + 3$ |

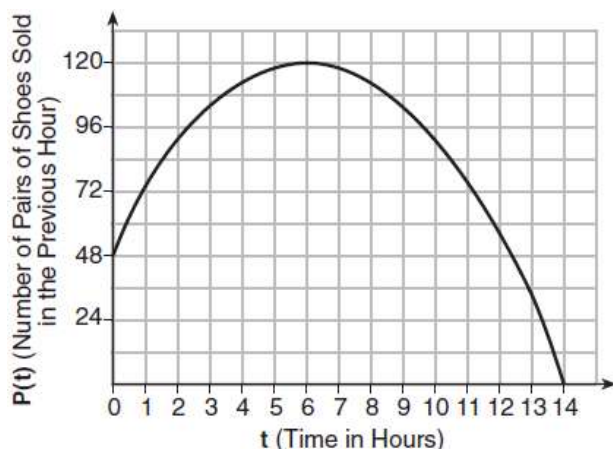
4. Joey's math class is studying the basic quadratic function,  $f(x) = x^2$ . Each student is supposed to make two new functions by adding or subtracting a constant to the function. Joey chooses the function  $g(x) = (x+2)^2 - 5$ . What transformations would map  $f(x)$  to  $g(x)$ ?

- |                                |                               |
|--------------------------------|-------------------------------|
| 1) shift left 2, shift down 5  | 3) shift right 5, shift up 2  |
| 2) shift right 2, shift down 5 | 4) shift left 5, shift down 2 |

State the transformations that are applied to  $f(x)$  to create  $g(x)$

- |   |                                |
|---|--------------------------------|
| 5. $g(x) = -f\left(\frac{1}{3}(x-5)\right) + 1$ | 6. $g(x) = -\frac{1}{2}f(x+5)$ |
|---|--------------------------------|

7. A manager wanted to analyze the online shoe sales for his business. He created a graph to model the data, as shown below. Determine the average rate of change between the sixth and fourteenth hours, and explain what it means in the context of the problem.



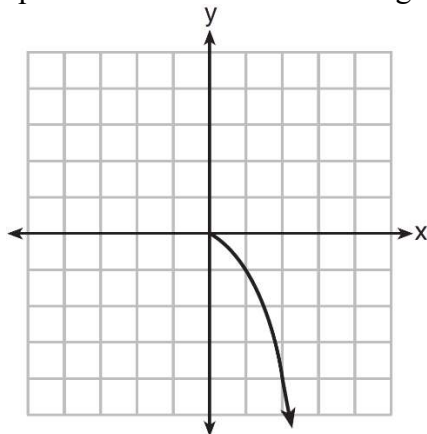
8. The population,  $P(t)$ , of a town increased according to the function  $P(t) = 12,000(1.03)^t$ , where  $t$  is the number of years since 2000. Find the average rate of change from  $t = 10$  to  $t = 20$  rounding to the nearest integer. Explain its meaning in the context of the problem.

9. The table below shows the number of hours of daylight on the first day of each month in Rochester, NY. Given the data, what is the average rate of change in hours of daylight per month from January 1st to April 1st? Interpret what this means in the context of the problem.

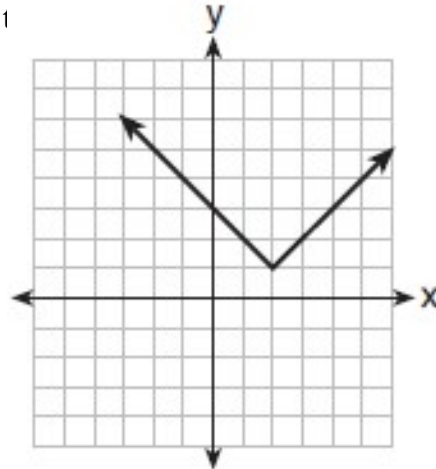
Month	Hours of Daylight
Jan.	9.4
Feb.	10.6
March	11.9
April	13.9
May	14.7
June	15.4
July	15.1
Aug.	13.9
Sept.	12.5
Oct.	11.1
Nov.	9.7
Dec.	9.0

Graph the inverse of the following functions on 1

7.



8.



9. Given  $f(x) = \frac{1}{2}x + 8$ , which equation represents the inverse,  $g(x)$ ?

1)  $g(x) = 2x - 8$

3)  $g(x) = -\frac{1}{2}x + 8$

2)  $g(x) = 2x - 16$

4)  $g(x) = -\frac{1}{2}x - 16$

10. What is the inverse of  $f(x) = 2x + 6$ ?

1)  $f^{-1}(x) = -2(x + 3)$

3)  $f^{-1}(x) = \frac{x}{2} - 3$

2)  $f^{-1}(x) = x - 3$

4)  $f^{-1}(x) = \frac{x}{2} + 3$

11. What is the inverse of  $y = \frac{1}{2}x + 2$ ?

12. Given  $f(x) = \frac{2}{3}x + 6$ , write the equation of  $f^{-1}(x)$ .

Determine whether the following are even functions, odd functions, or neither. Explain your answer.

13.  $f(x) = \left(\frac{1}{2}\right)^x$

14.  $f(x) = -x^2 + 4$

15.  $f(x) = \frac{2}{x}$

16.  $f(x) = -2x^3 + 6x$

17.  $f(x) = -|x| - 6$

18.  $f(x) = 2x^3 + 3$

19. The expression  $\left(\frac{m^2}{m^{\frac{1}{3}}}\right)^{-\frac{1}{2}}$  is equivalent to

1)  $-\sqrt[6]{m^5}$

3)  $-m^5\sqrt{m}$

2)  $\frac{1}{\sqrt[6]{m^5}}$

4)  $\frac{1}{m^5\sqrt{m}}$

20. The expression  $\sqrt[4]{81x^2y^5}$  is equivalent to

1)  $3x^{\frac{1}{2}}y^{\frac{5}{4}}$

2)  $3x^{\frac{1}{2}}y^{\frac{4}{5}}$

3)  $9xy^{\frac{5}{2}}$

4)  $9xy^{\frac{2}{5}}$

21. The solution to the equation  $6(2^{x+4}) = 36$  is

1)  $-1$

3)  $\ln(3) - 4$

2)  $\frac{\ln 36}{\ln 12} - 4$

4)  $\frac{\ln 6}{\ln 2} - 4$



22. Which is the solution to:  $5(3)^{2x} = 30$  ?

- |                              |                              |
|------------------------------|------------------------------|
| 1) $\frac{\log 6}{3 \log 2}$ | 3) $\frac{2 \log 6}{\log 3}$ |
| 2) $\frac{\log 6}{2 \log 3}$ | 4) $\frac{2 \log 3}{\log 6}$ |

Solve the following equations for all values of x:

23.  $3x^{\frac{4}{3}} - 5 = 43$

24.  $3x^{\frac{2}{5}} - 11 = 289$

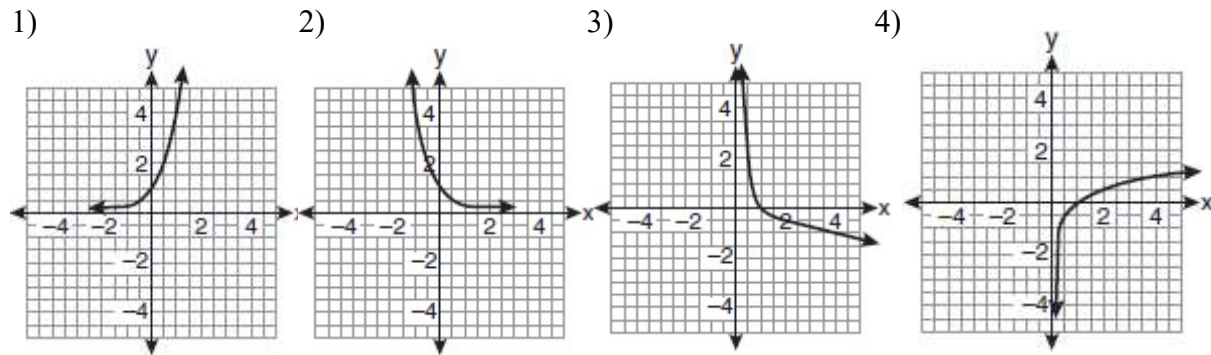
25. If (2,4) is included in  $f(x)$ , what point must be included in  $g(x)$  if  $g(x) = f(x) + 3$ .

26. If (2,4) is included in  $f(x)$ , what point must be included in  $g(x)$  if  $g(x) = f(x + 3)$ .

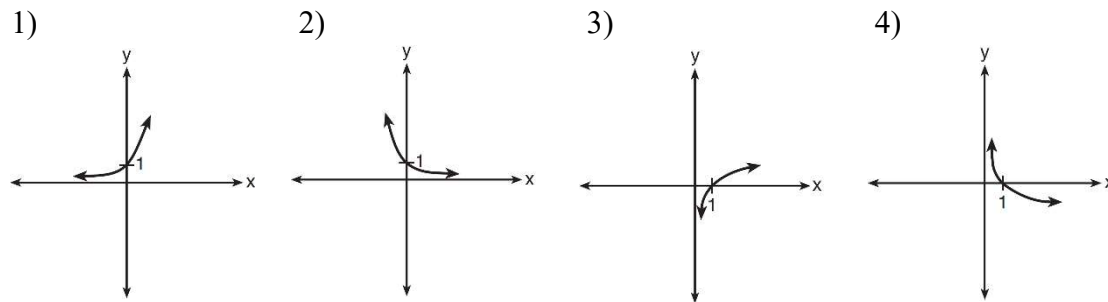
27. If (5,1) is included in  $f(x)$ , what point must be included in  $g(x)$  if  $g(x) = f(x - 5)$ .

28. If (4,7) is included in  $f(x)$ , what point must be included in  $g(x)$  if  $g(x) = f(x) - 2$ .

29. If a function is defined by the equation  $f(x) = \log_4 x$ , which graph represents the inverse of this function?



30. Which sketch shows the inverse of  $y = a^x$ , where  $a > 1$ ?



31. Which statement about the graph of  $f(x) = 2^x - 1$  is *true*?

- 1) It is always increasing with a y intercept of 0
- 2) It is always decreasing with a y intercept of 0
- 3) It is always increasing with a y intercept of -1
- 4) It is always decreasing with a y intercept of -1

32. Which statement about the graph of the equation  $f(x) = \frac{1}{3}^x + 2$  is *true*?

- 1) It is always increasing with a y intercept of 2
- 2) It is always decreasing with a y intercept of 2
- 3) It is always increasing with a y intercept of 3
- 4) It is always decreasing with a y intercept of 3

33. Given  $f(x) = 3^{x-1} + 2$ , as  $x \rightarrow -\infty$

1)  $f(x) \rightarrow -1$

2)  $f(x) \rightarrow 0$

3)  $f(x) \rightarrow 2$

4)  $f(x) \rightarrow -\infty$

34. For the equation  $f(x) = -\log_3(x+1) - 2$ , as  $x \rightarrow \infty$

1)  $f(x) \rightarrow -\infty$

3)  $f(x) \rightarrow \infty$

2)  $f(x) \rightarrow -1$

4)  $f(x) \rightarrow -2$

35.  $y = 2(3)^{x+1} - 8$

Domain:

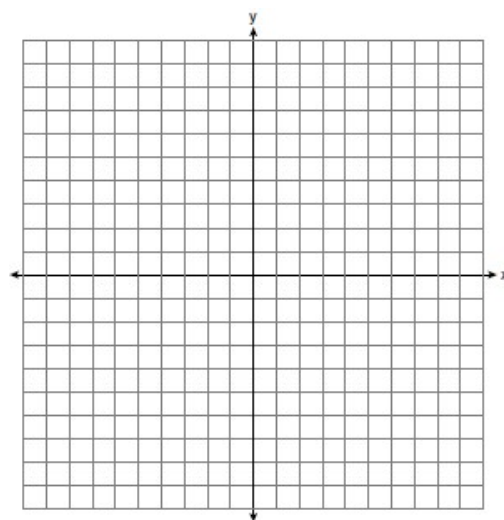
Range:

Asymptote:

End Behavior:

$x \rightarrow -\infty, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$



36.  $y = -2\left(\frac{1}{3}\right)^{x-5} + 9$

Domain:

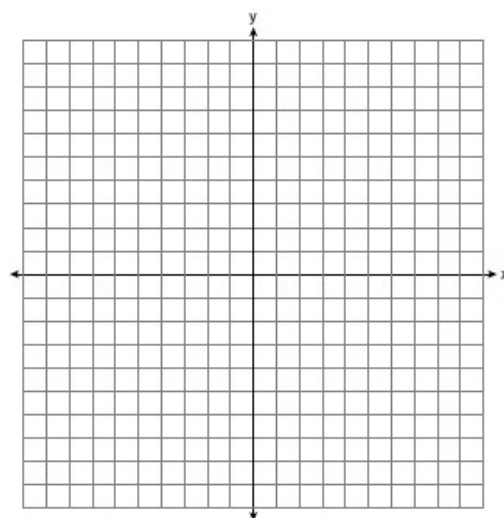
Range:

Asymptote:

End Behavior:

$x \rightarrow -\infty, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$



37.  $y = \log_3(x+2) - 1$

Domain:

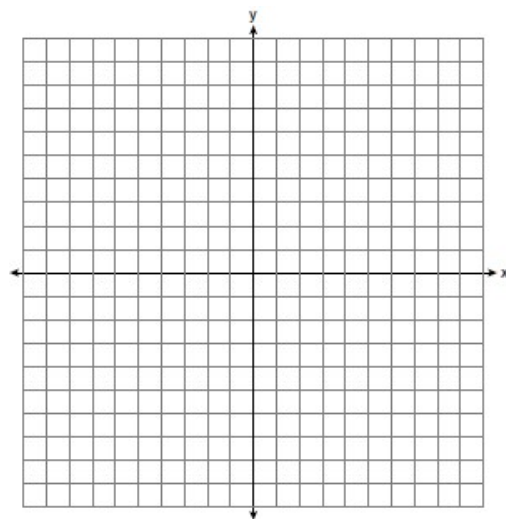
Range:

Asymptote:

End Behavior:

$x \rightarrow -2, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$



38.  $y = -2\log_2(x+6) - 4$

Domain:

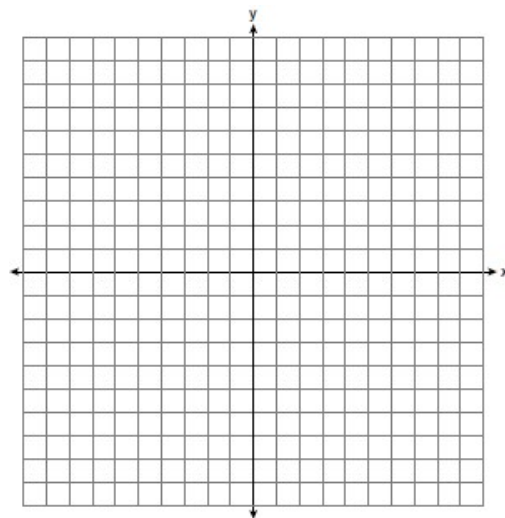
Range:

Asymptote:

End Behavior:

$x \rightarrow -6, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$



39.  $f(x) = 2 - \sqrt{x+3}$

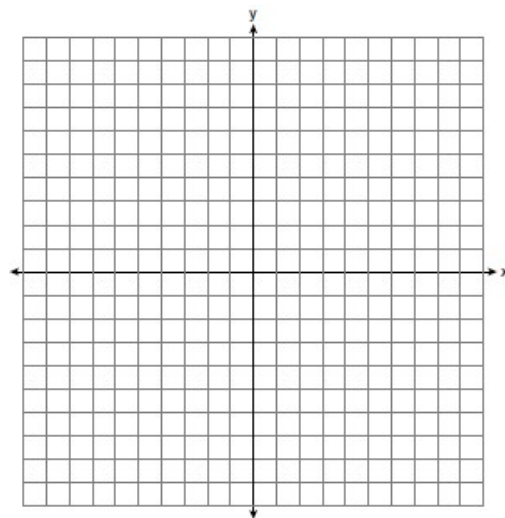
Domain:

Range:

End Behavior:

$x \rightarrow -3, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$



40.  $f(x) = -\sqrt[3]{2x+8}$

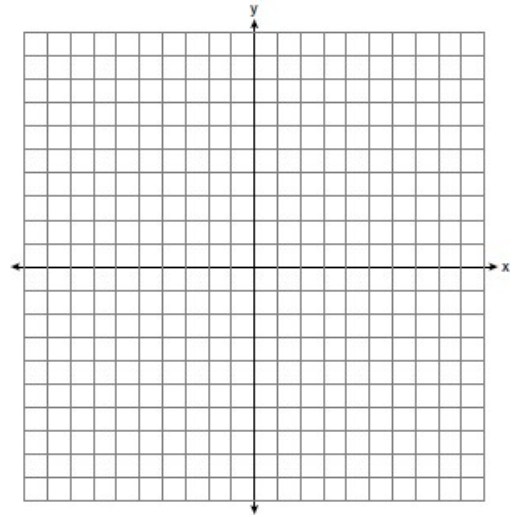
Domain:

Range:

End Behavior:

$x \rightarrow -\infty, f(x) \rightarrow$

$x \rightarrow \infty, f(x) \rightarrow$



Express the following in simplest form with a rational exponent

41.  $\frac{\sqrt[3]{x^2} \cdot \sqrt{x^5}}{\sqrt[6]{x}}$

42.  $\frac{x\sqrt{x^3}}{\sqrt[3]{x^5}}$

43. 200 grams of a radioactive substance decays according to the formula  $a(t) = 200(.094)^{2t}$  where  $a(t)$  is the amount of the radioactive substance remaining after  $t$  years. To the nearest hundredth of a year, how long will it take until there are 50 grams remaining?

44. Juliette deposits \$2500 into a bank account where the balance of the account  $b(t)$  after  $t$  years can be represented by  $b(t) = 2500(1.075)^t$ . To the nearest tenth of a year, how long will it take for Juliette's money to reach \$4000?

45. Empanadas are taken out of an oven when they reached a temperature of 168°F and put on the kitchen table at room temperature (68°F). After 8 minutes, the temperature of the empanadas is 125°F. The temperature of a cooled object can be given by the formula below:

$$T = T_a + (T_0 - T_a)e^{-kt}$$

$T$  = the temperature of the object after  $t$  minutes

$t$  = time in minutes

$T_a$  = the surrounding temperature

$T_0$  = the initial temperature of the object

$k$  = decay constant

Algebraically determine the value of  $k$ , rounded to the *nearest thousandth*. Using your value of  $k$ , to the *nearest minute*, algebraically determine how long will it take for the empanadas to reach 100°F?

46. After sitting out of the refrigerator for a while, a turkey at room temperature ( $68^{\circ}\text{F}$ ) is placed into an oven at 8 a.m., when the oven temperature is  $325^{\circ}\text{F}$ . Newton's Law of Heating explains that the temperature of the turkey will increase proportionally to the difference between the temperature of the turkey and the temperature of the oven, as given by the formula below:

$$T = T_a + (T_0 - T_a)e^{-kt}$$

$T_a$  = the temperature surrounding the object

$T_0$  = the initial temperature of the object

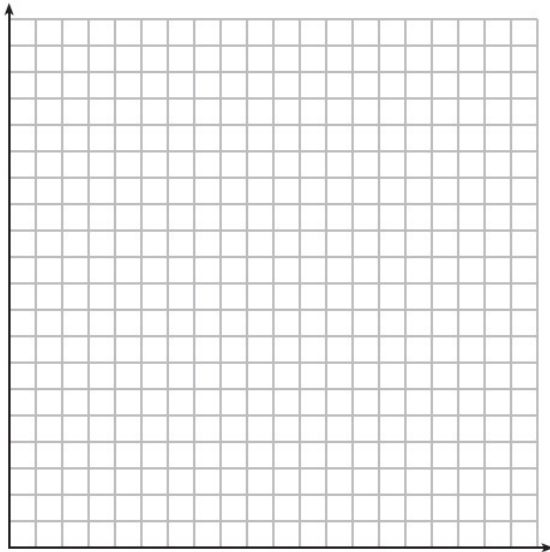
$t$  = the time in hours

$T$  = the temperature of the object after  $t$  hours

$k$  = decay constant

The turkey reaches the temperature of approximately  $100^{\circ}\text{F}$  after 2 hours. Algebraically determine the value of  $k$ , to the *nearest thousandth*. Using your value of  $k$ , algebraically determine how many hours after 8 a.m. the turkey will be  $160^{\circ}$  to the *nearest tenth of a year*?

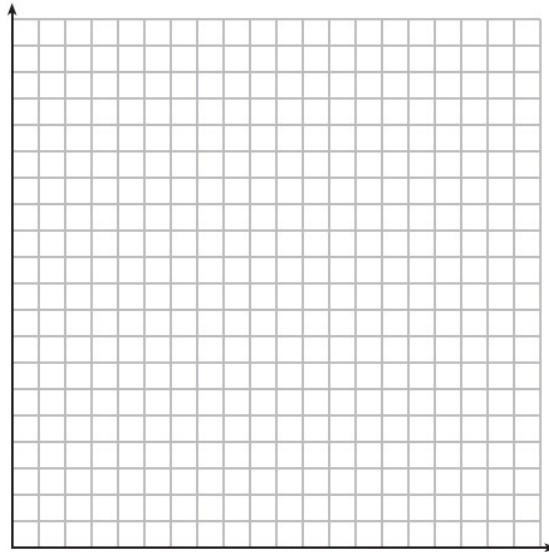
47. The value of Tom's bank account is currently 100000 and is decreasing according to the equation  $V(t) = 90000(.887)^t$ . The amount of money he has paid for his mortgage can be represented by the equation  $M(t) = 18000(1.152)^t$ . Graph and label  $V(t)$  and  $M(t)$  over the interval  $[0, 10]$ .



After how many years will the value of Tom's bank account be equal to the amount of money he paid for his mortgage? Round your answer to the *nearest tenth of a year*. Tom will open a new bank account when the value of his account is \$25,000. Algebraically, determine after how many years, to the *nearest hundredth of a year*, will that happen?



48. The value of a certain small passenger car based on its use in years is modeled by  $V(t) = 28482.698(0.684)^t$ , where  $V(t)$  is the value in dollars and  $t$  is the time in years. Zach had to take out a loan to purchase the small passenger car. The function  $Z(t) = 22151.327(0.778)^t$ , where  $Z(t)$  is measured in dollars, and  $t$  is the time in years, models the unpaid amount of Zach's loan over time. Graph  $V(t)$  and  $Z(t)$  over the interval  $0 \leq t \leq 5$ , on the set of axes below.



State when  $V(t) = Z(t)$ , to the *nearest hundredth*, and interpret its meaning in the context of the problem. Zach will cancel the collision policy when the value of his car equals \$3000. To the *nearest tenth of a year*, how long will it take Zach to cancel this policy? Justify your answer.