Name _____ KEY VIDEO Date _____
Mr. Schlansky Geometry

Schlansky's Guide to 65 Geometry!

1. Which of the following cannot make up the three sides of a triangle?

1) {3,5,4} 3+4>5 ✓
2) {2,2,3} 2+2>3 ✓
3) {9,7,5} 5+7>9 ✓
4) {6,1,4} 1+4>6 ✗

Triangle Inequality Theorem:
The two smallest sides of a triangle must add up to be greater than the third side.
If given two sides, add those two sides onto each of the choices and do the same thing.

2. Which of the following can make up the three sides of a triangle?

1) {2,4,2}
2) {1,7,4}
3) {8,1,6}
4) {5,5,7}

3. Which numbers could represent the lengths of the sides of a triangle?

1) **5,9,14**
2) **7,7,15**
3) **1,2,4**
4) **3,6,8**

4. In ΔBLA, $\overline{BL} = 12$ and $\overline{AL} = 8$. What is a possible values of $\overline{BA}$?

1) 17, 12, 8 8+12>17 ✓
2) 4, 12, 8 4+8>12 ✗
3) 2, 12, 8 2+8>12 ✗
4) 21, 12, 8 8+12>21 ✗

Triangle Inequality Theorem:
The two smallest sides of a triangle must add up to be greater than the third side.
If given two sides, add those two sides onto each of the choices and do the same thing.

5. In ΔCAM, $\overline{CM} = 10$ and $\overline{CA} = 4$. What is a possible value of $\overline{MA}$?

1) 4
2) 6
3) 15
4) 13

6. In ΔABC, $AB = 5$ feet and $BC = 3$ feet. Which *cannot* represent the value for the length of $\overline{AC}$, in feet?

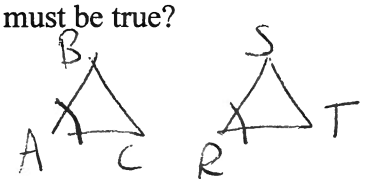
1) 3
2) 5
3) 7
4) 9

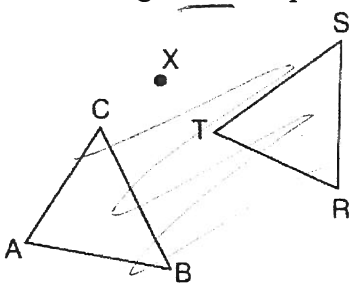
Corresponding Parts of Congruent Triangles are Congruent
Redraw the shapes so it is more clear to see what parts correspond to each other.
If algebraic expressions, set the corresponding sides equal to each other.

7. After a counterclockwise rotation about point X, scalene triangle ABC maps onto $\triangle RST$, as shown in the diagram below.

Which statement must be true?

1) $\angle A \cong \angle R$ ✓
2) $\angle A \cong \angle S$
3) $\overline{CB} \cong \overline{TR}$
4) $\overline{CA} \cong \overline{TS}$

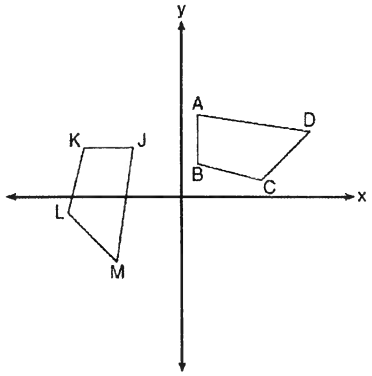




8. In the diagram below, a sequence of rigid motions maps $ABCD$ onto $JKLM$.

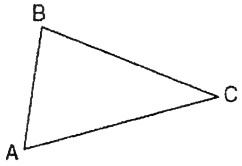
Which of the following statements must be true?

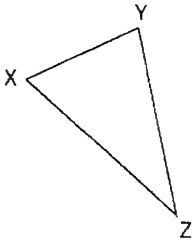
1) $\angle L \cong \angle B$
2) $\angle A \cong \angle J$
3) $\overline{JK} \cong \overline{AC}$
4) $\overline{JM} \cong \overline{AB}$



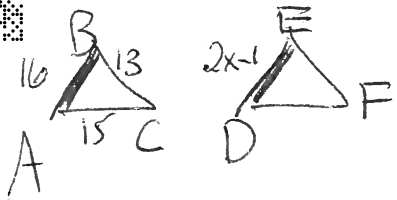
9. In the diagram below of $\triangle ABC$ and $\triangle XYZ$, a sequence of rigid motions maps $\angle A$ onto $\angle X$, $\angle C$ onto $\angle Z$, and $\overline{AC}$ onto $\overline{XZ}$.. Which of the following statements is *not* true?

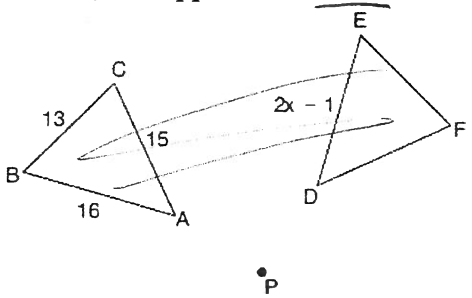
1) $\overline{AB} \cong \overline{XY}$
2) $\overline{BC} \cong \overline{XZ}$
3) $\angle B \cong \angle Y$
4) $\angle C \cong \angle Z$





10. In the diagram below, $\triangle ABC$ with sides 13, 15, and 16, is mapped onto $\triangle DEF$ after a clockwise rotation of 90° about point P.
If $DE = 2x - 1$, what is the value of x?

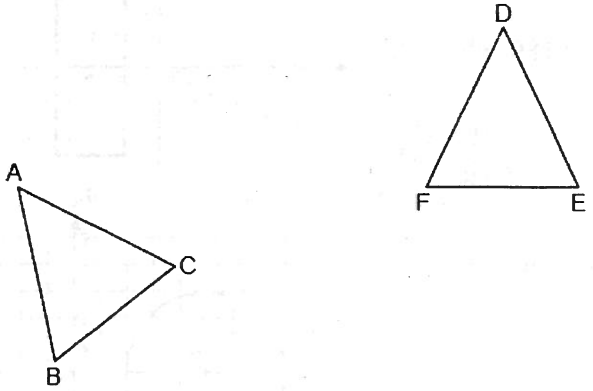




$2x+1=16$
$-1 \quad -1$
$\frac{2x}{2} = \frac{15}{2}$
$x = 8.5$

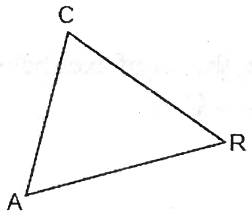
Corresponding Parts of Congruent Triangles are Congruent
Redraw the shapes so it is more clear to see what parts correspond to each other.
If algebraic expressions, set the corresponding sides equal to each other.

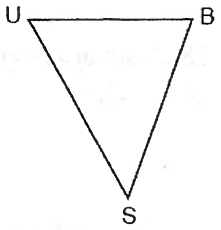
11. In the diagram below, ΔDEF is the image of ΔABC after a reflection. If $\overline{AB}=7$, $\overline{CB}=5$, $\overline{AC}=8$, and $\overline{DE}=5x-3$, find the value of x.



12. In the diagram below, $\triangle CAR$ is mapped onto $\triangle BUS$ after a sequence of rigid motions. If $AR = 3x + 4$, $RC = 5x - 10$, $CA = 2x + 6$, and $SB = 4x - 4$, what is the length of $\overline{SB}$?

1) 6
2) 16
3) 20
4) 28





6

13. Which of the following degree measures would map a regular hexagon onto itself?

1) 45
2) 90
3) 180 (a(3)
4) 270

$\frac{360}{6}=60$

Regular Polygon Rotations
To determine the minimum number of degrees a regular polygon must be rotated to be mapped onto itself:

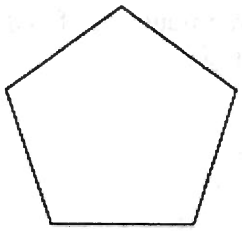
1) The minimum rotation is $\frac{360}{n}$.
2) **Any multiple of that will also map the regular polygon onto itself!**

14. Which of the following degree measures would map a regular nonagon onto itself?

1) 60
2) 90
3) 180
4) 240

15. The regular polygon below is rotated about its center. Which angle of rotation will carry the figure onto itself?

1) 60°
2) 108°
3) 216°
4) 540°

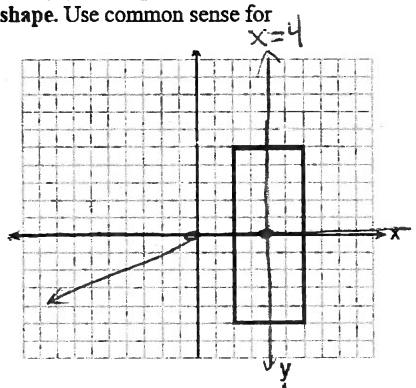


To map a shape onto itself:
Translation/Dilation: Never.
Reflection: **The line of reflection must be a line of symmetry** (cuts shape in half).
Rotation: **Center of rotation must be the center of the shape.** Use common sense for degree measure.

16. As shown in the graph below, the quadrilateral is a rectangle. Which transformation would *not* map the rectangle onto itself?

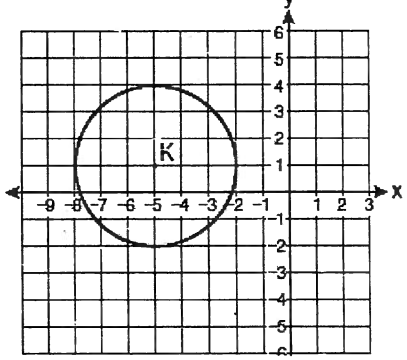
1) a reflection over the x-axis
2) a reflection over the line $x = 4$
3) a rotation of 180° about the origin
4) a rotation of 180° about the point (4, 0)

not the center of the shape



17. Circle K is shown in the graph below.
Which of the following transformations map circle K onto itself?

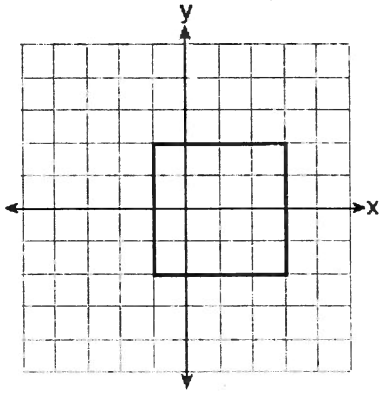
1) Reflection over the line $x-axis$
2) Reflection over the $y-axis$
3) Rotation of 90 centered at the origin
4) Rotation of 90 centered at K



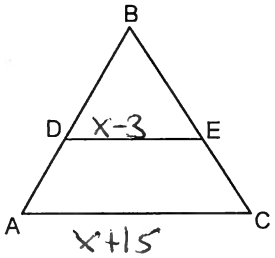
18. A square is graphed on the set of axes below, with vertices at (−1, 2), (−1, −2), (3, −2), and (3, 2).

Which transformation would *not* carry the square onto itself?

1) reflection over the y-axis
2) reflection over the x-axis
3) rotation of 180 degrees around point (1, 0)
4) reflection over the line $y = x - 1$



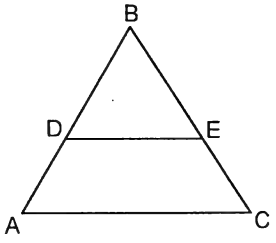
19. D and E are midpoints of $\overline{AB}$ and $\overline{BC}$ respectively. If $\overline{AC} = x + 15$ and $\overline{DE} = x - 3$, find the measure of $\overline{DE}$.



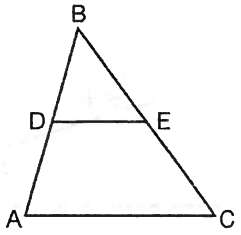
Joining the Midpoints of a Triangle
2(midsegment) = opposite side

$2(x-3) = x+15$
$2x-6 = x+15$
$-x \quad -x$
$x-6 = 15$
$+6 \quad +6$
$x = 21$
$\overline{DE} = x-3$
$\overline{DE} = 21-3$
$\overline{DE} = 18$

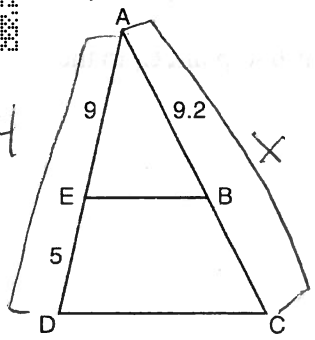
20. D and E are midpoints of $\overline{AB}$ and $\overline{BC}$ respectively. If $\overline{DE} = 2x + 5$ and $\overline{AC} = 7x + 1$, find the measure of $\overline{AC}$.



21. In ΔABC, D is the midpoint of $\overline{AB}$ and E is the midpoint of $\overline{BC}$. If $AC = 3x - 15$ and $DE = 6$, what is the value of x?



22. In the diagram of ΔADC below, $\overline{EB} \parallel \overline{DC}$. $AE = 9$. $ED = 5$. and $AB = 9.2$. What is the length of $\overline{AC}$, to the *nearest tenth*?



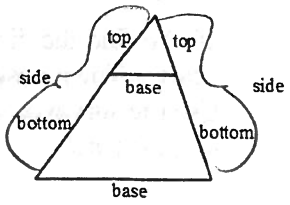
14

Candy Corn Problems

If the bases are not involved: $\frac{top}{top} = \frac{bottom}{bottom} = \frac{side}{side}$

If bases are involved: separate your triangles!

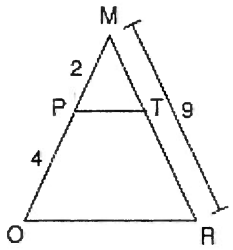
*Expect to have to add/subtract



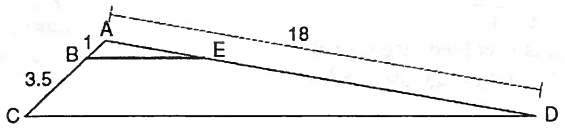
$\frac{top}{top} = \frac{side}{side}$ $\quad \frac{9}{9.2} = \frac{14}{x}$

$\frac{9x}{9} = \frac{128.8}{9}$ $\quad 9x = 128.8$ $\quad x = 14.3$

23. Given $\triangle MRO$ shown below, with trapezoid $PTRO$, $MR = 9$, $MP = 2$, and $PO = 4$. What is the length of $\overline{TR}$?



24. In the diagram below, triangle ACD has points B and E on sides $\overline{AC}$ and $\overline{AD}$, respectively, such that $\overline{BE} \parallel \overline{CD}$, $AB = 1$, $BC = 3.5$, and $AD = 18$.



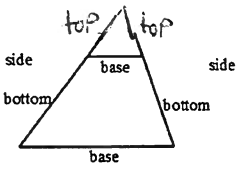
What is the length of $\overline{AE}$, to the *nearest tenth*?

Candy Corn Problems

If the bases are not involved: $\frac{top}{top} = \frac{bottom}{bottom} = \frac{side}{side}$

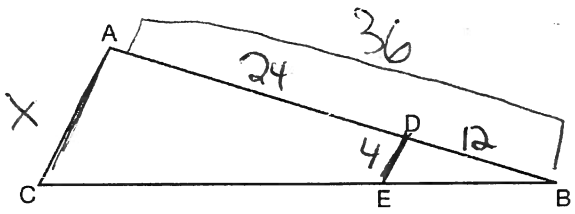
If bases are involved: separate your triangles!

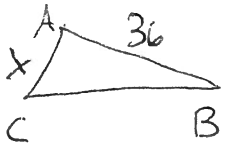
*Expect to have to add/subtract

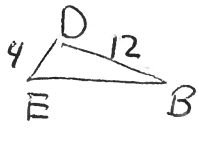


25. In the diagram of $\triangle ABC$, points D and E are on $\overline{AB}$ and $\overline{CB}$, respectively, such that $\overline{AC} \parallel \overline{DE}$. If $AD = 24$, $DB = 12$, and $DE = 4$, what is the length of $\overline{AC}$?

bases involved
Separate





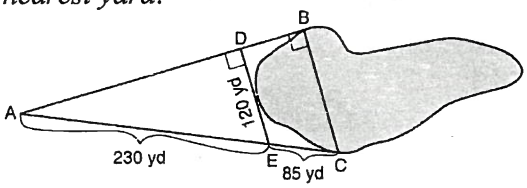


$\frac{X}{4} = \frac{36}{12}$

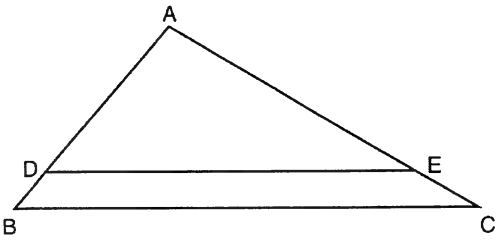
$\frac{12x}{12} = \frac{144}{12}$

$X = 12$

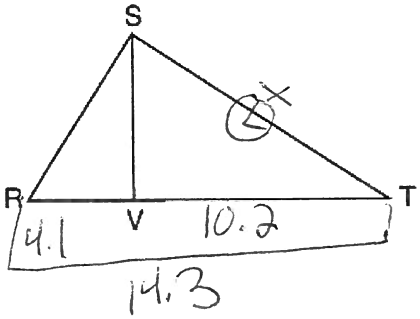
26. To find the distance across a pond from point B to point C, a surveyor drew the diagram below. The measurements he made are indicated on his diagram.
Use the surveyor's information to determine and state the distance from point B to point C, to the *nearest yard*.



27. In the diagram of $\triangle ABC$ shown below, $\overline{DE} \parallel \overline{BC}$.
If $\overline{AE} = 6$, $\overline{DE} = 10$, and $\overline{EC} = 3$, find $\overline{BC}$



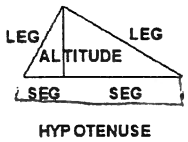
28. In right triangle RST below, altitude $\overline{SV}$ is drawn to hypotenuse $\overline{RT}$. If $RV = 4.1$ and $TV = 10.2$, what is the length of $\overline{ST}$, to the *nearest tenth*?



When an altitude is drawn to a right triangle
HLLS and SAAS

$\frac{H}{L} = \frac{L}{S}$ $\frac{S}{A} = \frac{A}{S}$

If L is involved, use HLLS
If A is involved, use SAAS



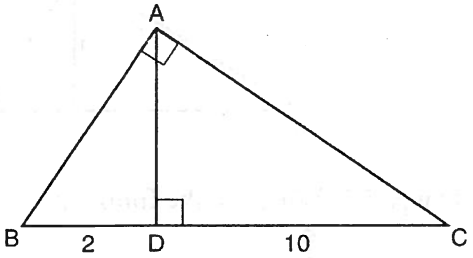
$\frac{H}{L} = \frac{L}{S}$

$\frac{14.3}{X} = \frac{X}{10.2}$

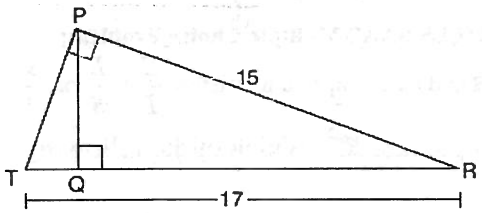
$\sqrt{X^2} = \sqrt{145.86}$

$X = 12.1$

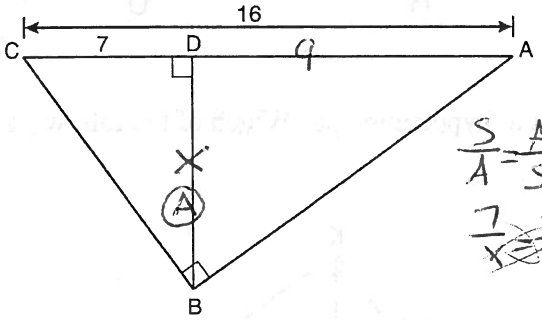
29. Triangle ABC shown below is a right triangle with altitude $\overline{AD}$ drawn to the hypotenuse $\overline{BC}$. If $BD = 2$ and $DC = 10$, what is the length of $\overline{AB}$ to the *nearest tenth*?



30. In right triangle PRT, $m\angle P = 90°$, altitude $\overline{PQ}$ is drawn to hypotenuse $\overline{RT}$, $RT = 17$, and $PR = 15$. Determine and state, to the *nearest tenth*, the length of $\overline{RQ}$.



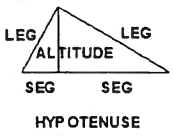
31. In the diagram below of right triangle ABC, altitude $\overline{BD}$ is drawn to hypotenuse $\overline{AC}$, $AC = 16$, and $CD = 7$. What is the length of $\overline{BD}$ to the *nearest tenth*?



When an altitude is drawn to a right triangle
HLLS and SAAS

$\frac{H}{L} = \frac{L}{S}$ $\frac{S}{A} = \frac{A}{S}$

If L is involved, use HLLS
If A is involved, use SAAS



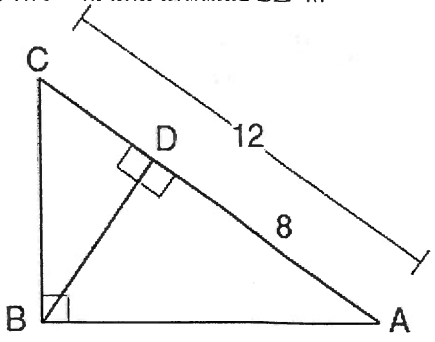
$\frac{S}{A} = \frac{A}{S}$

$\frac{7}{x} = \frac{x}{9}$

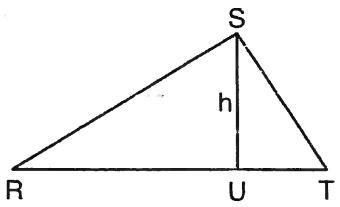
$\sqrt{x^2} = \sqrt{63}$

$x = 7.9$

32. In the diagram below of $\triangle ABC$, $\angle ABC$ is a right angle, $AC = 12$, $AD = 8$, and altitude $\overline{BD}$ is drawn. What is the length of $\overline{BD}$ to the *nearest tenth*?



33. In $\triangle RST$ shown below, altitude $\overline{SU}$ is drawn to $\overline{RT}$ at U.
If $SU = h$, $UT = 12$, and $RT = 42$, which value of h, to the *nearest tenth*, will make $\triangle RST$ a right triangle with $\angle RST$ as a right angle?



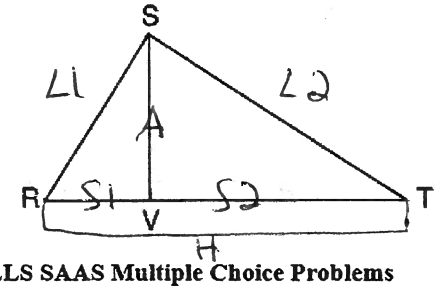
34. In right triangle RST below, altitude $\overline{SV}$ is drawn to hypotenuse $\overline{RT}$. Which of the following proportions is true?

1) $\frac{RV}{VS} = \frac{VT}{VS}$ $\frac{S1}{A} = \frac{S2}{A}$

2) $\frac{RT}{RS} = \frac{RS}{VT}$ $\frac{H}{L1} = \frac{L1}{S2}$

3) $\frac{RT}{SV} = \frac{SV}{VT}$ $\frac{H}{A} = \frac{A}{S2}$

4) $\frac{RT}{ST} = \frac{ST}{VT}$ $\frac{H}{L2} = \frac{L2}{S2}$



HLLS SAAS Multiple Choice Problems

See if each proportion satisfies $\frac{H}{L} = \frac{L}{S}$ or $\frac{S}{A} = \frac{A}{S}$.

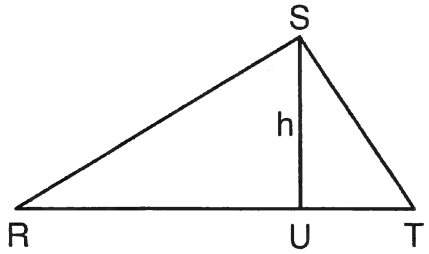
35. In right triangle RST below, altitude $\overline{SU}$ is drawn to hypotenuse $\overline{RT}$. Which of the following proportions is *not* true?

1) $\frac{RU}{SU} = \frac{SU}{UT}$

2) $\frac{SU}{RU} = \frac{RU}{UT}$

3) $\frac{RT}{RS} = \frac{RS}{RU}$

4) $\frac{TR}{ST} = \frac{ST}{UT}$



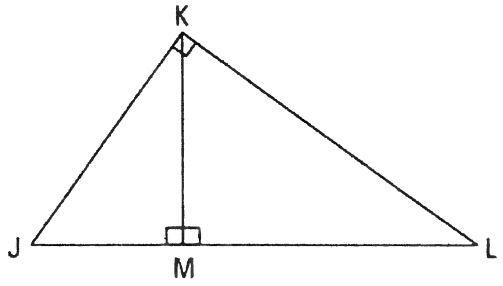
36. In right triangle JKL below, altitude $\overline{KM}$ is drawn to hypotenuse $\overline{JL}$. Which of the following proportions is *not* true?

1) $\frac{JL}{JK} = \frac{JK}{JM}$

2) $\frac{JM}{KM} = \frac{KM}{ML}$

3) $\frac{JL}{KL} = \frac{KL}{JM}$

4) $\frac{ML}{MK} = \frac{MK}{MJ}$



Scale Factor with Perimeter and Area
Perimeter(scale factor)
Area $(scale\ factor)^2$

37. Triangle JOY has a perimeter of 10 and an area of 12. What is the perimeter and area of triangle JOY after a dilation by a scale factor of 2?

Perimeter(scale factor) $10(2)=20$
area(scale factor)^2 $12(2)^2=48$

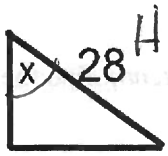
38. Quadrilateral CAMI has a perimeter of 20 and an area of 15. What is the perimeter and area of quadrilateral CAMI after a dilation by a scale factor of 4?

39 . Triangle *RJM* has an area of 6 and a perimeter of 12. If the triangle is dilated by a scale factor of 3 centered at the origin, what are the area and perimeter of its image, triangle *R'J'M'*?

1) area of 9 and perimeter of 15
2) area of 18 and perimeter of 36
3) area of 54 and perimeter of 36
4) area of 54 and perimeter of 108

Find x in each of the following pictures rounding to the *nearest integer*

40.



H
A
O

$\sin\theta = \frac{O}{H}$

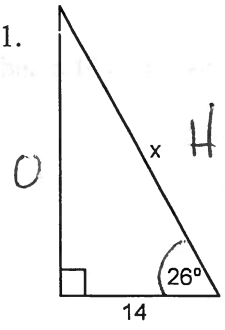
$\sin^{-1}\sin X = \sin^{-1}\frac{17}{28}$

$X = \sin^{-1}\left(\frac{17}{28}\right)$

$X = 37$

SOH CAH TOA

41.



O
H
A

$\cos\theta = \frac{A}{H}$

$\cos 26 = \frac{14}{x}$

$X\cos 26 = 14$

$\frac{X\cos 26}{\cos 26} = \frac{14}{\cos 26}$

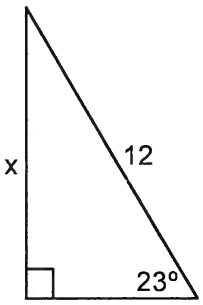
$X = 16$

OR

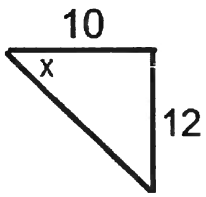
Math UP
Graph, Wi Graph!

Finding Sides and Angles with Trig (SOHCAHTOA)
1) Label each side with H, A, and O
2) Determine whether to use sine, cosine, or tangent (Which two are involved?)
3) Substitute into appropriate formula
*If finding a side, cross multiply and solve
*If finding an angle, use $\sin^{-1}$, $\cos^{-1}$, *or* $\tan^{-1}$

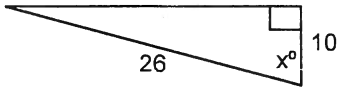
42.



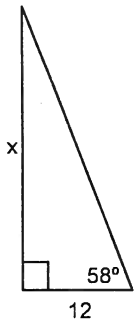
43.



44.

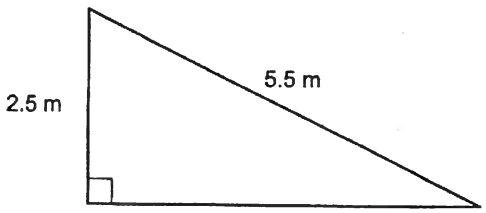


45.



46. In right triangle *JOE*, hypotenuse $JO = 31.8$ and $m\angle J = 38°$. To the *nearest tenth*, find the length of $\overline{EJ}$.

47. Many roofs are slanted to prevent the buildup of snow. As modeled below, the length of a roof is 5.5 meters and it rises to a height of 2.5 meters. What is the angle of elevation of the roof to the *nearest degree*?

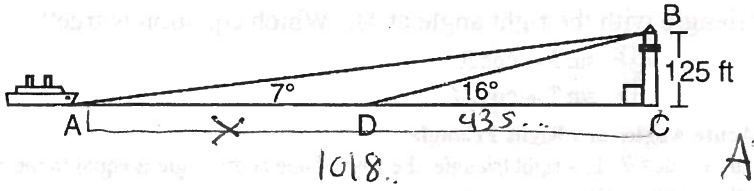


48. Triangle ABC has a right angle at C. If $AC = 7.7$ and $m\angle B = 24°$, what is AB, to the *nearest tenth*?

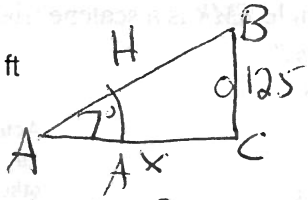
Procedure 1: Subtraction: Find corresponding parts of the two right triangles and subtract them.

49. As shown in the diagram below, a ship is heading directly toward a lighthouse whose beacon is 125 feet above sea level. At the first sighting, point A, the angle of elevation from the ship to the light was 7°. A short time later, at point D, the angle of elevation was 16°.

To the *nearest foot*, determine and state how far the ship traveled from point A to point D.



1018..
− 435.
582



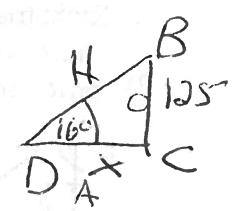
$\tan\theta = \frac{O}{A}$

$\frac{\tan 7}{1} = \frac{125}{x}$

$x \tan 7 = 125$

$\frac{x \tan 7}{\tan 7} = \frac{125}{\tan 7}$

$x = 1018...$



$\tan\theta = \frac{O}{A}$

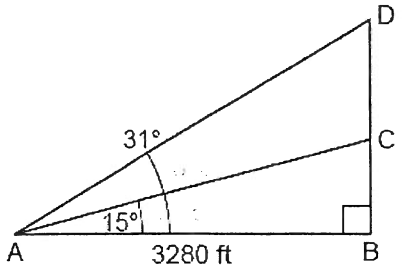
$\frac{\tan 16}{1} = \frac{125}{x}$

$x \tan 16 = 125$

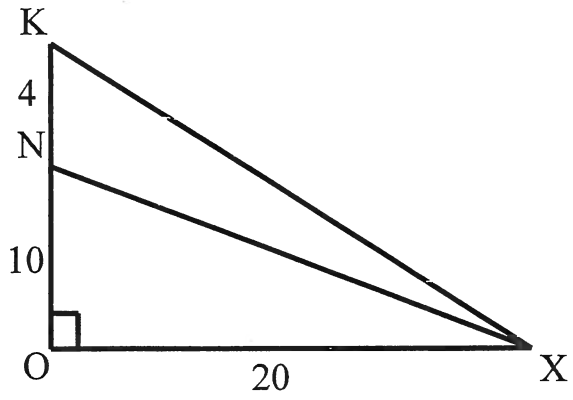
$\frac{x \tan 16}{\tan 16} = \frac{125}{\tan 16}$

$x = 435...$

50. Cape Canaveral, Florida is where NASA launches rockets into space. As modeled in the diagram below, a person views the launch of a rocket from observation area A, 3280 feet away from launch pad B. After launch, the rocket was sighted at C with an angle of elevation of 15°. The rocket was later sighted at D with an angle of elevation of 31°. Determine and state, to the *nearest foot*, the distance the rocket traveled between the two sightings, C and D.

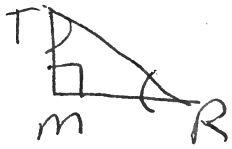


51. Find the measure of $\angle KXN$ below the *nearest degree*.



52. Right triangle *TMR* is a scalene triangle with the right angle at *M*. Which equation is true?

1) $\sin M = \cos T$
2) $\sin R = \cos R$
3) $\sin T = \cos R$
4) $\sin T = \cos M$



Acute Angles in a Right Triangle
$\sin A = \cos B$: In a right triangle, the sine of one acute angle is equal to the cosine of the other acute angle
$A + B = 90$: The two acute angles in a right triangle are complementary
*A and B must be the ACUTE angles, not the right angle!

53. Right triangle *HWS* has $m\angle H = 90°$. Which equation is always true?

1) $\sin H = \cos W$
2) $\cos S = \sin W$
3) $\cos H = \sin S$
4) $\sin W = \cos H$

54. Right triangle *BIL* has $m\angle L = 90°$. Which equation is always true?

1) $\sin B = \cos I$
2) $\cos L = \sin I$
3) $\sin B = \cos L$
4) $\cos B = \sin L$

55. If $\sin(2x+7)° = \cos(4x-7)°$, what is the value of x?

1) 7
2) 15
3) 21
4) 30

$\sin A = \cos B$

$A + B = 90$

$2x + 7 + 4x - 7 = 90$

$\frac{6x}{6} = \frac{90}{6}$

$x = 15$

Acute Angles in a Right Triangle
$\sin A = \cos B$: In a right triangle, the sine of one acute angle is equal to the cosine of the other acute angle
$A + B = 90$: The two acute angles in a right triangle are complementary
*A and B must be the ACUTE angles, not the right angle!

56. In a right triangle, $\sin(40 - x)° = \cos(3x)°$. What is the value of x?

1) 10
2) 15
3) 20
4) 25

57. In a right triangle, the acute angles have the relationship $\sin(2x + 4) = \cos(46)$. What is the value of x?

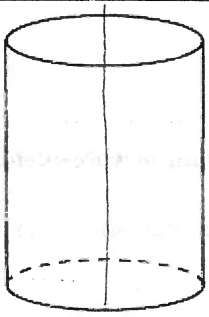
1) 20
2) 21
3) 24
4) 25

Plane Sections (2 dimensional slice of a 3 dimensional object):

Parallel to the Base (Horizontal)	Perpendicular to the Base (Vertical)	
The base	Prism/Cylinder: Rectangle	Pyramid/Cone: Triangle

58. A plane intersects a cylinder perpendicular to its bases.

Vertical

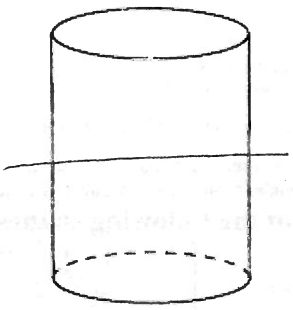


This cross section can be described as a

1) rectangle
2) parabola
3) triangle
4) circle

59. A plane intersects a cylinder parallel to its bases.

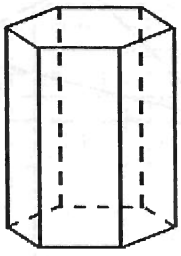
horizontal



This cross section can be described as a

1) rectangle
2) parabola
3) triangle
4) circle

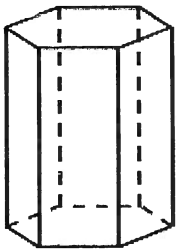
60. A right hexagonal prism is shown below. A two-dimensional cross section that is perpendicular to the base is taken from the prism.



Which figure describes the two-dimensional cross section?

1) triangle
2) rectangle
3) pentagon
4) hexagon

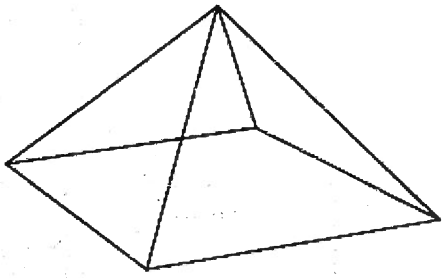
61. A right hexagonal prism is shown below. A two-dimensional cross section that is parallel to the base is taken from the prism.



Which figure describes the two-dimensional cross section?

1) triangle
2) rectangle
3) pentagon
4) hexagon

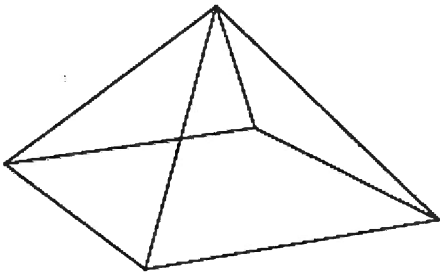
62. A square pyramid is intersected by a plane passing through the vertex and parallel to the base.



Which two-dimensional shape describes this cross section?

1) square
2) triangle
3) pentagon
4) rectangle

63. A square pyramid is intersected by a plane passing through the vertex and perpendicular to the base.



Which two-dimensional shape describes this cross section?

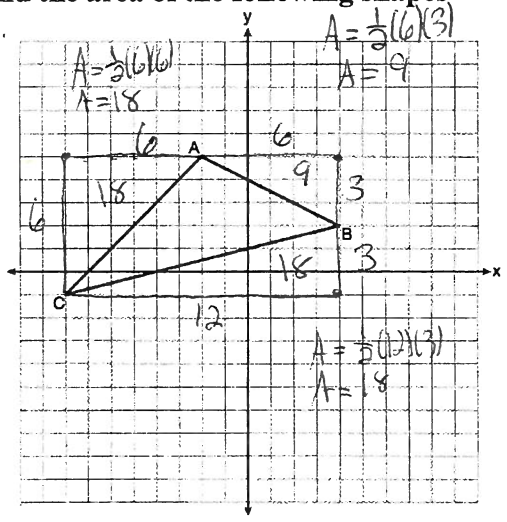
1) square
2) triangle
3) pentagon
4) rectangle

Area on the Grid
Box Method

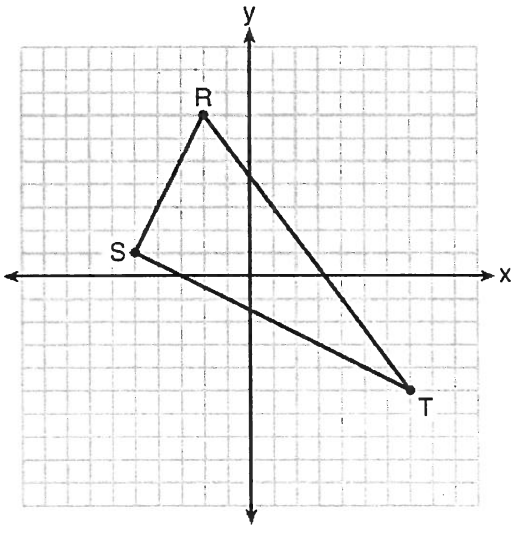
1) Build a rectangle around the shape
2) Find the area of the rectangle ($A = bh$)
3) Find the area of the triangles outside of the shape ($A = \frac{1}{2}bh$)
4) Add the triangle areas and subtract the triangle areas from the rectangle area
   *Triangles have 3 triangles to subtract away, quadrilaterals have 4.

Find the area of the following shapes

64.



65.



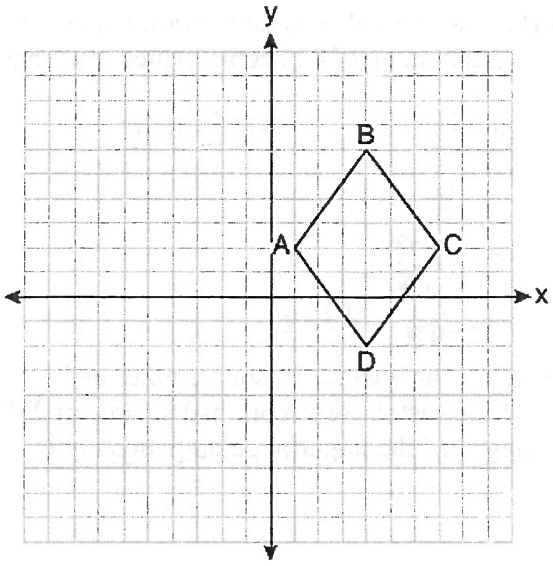
$A_R = 12(6)$
$A_R = 72$

18
+18
+9
45

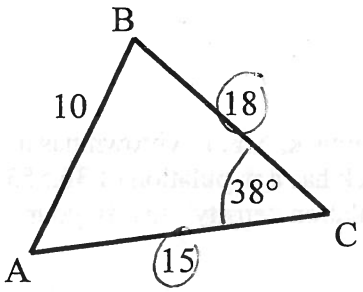
72
−45
27

#

66.



67.

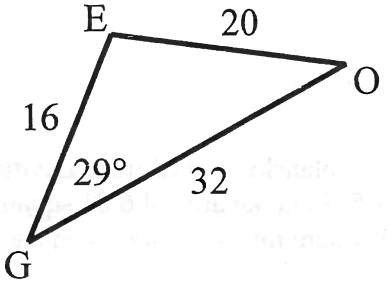


$A = \frac{1}{2}ab\sin C$

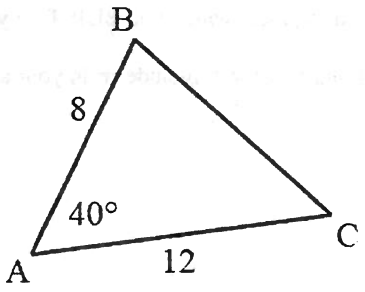
$A = \frac{1}{2}(15)(18)\sin 38$

$A = 83.1$

68.



69.



Triangle Area:

Area of a Triangle: $A = \frac{1}{2}ab\sin C$

a,b are sides and C is the INCLUDED angle

You may need to add and subtract from 180 in order to find the third angle of the triangle.

$$Population\ Density = \frac{Population}{Area}$$

70. Town A has an area of 12 square miles. Town B has an area of 10 square miles. If town A has a population of 8,198 people and town B has a population of 7,384 people, which town has a greater population density? Justify your answer.

Town A
$Pd = \frac{P}{a}$
$Pd = \frac{8198}{12}$
$Pd = 683...$

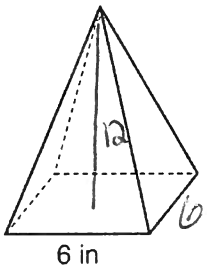
Town B
$Pd = \frac{P}{a}$
$Pd = \frac{7384}{10}$
$Pd = 738.4...$

Town B

71. In the year 2020, the village of Depew, New York had an area of 5.1 square miles and a population of 15,069. In the same year, the village of Lancaster, New York had an area of 2.7 square miles and a population of 10,087. Which village had the larger population density in 2020? Justify your answer.

72. In 2017, Mr. Schlansky moved from Levittown, NY to Commack, NY. Levittown has a population of 51,557 and an area of 6.81 square miles. Commack has a population of 36,553 and an area of 12.25 square miles. Which town has the greater population density? Justify your answer.

73. As shown in the diagram below, a regular pyramid has a square base whose side measures 6 inches. If the altitude of the pyramid measures 12 inches, find its volume.



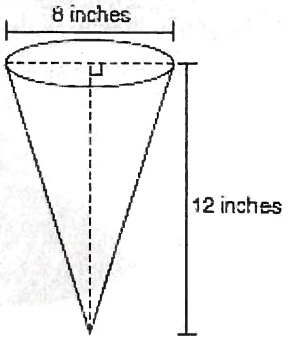
Calculating Volume
Substitute into the appropriate volume formula which you BUILT on your reference sheet.
In terms of π: don't type π in but make sure you include π in your answer.
Rounded: Type π in

$V = \frac{1}{3}lwh$
$V = \frac{1}{3}(6)(6)(12)$
$V = 144\ in^3$

74. In the diagram below, a right circular cone has a diameter of 8 inches and a height of 12 inches.

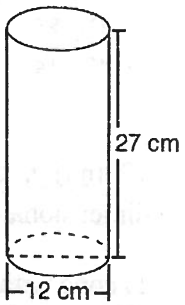
What is the volume of the cone to the *nearest cubic inch*?

1) 201
2) 481
3) 603
4) 804



75. What is the volume, in cubic centimeters, of a cylinder that has a height of 15 cm and a diameter of 12 cm?

1) 180π
2) 540π
3) 675π
4) $2,160\pi$



76. A spherical balloon is fully inflated with helium to a diameter of 1.7 feet. If helium costs $0.80 per cubic foot, what is the cost to completely fill the balloon with helium?

1) $2.06
2) $2.42
3) $3.22
4) $16.46

$r \approx .85$

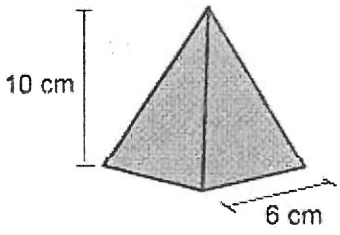
$V = \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi(.85)^3$

$V = 2.57\ldots \text{ft}^3 \cdot \frac{\$.80}{1\ \text{ft}^3} = \$2.06$

Calculating Volume with Basic Dimensional Analysis
-Calculate the volume using your volume formulas
-Perform basic dimensional analysis

77. A candle can be modeled by a pyramid with a square base, as shown below. The height of the candle is 10 cm, and each side of the base measures 6 cm. If the candle wax burns at a rate of 3.5 cubic centimeters per hour, to the *nearest* hour, how long could this candle burn?



78. Lucy wants to use a wagon to move a pile of sand in as few trips as possible. The pile of sand can be modeled by a cone, as shown below. The height and radius of the sandpile are both 3.5 feet.

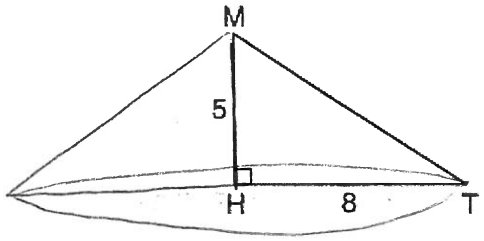
If the wagon holds 5 cubic feet of sand, what is the fewest number of trips Lucy needs to make to move the entire pile of sand?

(1) 27
(2) 14
(3) 3
(4) 9

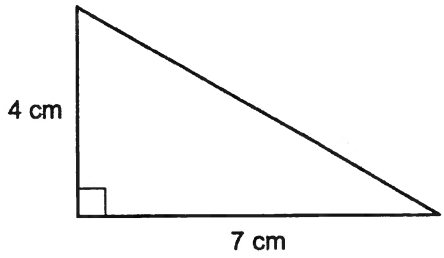
3 dimensional rotations ALMOST ALWAYS form a cylinder or cone
Reflect the shape in 2 dimensions and connect the images with curves

79. In right triangle MTH shown below, $m\angle H = 90°$, $HT = 8$, and $HM = 5$. What three-dimensional solid is formed by rotating $\triangle MTH$ continuously around $\overline{MH}$.

1) cone with a radius of 5 and a height of 8
2) cone with a radius of 8 and a height of 5
3) square pyramid with edge length 5 and height of 8
4) square pyramid with edge length of 8 and height of 5



80. The right triangle below is continuously rotated about the 4 cm side



The solid formed is

(1) a cone with a height of 4 cm and a radius of 7 cm
(2) a cone with a height of 4 cm and a radius of 14 cm
(3) a pyramid with a height of 4 cm and a base length of 7 cm
(4) a pyramid with a height of 4 cm and a base length of 14 cm

81. A rectangle with dimensions of 4 feet by 7 feet is continuously rotated about one of its 4-foot sides. The resulting three-dimensional object is a

1) cylinder with a height of 7 feet and a base radius of 4 feet.
2) cylinder with a height of 4 feet and a base radius of 7 feet.
3) cone with a height of 7 feet and a base radius of 7 feet.
4) cone with a height of 4 feet and a base radius of 7 feet.

multiply scale factor and b

82. The line $y = 3x - 2$ is dilated by a scale factor of 2 and centered at the origin. Write an equation that represents the image of the line after the dilation.

1) $y = 3x - 2$
2) $y = 3x - 4$
3) $y = 6x - 2$
4) $y = 6x - 4$

m=3
b = 2(-2) = -4

Line Dilations
THE IMAGE IS ALWAYS PARALLEL! SLOPE IS ALWAYS THE SAME!
CHEAT:
If center is origin: Multiply scale factor and original b to find new b
If center is on not the origin: The image is the same equation!

Same line

83. The line $y = 3x - 2$ is dilated by a scale factor of 2 and centered at (-1,-5). Write an equation that represents the image of the line after the dilation.

1) $y = 3x - 2$
2) $y = 3x - 4$
3) $y = 6x - 2$
4) $y = 6x - 4$

m=3

84. The line $y = -\frac{1}{2}x + 6$ is dilated by a scale factor of 4 and centered at (2,5). Write an equation that represents the image of the line after the dilation.

1) $y = -\frac{1}{2}x + 6$
2) $y = -\frac{1}{2}x + 24$
3) $y = -2x + 6$
4) $y = -2x + 24$

85. The line $y = -\frac{1}{2}x + 6$ is dilated by a scale factor of 4 and centered at the origin. Write an equation that represents the image of the line after the dilation.

1) $y = -\frac{1}{2}x + 6$
2) $y = -\frac{1}{2}x + 24$
3) $y = -2x + 6$
4) $y = -2x + 24$

86. Line $y = 3x - 1$ is transformed by a dilation with a scale factor of 2 and centered at $(3, 8)$. The line's image is

1) $y = 3x - 8$
2) $y = 3x - 4$
3) $y = 3x - 2$
4) $y = 3x - 1$

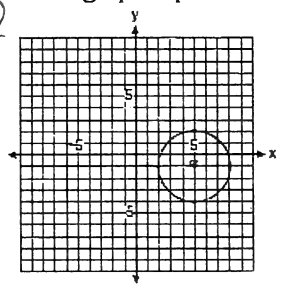
87. The line $y = 2x - 4$ is dilated by a scale factor of $\frac{3}{2}$ and centered at the origin. Which equation represents the image of the line after the dilation?

1) $y = 2x - 4$
2) $y = 2x - 6$
3) $y = 3x - 4$
4) $y = 3x - 6$

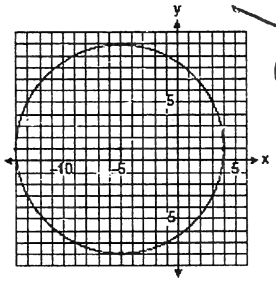
Graphing Circles Given Center-Radius Form
Negate what's in the parenthesis for the center. If there are no parenthesis, the coordinate is 0.
Take the square root of the right hand side for the radius.

88. Which graph represents a circle with the equation $(x-5)^2 + (y+1)^2 = 9$?

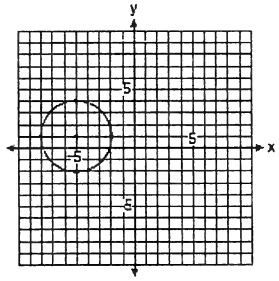
1)



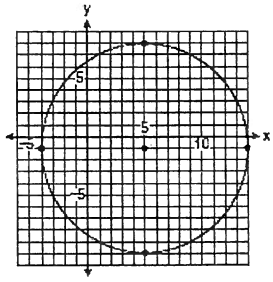
3)



2)

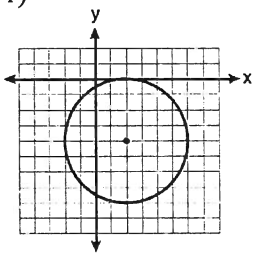


4)

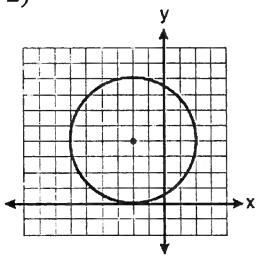


89. The equation of a circle is $(x-2)^2 + (y+4)^2 = 4$. Which diagram is the graph of the circle?

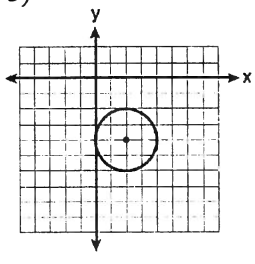
1)



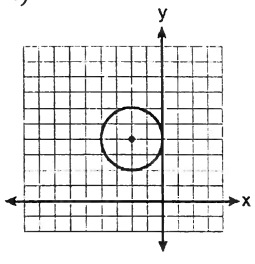
2)



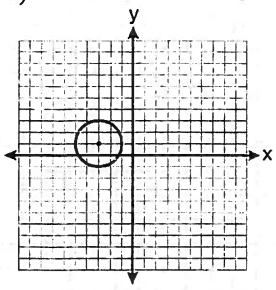
3)

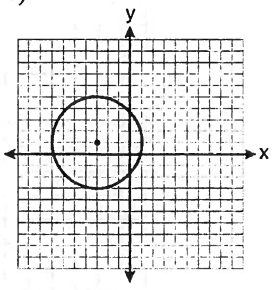


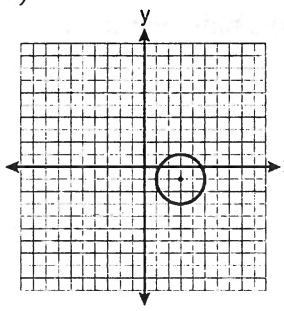
4)

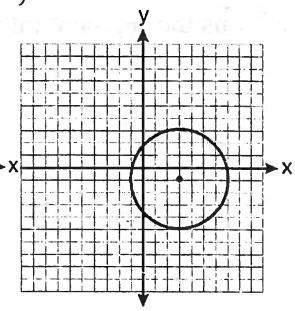


90. Which graph represents a circle with the equation $(x-3)^2+(y+1)^2=4$?

1) 

2) 

3) 

4) 

91. What are the coordinates of the center and length of the radius of the circle whose equation is $x^2+6x+y^2-4y=23$?

$-23\ \ -23$

1) (3,−2) and 36
2) (3,−2) and 6
3) (−3,2) and 36
4) (−3,2) and 6

$A=1$
$B=6$
$C=-4$
$D=-23$
Alpha Enter

Conics App
1) Apps, Conics, Circles, 2
2) Make sure the equation is equal to 0! If not, move D to the other side
3) ALPHA SOLVE for center and radius

92. An equation of circle M is $x^2+y^2+6x-2y+1=0$. What are the coordinates of the center and the length of the radius of circle M?

1) center (3,−1) and radius 9
2) center (3,−1) and radius 3
3) center (−3,1) and radius 9
4) center (−3,1) and radius 3

93. Find the coordinates of the center and radius of a circle whose equation is $x^2+y^2-16x+6y+53=0$?

1) center: (-8,3), radius: 20
2) center: (-8,3), radius: $\sqrt{20}$
3) center: (8,-3), radius: 20
4) center: (8,-3), radius: $\sqrt{20}$

Partitions
1) Find $\frac{\Delta x}{p}$ *and* $\frac{\Delta y}{p}$ where p is the number of partitions.
2) Count those values out on the graph between the two endpoints
3) Circle and state the point that matches the given ratio.
BE CAREFUL WHICH POINT YOU START FROM!
*Expect to have to use your scrap graph paper

94. The coordinates of the endpoints of $\overline{AB}$ are $A(-6,-5)$ and $B(4,0)$. Point P is on $\overline{AB}$. Determine and state the coordinates of point P, such that $AP:PB$ is 2:3.

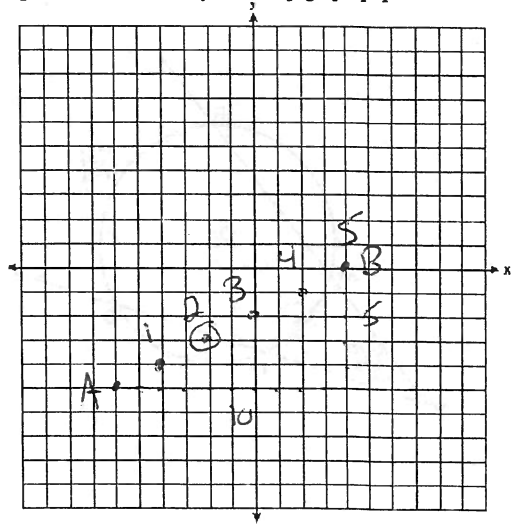
$\frac{\Delta x}{p}\quad\frac{\Delta y}{p}$

$\frac{10}{5}\quad\frac{5}{5}$

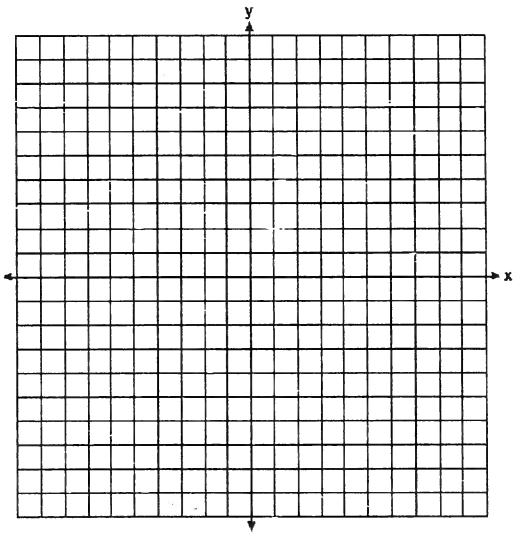
$2\quad 1$

$p=2+3=5$

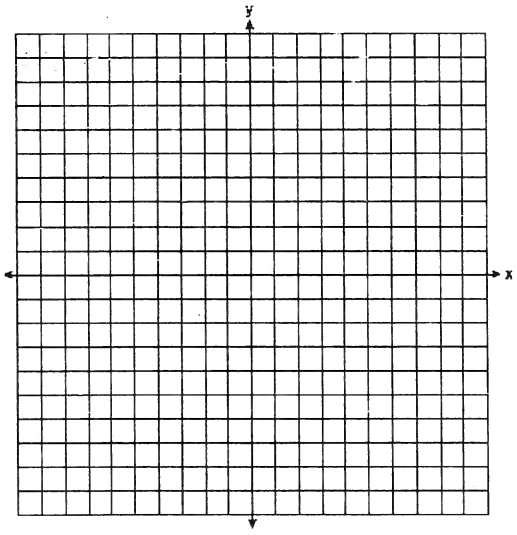
$(-2,-3)$



95. What are the coordinates of the point on the directed line segment from $G(-4,-7)$ to $O(4,5)$ that partitions the segment into a ratio of 3 to 1?

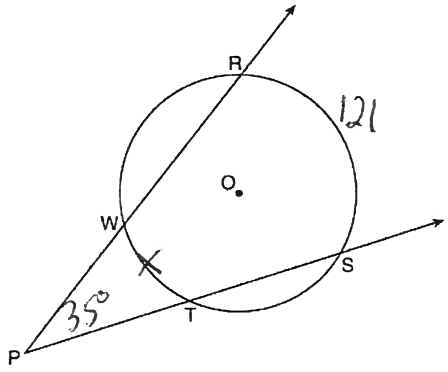


96. Directed line segment IQ has endpoints whose coordinates are $I(-7,8)$ and $Q(-1,-4)$. Determine the coordinates of point J that divides the segment in the ratio 1 to 5.



97. As shown in the diagram below, secants $\overrightarrow{PWR}$ and $\overrightarrow{PTS}$ are drawn to circle O from external point P.

If $m\angle RPS = 35°$ and $m\overset{\frown}{RS} = 121°$, determine and state $m\overset{\frown}{WT}$.



arcs and angles

2(EA) = major-minor

~~2(WT)~~

$2(35) = 121 - x$

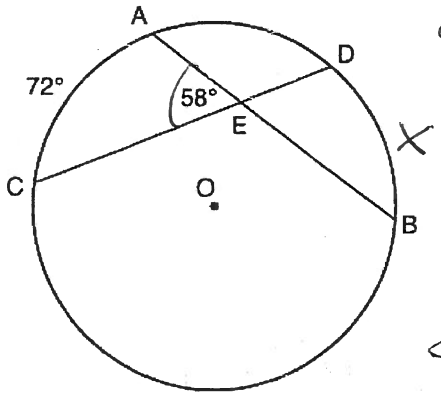
$70 = 121 - x$

$-121 \quad -121$

$\frac{-51}{-1} = \frac{-x}{-1}$

$51 = x$

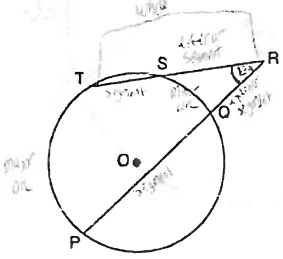
98. In the diagram below of circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E.
If $m\overset{\frown}{AC} = 72°$ and $m\angle AEC = 58°$, how many degrees are in $m\overset{\frown}{DB}$?

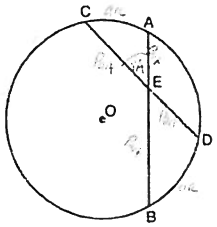


arcs and angles
~~2(EA)~~
2(VA) = arc + arc
2(58) = 72 + x
116 = 72 + x
-72 -72
44 = x

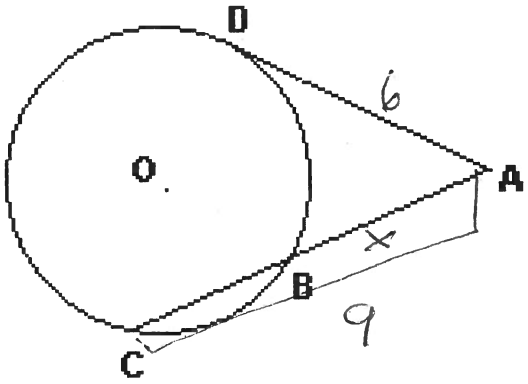
Arcs and Angles:
2(EA) = Major – Minor
2(VA) = arc + arc
2(exterior angle) = major arc – minor arc
2(vertical angle) = arc + arc

Segments:
PP=PP
WE=WE
(part)(part) = (part)(part)
(whole)(exterior)=(whole)(exterior)



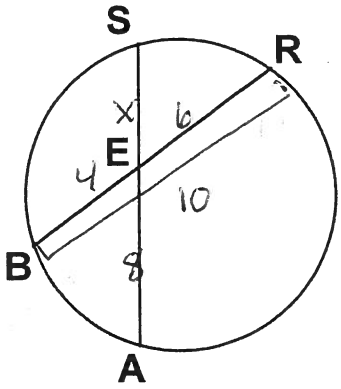


99. In the diagram, $\overline{AD}$ is tangent to circle O at D, and $\overline{CBA}$ is a secant. If $AD = 6$ and $AC = 9$, what is AB?



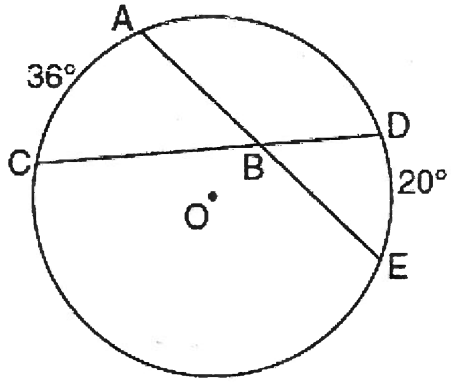
Segments
~~P·P=P·P~~
w·e=w·e
whole·exterior=whole·exterior
6·6 = 9·x
$\frac{36}{9} = \frac{9x}{9}$
4 = x

100. If $\overline{BR} = 10$, $\overline{BE} = 4$, $\overline{AE} = 8$, find $\overline{ES}$

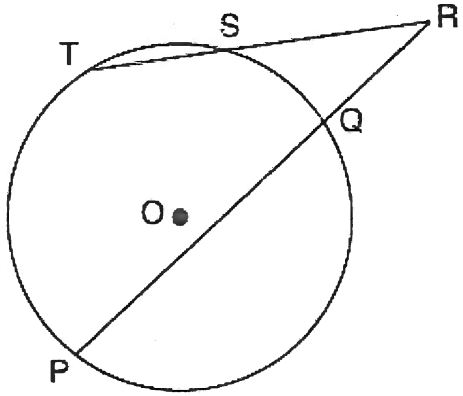


Segments
P·P=P·P
~~w·e=w·e~~
part·part=part·part
4·6 = 8·x
$\frac{24}{8} = \frac{8x}{8}$
3 = x

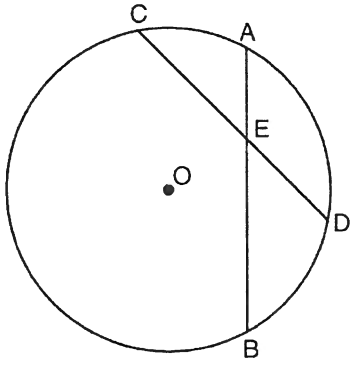
101. In the diagram below of circle O, chords $\overline{AE}$ and $\overline{DC}$ intersect at point B, such that $\text{m}\overset{\frown}{AC} = 36$ and $\text{m}\overset{\frown}{DE} = 20$. What is $\text{m}\angle ABC$?



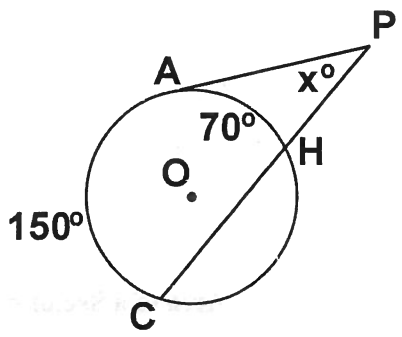
102. In the diagram below, secants $\overline{RST}$ and $\overline{RQP}$, drawn from point R, intersect circle O at S, T, Q, and P.
If $RS = 6$, $ST = 4$, and $RP = 15$, what is the length of $\overline{RQ}$?



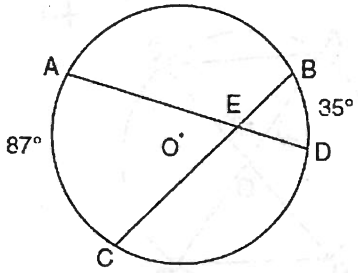
103. In the diagram below of circle O, chords $\overline{AB}$ and $\overline{CD}$ intersect at E.
If $CE = 10$, $ED = 6$, and $AE = 4$, what is the length of $\overline{EB}$?



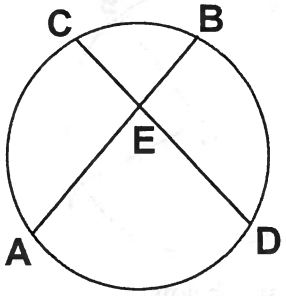
104. In Circle O, $m\overset{\frown}{AC} = 150$ and $m\overset{\frown}{AH} = 70$. Find $m\angle P$



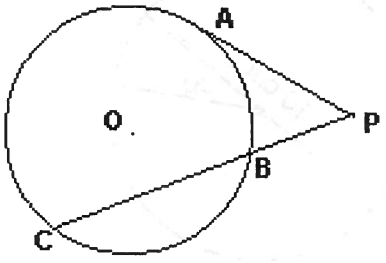
105. In the diagram below of circle O, chords $\overline{AD}$ and $\overline{BC}$ intersect at E, $m\overset{\frown}{AC} = 87$, and $m\overset{\frown}{BD} = 35$. What is the degree measure of $\angle CEA$?



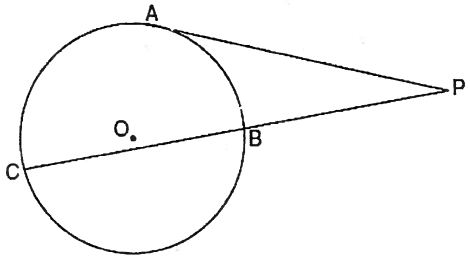
106. If $\overline{ED} = 10$, $\overline{EB} = 2$, $\overline{CE} = 4$, find $\overline{AB}$



107. In the diagram, $\overline{AP}$ is a tangent and $\overline{PBC}$ is a secant to circle O. If $\overline{PC} = 12$ and $\overline{BC} = 9$, what is $\overline{AP}$?



108. In the diagram below, tangent $\overline{PA}$ and secant $\overline{PBC}$ are drawn to circle O from external point P. If $\overset{\frown}{AC} = 120$ and $\overset{\frown}{AB} = 80$, find $m\angle APB$



Area of a Sector $= \frac{\theta \pi r^2}{360}$

θ = *central angle*, r = *radius*

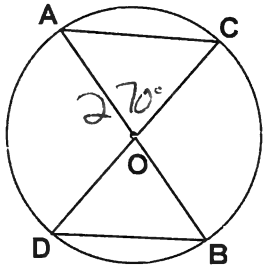
109. In circle O, $m\angle AOC = 70$ and $\overline{AO} = 2\ in$. Find the area of sector COA to the *nearest tenth of a square inch.*

type π in

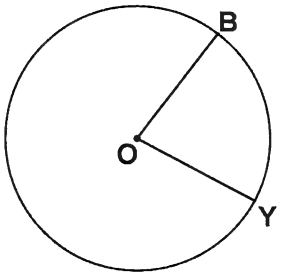
$A = \frac{\theta \pi r^2}{360}$

$A = \frac{70\pi(2)^2}{360}$

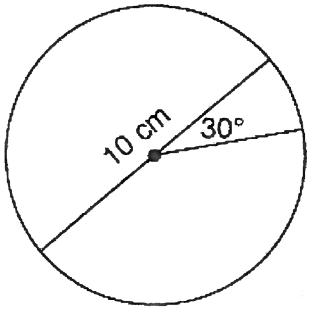
$A = 2.4$



110. In circle O, if $\angle$ BOY = 60° and $\overline{BO} = 8\ cm$, find the area of sector BOY in terms of π.

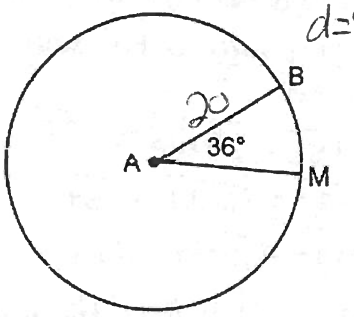


111. A circle with a diameter of 10 cm and a central angle of 30° is drawn below.
What is the area, to the *nearest tenth of a square centimeter*, of the sector formed by the 30° angle?



Arc Length: $L=\frac{\theta \pi d}{360}$, where L = arc length, θ = central angle, d = diameter

112. In circle A below, $m\angle BAM = 36°$. If $AB = 20$, determine and state the length of $\overset{\frown}{MB}$. [Leave your answer in terms of π.] don't type π in



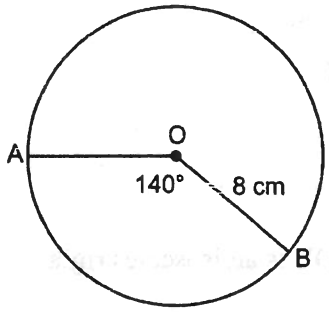
$d=40$

$L=\frac{\theta \pi d}{360}$

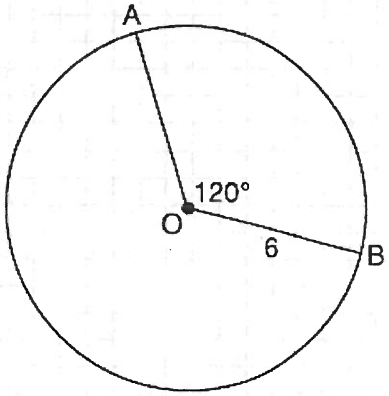
$L=\frac{36\pi(40)}{360}$

$L=4\pi$

113. In the diagram below, circle O has a radius of 8 cm and a central angle that measures 140°. What is the length of $\overset{\frown}{AB}$ to the *nearest centimeter*?



114. The diagram below shows circle O with radii $\overline{OA}$ and $\overline{OB}$. The measure of angle AOB is 120°, and the length of a radius is 6 inches. Find the length of arc AB, to the *nearest inch.*



Coordinate Geometry Proofs

Distance (Length) = $\sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

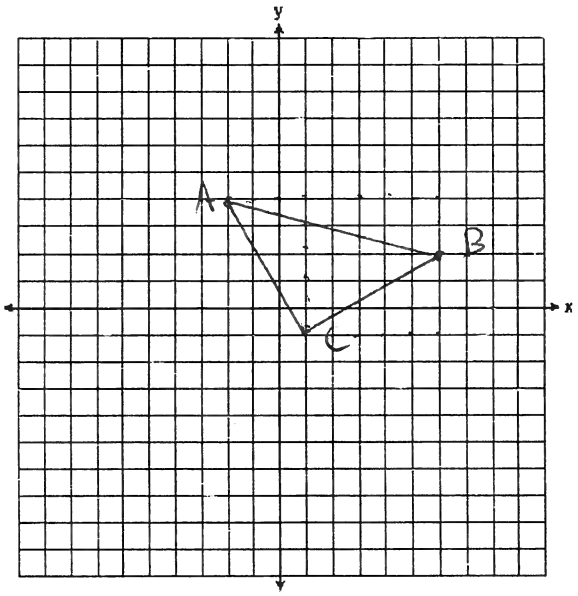
Slope = $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Midpoint = (*average x, average y*) = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

How do you prove...?

...an **isosceles triangle**? (2 Distances)
Two Congruent Sides
. ... a **right triangle**? (3 Distances)
The sides fit into Pythagorean Theorem
... a **parallelogram**? (4 Distances)
Two Pairs of Opposite Sides Congruent
... a **rhombus**? (4 Distances)
All Sides Congruent
... a **rectangle**? (6 Distances)
1) Two Pairs of Opposite Sides Congruent
2) Diagonals Congruent
... a **square**? (6 Distances)
1) All Sides Congruent
2) Diagonals Congruent
...a **trapezoid**? (4 Slopes)
1) 1 pair of opposite sides parallel
...an **isosceles trapezoid**? (4 Slopes, 2 Distances)
1) 1 pair of opposite sides parallel
2) Congruent Legs

115. A triangle has vertices $A(-2,4)$, $B(6,2)$, and $C(1,-1)$. Prove that $\triangle ABC$ is an isosceles right triangle. [The use of the set of axes below is optional.]



1) $\triangle ABC$ is an isosceles right triangle because it has two congruent sides and its sides fit into Pythagorean Theorem.

2) $d\overline{AC} = \sqrt{3^2+5^2} = \sqrt{9+25} = \sqrt{34}$

$d\overline{CB} = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$

$d\overline{AB} = \sqrt{8^2+2^2} = \sqrt{64+4} = \sqrt{68}$

3) $\overline{AC} \cong \overline{BC}$ because they have the same distance

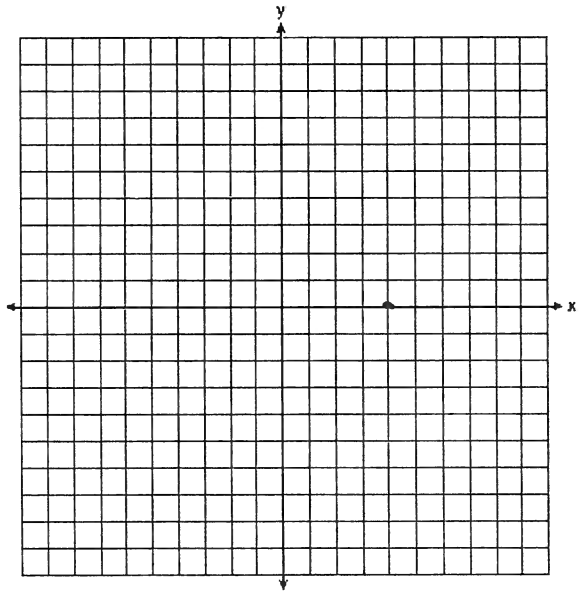
$a^2+b^2=c^2$

$\sqrt{34}^2+\sqrt{34}^2=\sqrt{68}^2$

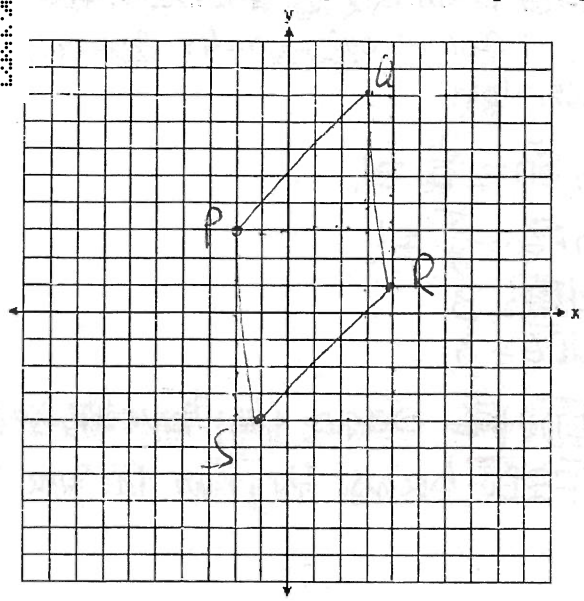
$34+34=68$

$68=68$ ✓

116. Triangle JOY has vertices J(4,0), O(5,4) and Y(1,5). Prove that JOY is an isosceles right triangle.



117. Quadrilateral *PQRS* has vertices *P*(−2, 3), *Q*(3, 8), *R*(4, 1), and *S*(−1, −4). Prove that *PQRS* is a rhombus. Prove that *PQRS* is *not* a square. [The use of the set of axes below is optional.]



1) PQRS is a rhombus because all sides are congruent. It is not a square because the diagonals are not congruent.

2) $d\overline{PQ} = \sqrt{5^2+5^2} = \sqrt{25+25} = \sqrt{50}$

$d\overline{QR} = \sqrt{1^2+7^2} = \sqrt{1+49} = \sqrt{50}$

$d\overline{RS} = \sqrt{5^2+5^2} = \sqrt{25+25} = \sqrt{50}$

$d\overline{SP} = \sqrt{1^2+7^2} = \sqrt{1+49} = \sqrt{50}$

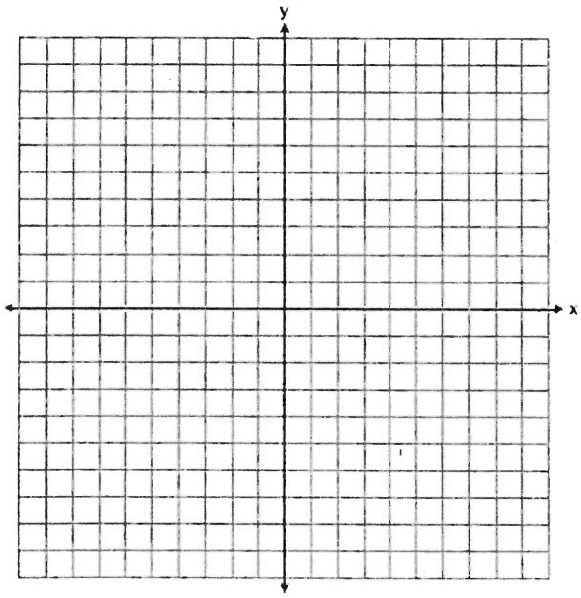
$d\overline{PR} = \sqrt{6^2+2^2} = \sqrt{36+4} = \sqrt{40}$

$d\overline{QS} = \sqrt{4^2+12^2} = \sqrt{16+144} = \sqrt{160}$

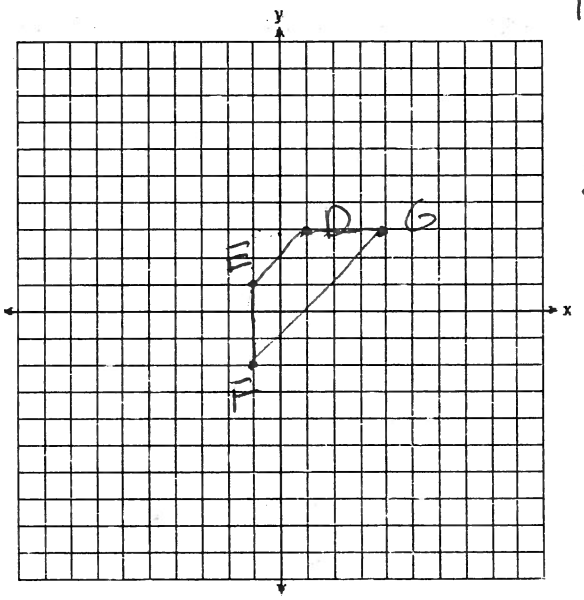
3) $\overline{PQ} = \overline{QR} \cong \overline{RS} \cong \overline{SP}$ because they have the same distance.

$\overline{PR} \ncong \overline{QS}$ because they don't have the same distance.

118. Quadrilateral *FRDY* has vertices F(-2, -8), R(7,-1), D(10,10) and Y(1,3). Using coordinate geometry, prove that quadrilateral *FRDY* is a rhombus but *not* a square.



119. Quadrilateral DEFG has vertices D(1,3) E(-1,1) F(-1,-2) G(4,3). Prove that DEFG is an isosceles trapezoid.



1) DEFG is an isosceles trapezoid because it has 1 pair of opposite sides parallel and congruent legs.

2) $m\overline{ED} = \frac{2}{2} = 1$

$m\overline{FG} = \frac{5}{5} = 1$

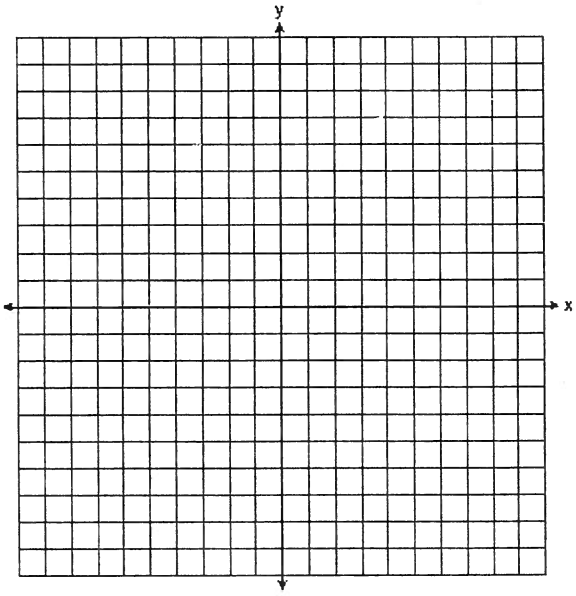
$d\overline{EF} = 3$

$d\overline{DG} = 3$

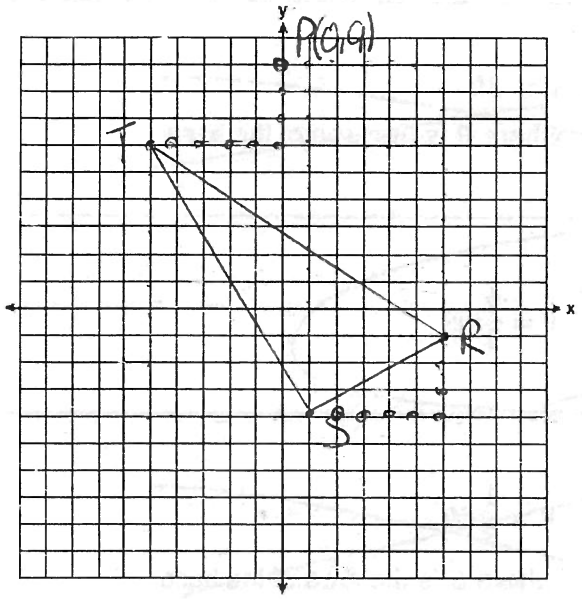
3) $\overline{ED} \parallel \overline{FG}$ because they have the same slope.

$\overline{EF} \cong \overline{DG}$ because they have the same distance.

120. Quadrilateral ABCD has vertices A(3,1) B(-3,5) C(5,4) and D(2,6). Prove quadrilateral ABCD is a trapezoid but *not* an isosceles trapezoid.



121. In the coordinate plane, the vertices of $\triangle RST$ are $R(6,-1)$, $S(1,-4)$, and $T(-5,6)$. Prove that $\triangle RST$ is a right triangle. State the coordinates of point P such that quadrilateral $RSTP$ is a rectangle. Prove that your quadrilateral $RSTP$ is a rectangle. [The use of the set of axes below is optional.]



1) $\triangle RST$ is a right triangle because its sides fit into Pythagorean Theorem.

2) $d\overline{SR} = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$

$d\overline{ST} = \sqrt{6^2+10^2} = \sqrt{36+100} = \sqrt{136}$

$d\overline{TR} = \sqrt{11^2+7^2} = \sqrt{121+49} = \sqrt{170}$

3) $a^2+b^2=c^2$

$\sqrt{34}^2+\sqrt{136}^2=\sqrt{170}^2$

$34+136=170$

$170=170$ ✓

1) RSTP is a rectangle because it has 2 pairs of opposite sides congruent and diagonals congruent.

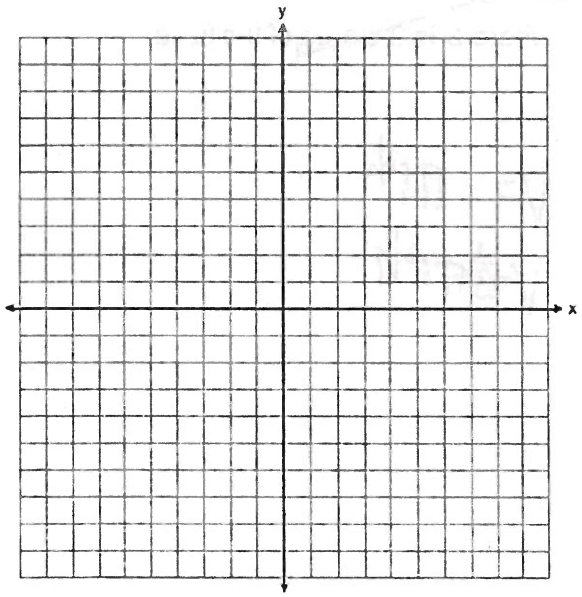
2) $d\overline{PT} = \sqrt{5^2+3^2} = \sqrt{25+9} = \sqrt{34}$

$d\overline{PR} = \sqrt{6^2+10^2} = \sqrt{36+100} = \sqrt{136}$

$d\overline{PS} = \sqrt{1^2+13^2} = \sqrt{1+169} = \sqrt{170}$

3) $\overline{TP} \cong \overline{SR}$, $\overline{TS} \cong \overline{PR}$, $\overline{TR} \cong \overline{PS}$ because they have the same distance.

122. Triangle PET has vertices with coordinates $P(-6,4)$, $E(6,8)$, and $T(-4,-2)$. Prove $\triangle PET$ is a right triangle. State the coordinates of N, the image of P, after a 180° rotation centered at $(1,3)$. Prove $PENT$ is a rectangle. [The use of the set of axes below is optional.]



Reference Sheet for Geometry (NGLS)

Volume	Cylinder	$V = Bh$ where B is the area of the base
	General Prism	$V = Bh$ where B is the area of the base
	Sphere	$V = \frac{4}{3}\pi r^3$
	Cone	$V = \frac{1}{3}Bh$ where B is the area of the base
	Pyramid	$V = \frac{1}{3}Bh$ where B is the area of the base

Rectangular Prism $V = lwh$

Triangular Prism $V = \frac{1}{2}lwh$

Pyramid $V = \frac{1}{3}lwh$

Cylinder $V = \pi r^2 h$

Cone $V = \frac{1}{3}\pi r^2 h$