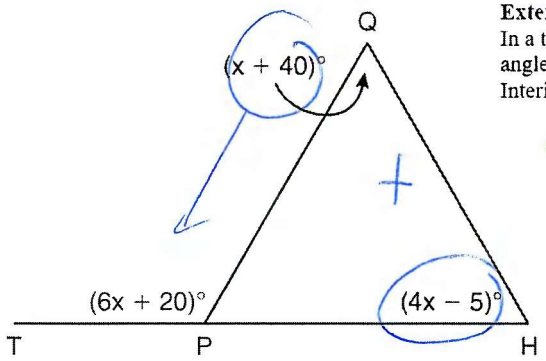
Name Schlansky
Mr. Schlansky

Date _____
Geometry

Video

# *Geometry Schlansky's Not As Common Guide to 65*

1. In the diagram below of $\triangle HQP$, side $\overline{HP}$ is extended through $P$ to $T$, $m\angle QPT = 6x+20$, $m\angle HQP = x+40$, and $m\angle PHQ = 4x-5$. Find $m\angle QPT$.



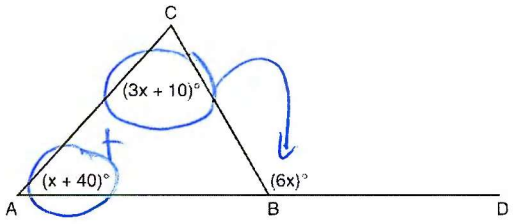

(Not drawn to scale)

**Exterior Angle Theorem**
In a triangle, the sum of the two non-adjacent interior angles add to equal the exterior angle.
Interior + Interior = Exterior

$4x-5+x+40=6x+20$
$5x+35=6x+20$
$-5x \quad -5x$
$35=x+20$
$-20 \quad -20$
$15=x$

$\angle QPT$
$6x+20$
$6(15)+20$
$110°$

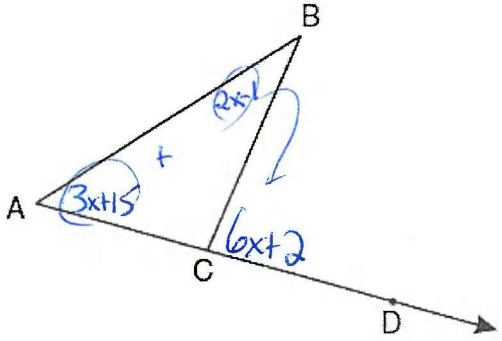
2. In the diagram of $\triangle ABC$ below, $\overline{AB}$ is extended to point $D$. If $m\angle CAB = x+40$, $m\angle ACB = 3x+10$, $m\angle CBD = 6x$, what is $m\angle CAB$?




$x+40+3x+10=6x$
$4x+50=6x$
$-4x \quad -4x$
$\frac{50}{2}=\frac{2x}{2}$
$25=x$

$\angle CAB$
$x+40$
$25+40$
$65$

3. In the diagram below, $\triangle ABC$ is shown with $\overline{AC}$ extended through point $D$.
If $m\angle BCD = 6x+2$, $m\angle BAC = 3x+15$, and $m\angle ABC = 2x-1$, what is the value of $x$?




$3x+15+2x-1=6x+2$
$5x+14=6x+2$
$-5x \quad -5x$
$14=x+2$
$-2 \quad -2$
$12=x$

**Intersecting Medians**

2x + 1x = median (the median is cut in a ratio of 2:1).

Once you solve, substitute into 1x and 2x to find the small and big pieces.

*If given the large piece, divide by 2 for the small piece.

*If given the small piece, multiply by 2 for the large piece.

4. In the given triangle, all three medians are drawn in. If $\overline{YN} = 30$, find

a) $\overline{YS}$ 10

b) $\overline{SN}$ 20

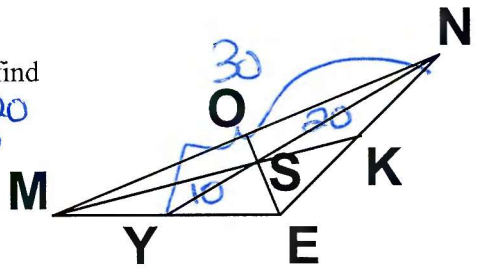
$2x+1x=30$

$\frac{3x}{3}=\frac{30}{3}$

$x=10$

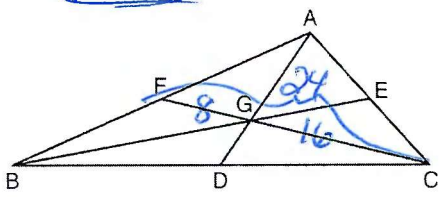
$2(10)=20$

$1(10)=10$




5. In the diagram below of $\triangle ABC$, medians $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$ intersect at $G$. If $CF = 24$, what is the length of $\overline{FG}$?

$\overline{FG}=8$




$2x+1x=24$

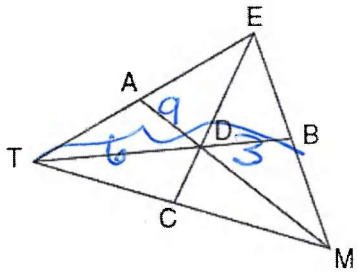
$\frac{3x}{3}=\frac{24}{3}$

$x=8$

$2(8)=16$

$1(8)=8$

6. In the diagram below of $\triangle TEM$, medians $\overline{TB}$, $\overline{EC}$, and $\overline{MA}$ intersect at $D$, and $TB = 9$. Find the length of $\overline{TD}$.




$2x+1x=9$

$\frac{3x}{3}=\frac{9}{3}$

$x=3$

$\overline{TD}=6$

$2(3)=6$

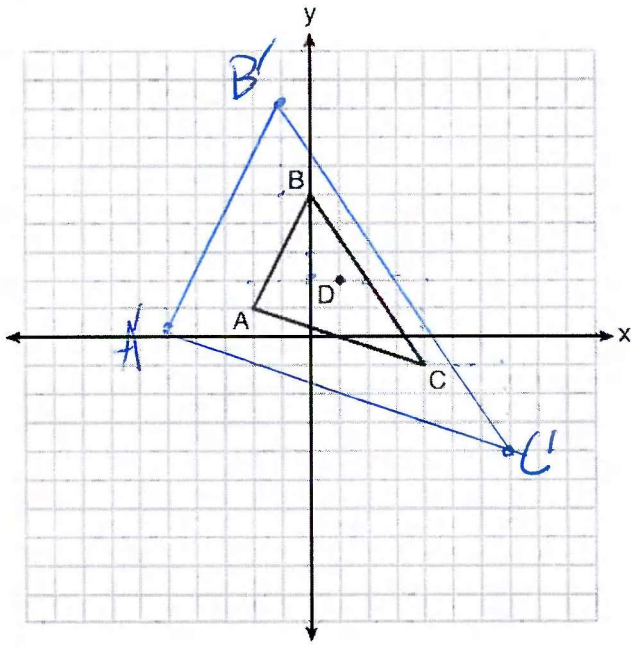
$1(3)=3$

**Performing Dilations**

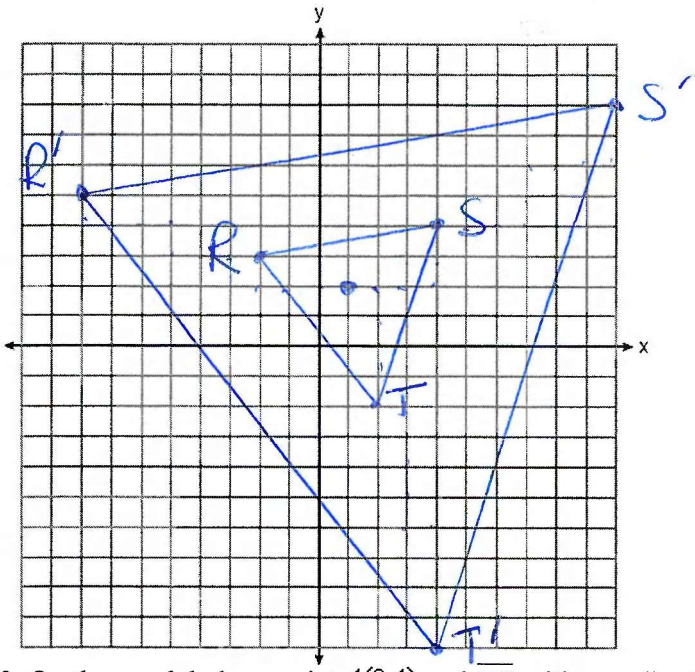
Count FROM the center of dilation to each point and do that the scale factor number of times.

7. Triangle $ABC$ and point $D(1, 2)$ are graphed on the set of axes below.

Graph and label $\triangle A'B'C'$, the image of $\triangle ABC$, after a dilation of scale factor 2 centered at point $D$.



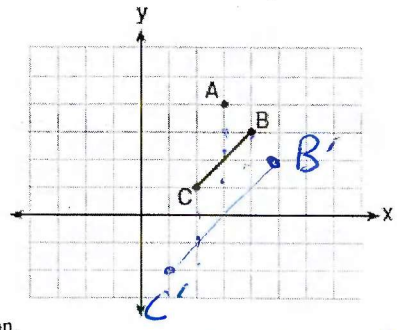
8. The coordinates of the vertices of $\triangle RST$ are $R(-2,3)$, $S(4,4)$, and $T(2,-2)$. Graph $\triangle RST$ and $\triangle R'S'T'$, the image of $\triangle RST$ after a dilation of 3 centered at (1,2).




9. On the graph below, point $A(3,4)$ and $\overline{BC}$ with coordinates $B(4,3)$ and $C(2,1)$ are graphed.

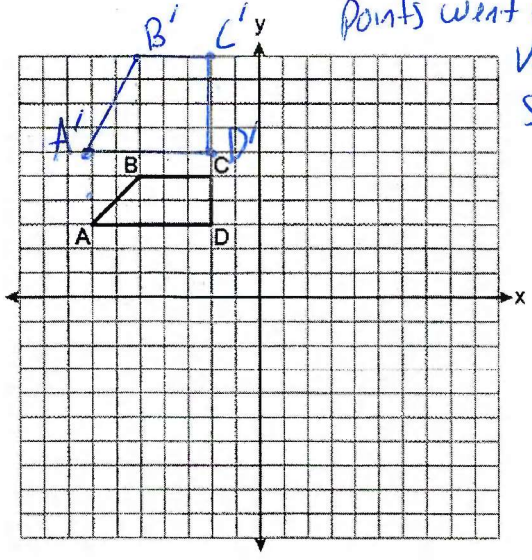
What are the coordinates of $B'$ and $C'$ after $\overline{BC}$ undergoes a dilation centered at point $A$ with a scale factor of 2?

1) $B'(5,2)$ and $C'(1,-2)$
2) $B'(6,1)$ and $C'(0,-1)$
3) $B'(5,0)$ and $C'(1,-2)$
4) $B'(5,2)$ and $C'(3,0)$




**Identifying Vertical Stretches, Horizontal Stretches, and Dilations**
If a point's image is moved left/right only, horizontal stretch.
If a point's image is moved up/down only, vertical stretch.
If a point's image is moved left/right and up/down (proportionally), dilation.

10. Trapezoid $ABCD$ is graphed on the set of axes below. Trapezoid $A'B'C'D'$, whose vertices are $A'(-7,6)$, $B'(-5,10)$, $C'(-2,10)$, and $D'(-2,6)$ is the image of trapezoid $ABCD$. What transformation maps trapezoid $ABCD$ on trapezoid $A'B'C'D'$?

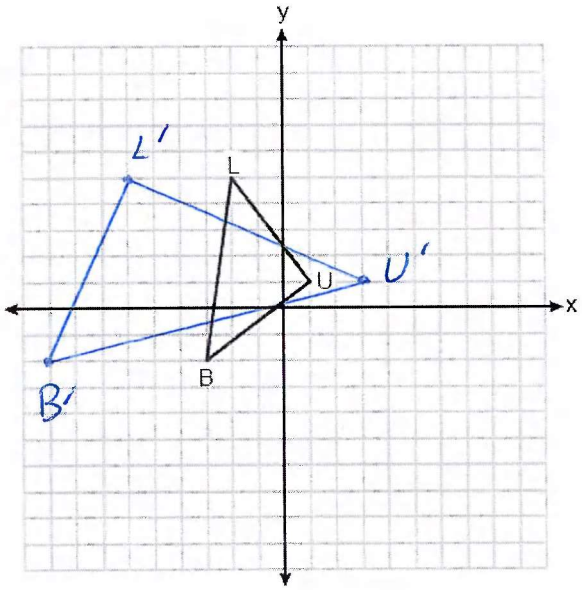



Points went up! Vertical stretch

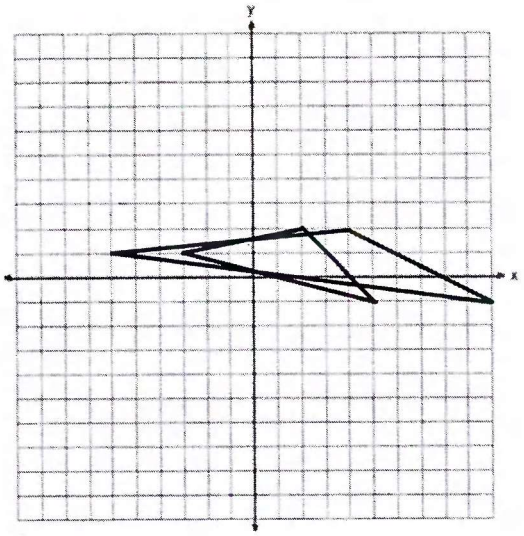
1) dilation
2) translation
3) vertical stretch
4) horizontal stretch

11. On the set of axes below, $\triangle BLU$ has vertices with coordinates $B(-3,-2)$, $L(-2,5)$, and $U(1,1)$. $\Delta\, B'L'U'$ whose vertices are , $B'(-9,-2)$ , $L'(-6,5)$ , and $U'(3,1)$ is the image of $\triangle BLU$. What transformation maps $\triangle BLU$ onto $\Delta\, B'L'U'$?

1) dilation
2) translation
3) vertical stretch
4) horizontal stretch

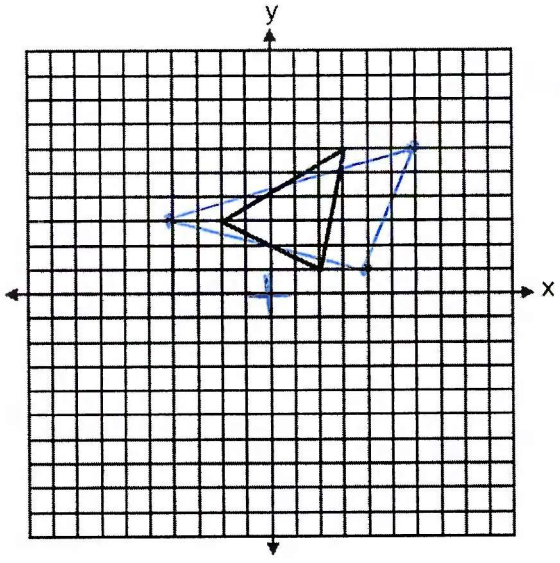



12. Which type of transformation is shown below?
1) horizontal stretch
2) vertical stretch
3) point reflection
4) dilation




**Performing Vertical and Horizontal Dilations**
Multiply the given variable by the given scale factor. Leave the other variable.

13. A triangle with vertices at (-2,3), (3,6), and (2,1), is graphed on the set of axes below. A horizontal stretch of scale factor 2 with respect to x = 0 is represented by $(x, y) \rightarrow (2x, y)$. Graph the image of this triangle, after the horizontal stretch on the same set of axes.

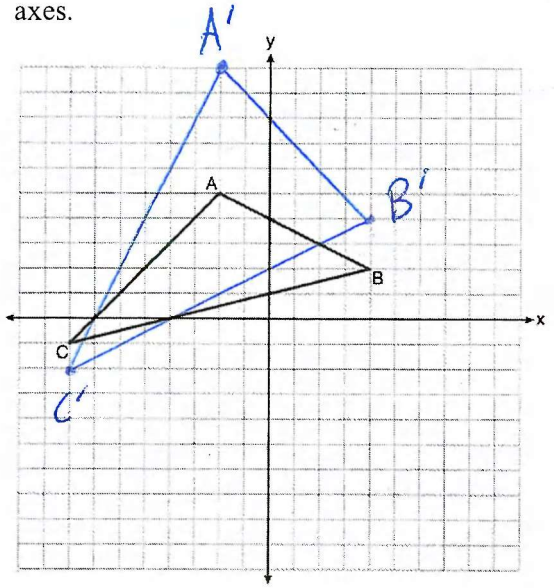



$(-2,3) \xrightarrow{2x,y} (-4,3)$

$(3,6) \rightarrow (6,6)$

$(2,1) \rightarrow (4,1)$

14. The triangle graphed below with vertices at A(-2,5), B(4,2), and C(-8,-1), is graphed on the set of axes below. A vertical stretch of scale factor 2 with respect to y = 0 is represented by $(x, y) \rightarrow (x, 2y)$. Graph the image of this triangle, after the vertical stretch on the same set of axes.

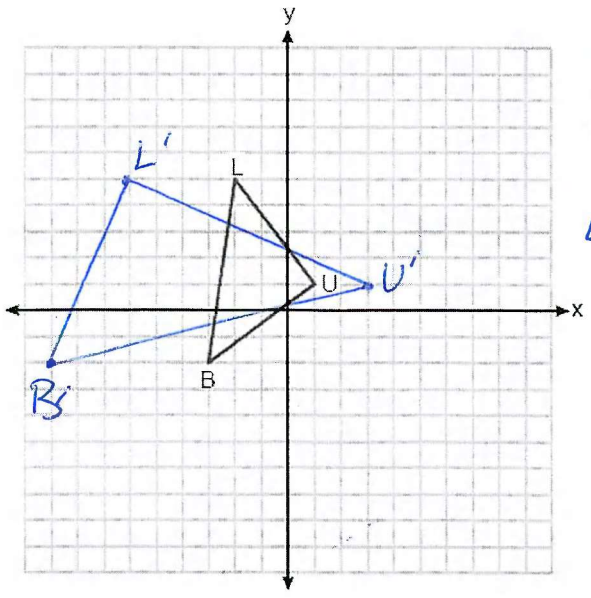



$A(-2,5) \xrightarrow{(x,2y)} (-2,10)$

$B(4,2) \rightarrow (4,4)$

$C(-8,-1) \rightarrow (-8,-2)$

15. The triangle graphed below with vertices at B(-3,-2), U(1,1), and L(-2,5), is graphed on the set of axes below. A horizontal stretch of scale factor 3 with respect to x = 0 is represented by $(x, y) \rightarrow (3x, y)$. Graph the image of this triangle, after the horizontal stretch on the same set of axes.




$B(-3,-2) \xrightarrow{3x,y} (-9,-2)$

$U(1,1) \rightarrow (3,1)$

$L(-2,5) \rightarrow (-6,5)$

**Rigid Motion Properties**
**A rigid motion preserves size and angle measure producing a congruent figure**
**They all produce a congruent figure except dilation.**

16. The vertices of $\Delta JKL$ have coordinates $J(5,1)$, $K(-2,-3)$, and $L(-4,1)$. Under which transformation is the image $\Delta J'K'L'$ *not* congruent to $\Delta JKL$?
1) a translation of two units to the right and two units down
2) a counterclockwise rotation of 180 degrees around the origin
3) a reflection over the $x$-axis
4) a dilation with a scale factor of 2 and centered at the origin

17. If $\triangle A'B'C'$ is the image of $\triangle ABC$, under which transformation will the triangles *not* be congruent?
1) reflection over the $x$-axis
2) translation to the left 5 and down 4
3) dilation centered at the origin with scale factor 2
4) rotation of 270° counterclockwise about the origin

18. Under which transformation would $\triangle A'B'C'$, the image of $\triangle ABC$, *not* be congruent to $\triangle ABC$?
1) reflection over the $y$-axis
2) rotation of 90° clockwise about the origin
3) translation of 3 units right and 2 units down
4) dilation with a scale factor of 2 centered at the origin

19. Triangle $A'B'C'$ is the image of triangle $ABC$ after a translation of 2 units to the right and 3 units up. Is triangle $ABC$ congruent to triangle $A'B'C'$? Explain why.

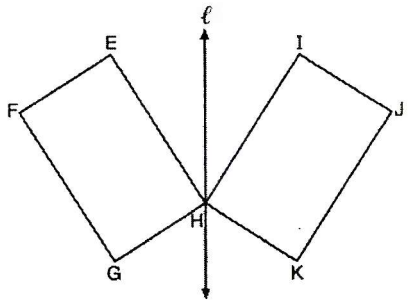
Yes, a translation is a rigid motion. A rigid motion preserves size and angle measure producing a congruent figure.

**Rigid Motion Proofs**
**To prove triangles are congruent using rigid motions/transformations**
1) A _____ is a rigid motion.
2) **A rigid motion preserves size and angle measure producing a congruent figure.**

20. After a reflection over a line, $\Delta A'B'C'$ is the image of $\Delta ABC$. Explain why triangle $ABC$ is congruent to triangle $\Delta A'B'C'$.

A reflection is a rigid motion. A rigid motion preserves size and angle measure producing a congruent figure.

21. In the diagram below, parallelogram $EFGH$ is mapped onto parallelogram $IJKH$ after a reflection over line $\ell$. Use the properties of rigid motions to explain why parallelogram $EFGH$ is congruent to parallelogram $IJKH$.



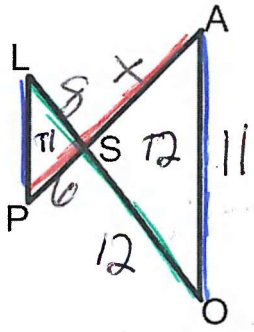

A reflection is a rigid motion. A rigid motion preserves size and angle measure producing a congruent figure.

**Bow Tie Problems**

Corresponding sides of similar triangles are in proportion. Put the corresponding sides on top of each other to create the proportion.

**With parallel lines, the corresponding sides are diagonal from each other.**

22. In the diagram below, $\overline{LP} \parallel \overline{AO}$. If $LS = 8$, $SO = 12$, $AO = 11$, and $PS = 6$, find $SA$.

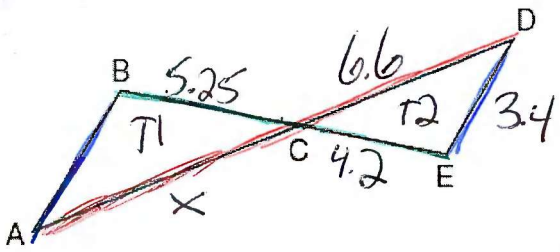



$\frac{8}{12} = \frac{6}{x}$

$\frac{8x}{8} = \frac{72}{8}$

$x = 9$

23. In the diagram below, $\overline{AD}$ intersects $\overline{BE}$ at $C$, and $\overline{AB} \parallel \overline{DE}$. If $CD = 6.6$ cm, $DE = 3.4$ cm, $CE = 4.2$ cm, and $BC = 5.25$ cm, what is the length of $\overline{AC}$, to the *nearest hundredth of a centimeter*?

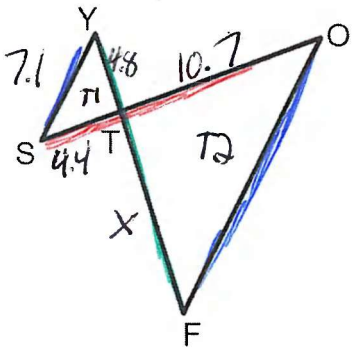



$\frac{5.25}{4.2} = \frac{x}{6.6}$

$\frac{4.2x}{4.2} = \frac{34.65}{4.2}$

$x = 8.25$

24. In the diagram below, $\overline{SO}$ intersects $\overline{YF}$ at $T$, and $\overline{SY} \parallel \overline{FO}$. If $\overline{ST} = 4.4$, $\overline{TO} = 10.7$, $\overline{TY} = 4.8$, and $\overline{SY} = 7.1$, what is the length of $\overline{TF}$, to the *nearest tenth*?




$\frac{4.8}{x} = \frac{4.4}{10.7}$

$\frac{4.4x}{4.4} = \frac{51.36}{4.4}$

$x = 11.7$

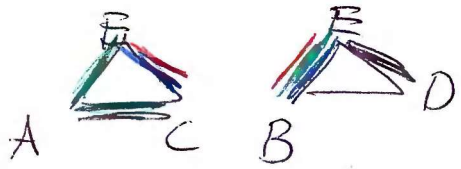
25. As shown in the diagram below, $\overline{AB}$ and $\overline{CD}$ intersect at $E$, and $\overline{AC} \parallel \overline{BD}$. Given $\triangle AEC \sim \triangle BED$, which equation is true?

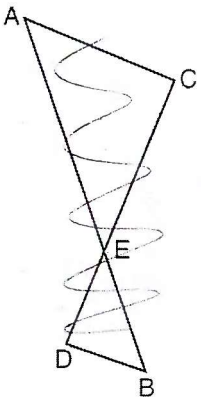
1) $\frac{CE}{DE} = \frac{EB}{EA}$

2) $\frac{AE}{BE} = \frac{AC}{BD}$

3) $\frac{EC}{AE} = \frac{BE}{ED}$

4) $\frac{ED}{EC} = \frac{AC}{BD}$



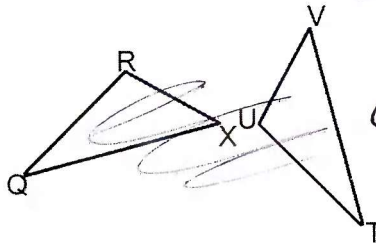

26. In the diagram below, $\Delta QRX \sim \Delta TUV$. Which of the following statements is *not* true?

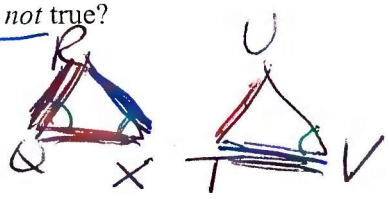
1) $\frac{QR}{TU} = \frac{QX}{TV}$

2) $\frac{\angle X}{\angle V} = \frac{\angle Q}{\angle T}$

3) $\frac{RX}{UV} = \frac{VT}{XQ}$

4) $\frac{QX}{QR} = \frac{TV}{TU}$



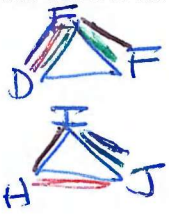

27. Given that $\Delta DEF \sim \Delta HIJ$, which is the correct statement about their corresponding sides?

1) $\frac{EF}{IJ} = \frac{DE}{HI}$

2) $\frac{EF}{HI} = \frac{IJ}{DE}$

3) $\frac{DE}{HJ} = \frac{EF}{HI}$

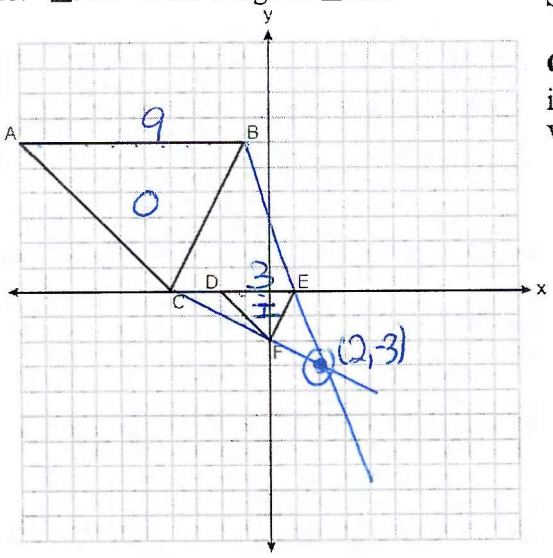
4) $\frac{DE}{JI} = \frac{EF}{HJ}$




**To determine if a proportion is correct**
Look at the letters vertically and horizontally
One direction, the letters should correspond
Second direction, the letters should be in the same triangle
*It does not matter which direction does which

Find the center of dilation AND scale factor if

28. $\triangle DEF$ is the image of $\triangle ABC$



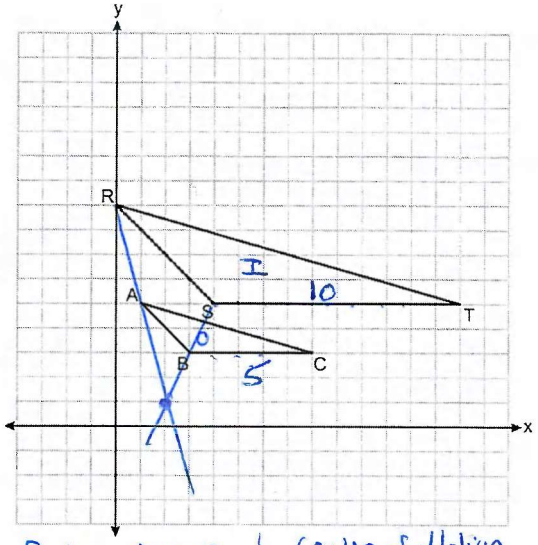

**Scale factor** = $\frac{image}{original}$

**Center of Dilation:** Extend a line from any point to its image. Repeat the process for a second point and its image. Where the lines intersect is the center of dilation.

Scale factor = $\frac{image}{original}$ = $\frac{3}{9}$ = $\frac{1}{3}$

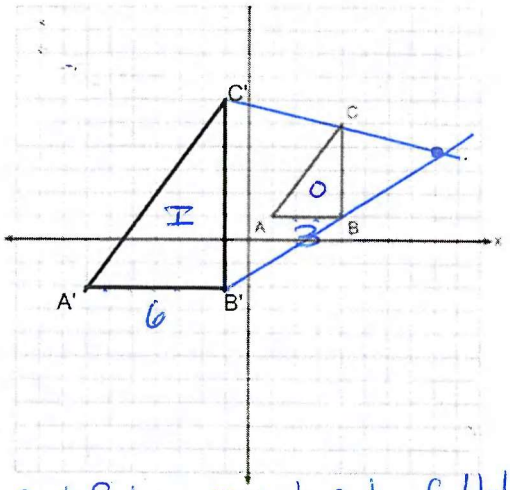
Center of dilation (2, -3)

29. $\triangle RST$ is the image of $\triangle ABC$




Scale factor = $\frac{\text{image}}{\text{original}} = \frac{10}{5} = 2$ | Center of dilation: (2, 1)

30. $\triangle A'B'C'$ is the image of $\triangle ABC$




Scale factor = $\frac{\text{image}}{\text{original}} = \frac{6}{3} = 2$ | Center of dilation: (8, 4)

31. A brick that weighs 1824 grams has dimensions that measure 4 cm by 3 cm by 8 cm. To the nearest tenth, what is the density of the brick?

**Density**

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

1) Find the volume
2) Divide the mass by the volume

$d = \frac{\text{mass}}{\text{volume}}$

$d = \frac{1824\text{ g}}{96\text{ cm}^3}$

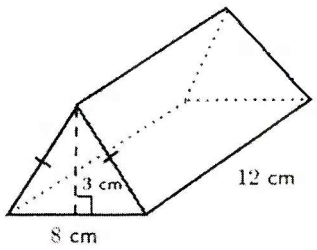
$d = 19\text{ g/cm}^3$

$V = lwh$

$V = 4(3)(8)$

$V = 96\text{ cm}^3$

32. Clay in the shape of a triangular prism shown below has a mass of 1260 grams. What is its density?




$d = \frac{\text{mass}}{\text{volume}}$

$d = \frac{1260\text{ g}}{144\text{ cm}^3}$

$d = 8.75\text{ g/cm}^3$

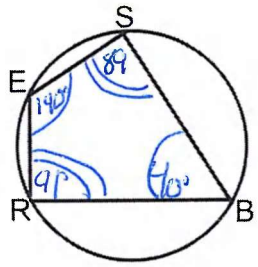
$V = \frac{1}{2}lwh$

$V = \frac{1}{2}(12)(8)(3)$

$V = 144\text{ cm}^3$

**Quadrilateral Inscribed In a Circle**
**Opposite angles are supplementary (add to 180)**

33. In the diagram below, quadrilateral *SBRE* is inscribed in the circle. If $m\angle BRE = 91°$ and $m\angle SBR = 40°$, find $m\angle BSE$ and $m\angle SER$

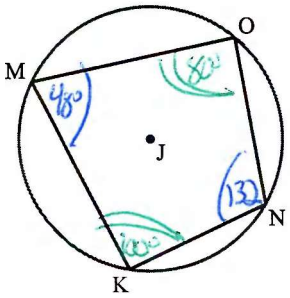



89 140°

180 − 40 = 140

180 − 91 = 89

34. In the diagram below, quadrilateral MONK is inscribed in circle J, $m\angle KMO = 48°$ and $m\angle MON = 80°$. Find the measures of $m\angle KNO$ and $m\angle MKN$.

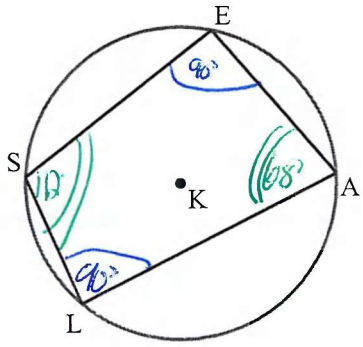



180 − 48 = 132

180 − 80 = 100

35. In the diagram below, quadrilateral SEAL is inscribed in circle K, $\overline{SE} \perp \overline{EA}$ and $m\angle EAL = 68°$. Find the measures of $m\angle SLA$ and $m\angle ESL$.

90°

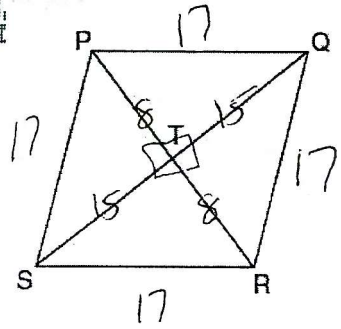



180 − 90 = 90

180 − 68 = 112

**Perimeter of a Rhombus**
Diagonals bisect each other and are perpendicular.
1) Cut the diagonals in half to get sides of the right triangle.
2) Use Pythagorean Theorem to find the side of the rhombus
3) Multiply the side by 4 to find the perimeter of the rhombus

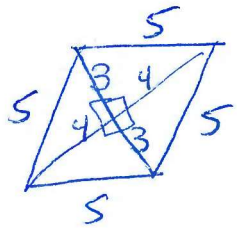
36. In the diagram of rhombus $PQRS$ below, the diagonals $\overline{PR}$ and $\overline{QS}$ intersect at point $T$, $PR = 16$, and $QS = 30$. Determine and state the perimeter of $PQRS$.




$a^2+b^2=c^2$
$8^2+15^2=c^2$
$64+225=c^2$
$\sqrt{289}=\sqrt{c^2}$
$17=c$

$17(4)=68$

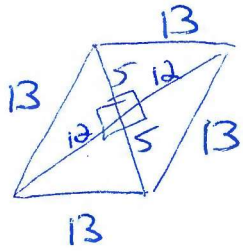
37. A rhombus has diagonals that measure 6 and 8. Find the perimeter of the rhombus.




$a^2+b^2=c^2$
$3^2+4^2=c^2$
$9+16=c^2$
$\sqrt{25}=\sqrt{c^2}$
$5=c$

$5(4)=20$

38. A rhombus has diagonals that measure 10 and 24. Find the perimeter of the rhombus.




$a^2+b^2=c^2$
$5^2+12^2=c^2$
$25+144=c^2$
$\sqrt{169}=\sqrt{c^2}$
$13=c$

$13(4)$
52

DDYV CCA

# Reference Sheet for Geometry (NGLS)

Density $d = \frac{\text{mass}}{\text{volume}}$

Population Density $pd = \frac{\text{Population}}{\text{Area}}$

$y - y_1 = m(x - x_1)$

$A = \frac{\theta \pi r^2}{360}$ $L = \frac{\theta \pi d}{360}$

Area of a Circle — Arc Length

Volume

| | |
|---|---|
| Cylinder | $V = Bh$ where $B$ is the area of the base |
| General Prism | $V = Bh$ where $B$ is the area of the base |
| Sphere | $V = \frac{4}{3}\pi r^3$ |
| Cone | $V = \frac{1}{3}Bh$ where $B$ is the area of the base |
| Pyramid | $V = \frac{1}{3}Bh$ where $B$ is the area of the base |

Rectangular Prism $V = lwh$

Triangular Prism $V = \frac{1}{2}lwh$

Pyramid $V = \frac{1}{3}lwh$

Cylinder $V = \pi r^2 h$

Cone $V = \frac{1}{3}\pi r^2 h$

$2(EA) = \text{major} - \text{minor}$ } Area and angles
$2(VA) = \text{arc} + \text{arc}$

Part-part = part-part } Segments
whole-exterior = whole-exterior

Triangle Area

$A = \frac{1}{2}ab \sin C$