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Algebra II

Sequence/Series Review Sheet

Write an equation for each of the following sequences explicitly and recursively

1. Which of the following represents an explicit formula for -4, -7, -10, …

-3 -3

$a_n = a_1 + d(n-1)$

1) $a_n = 3n-7$
2) $a_n = 3n-1$
3) $a_n = -3n-7$
4) $a_n = -3n-1$

$a_1 = -4$
$d = -3$

$a_n = -4 - 3(n-1)$
$a_n = -4 - 3n + 3$
$a_n = -3n - 1$

-2 -2

2. Which of the following represents an explicit formula for -9, -11, -13, …

1) $a_n = 2n-11$
2) $a_n = 2n-7$
3) $a_n = -2n-11$
4) $a_n = -2n-7$

$a_1 = -9$
$d = -2$

$a_n = a_1 + d(n-1)$
$a_n = -9 - 2(n-1)$
$a_n = -9 - 2n + 2$
$a_n = -2n - 7$

3. Write a recursive formula for the sequence 6, 9, 13.5, 20.25, . . .

$\frac{9}{6} = 1.5$

$\frac{13.5}{9} = 1.5$

$a_1 = 6$
$a_n = 1.5a_{n-1}$

4. Write a recursive formula for the sequence 189, 63, 21, 7, ...

$\frac{63}{189} = \frac{1}{3}$

$\frac{21}{63} = \frac{1}{3}$

$a_1 = 189$
$a_n = \frac{1}{3}a_{n-1}$

5. If $a_n = 3a_{n-1} - 4$ and $a_1 = 9$, find a_5

$a_2 = 3(9) - 4$
$a_2 = 23$

$a_3 = 3(23) - 4$
$a_3 = 65$

$a_4 = 3(65) - 4$
$a_4 = 191$

$a_5 = 3(191) - 4$
$a_5 = 569$

6. Find the 8th term for the sequence where $a_n = 5a_{n-1} + 2$ where $a_5 = 3$

$a_6 = 5(3)+2$ $\quad a_7 = 5(17)+2$ $\quad a_8 = 5(87)+2$

$a_6 = 17$ $\quad a_7 = 87$ $\quad a_8 = 437$

7. After Roger's surgery, his doctor administered pain medication in the following amounts in milligrams over four days.
How can this sequence best be modeled recursively?

Day (n)	1	2	3	4
Dosage (m)	2000	1680	1411.2	1185.4

1) $m_1 = 2000$

$m_n = m_{n-1} - 320$

2) ~~$m_n = 2000(0.84)^{n-1}$~~

3) $m_1 = 2000$

$m_n = (0.84)m_{n-1}$

4) ~~$m_n = 2000(0.84)^{n+1}$~~

$\frac{1680}{2000} = .84$ $\quad \frac{1411.2}{1680} = .84$

8. Samantha logged her weekly running distances in the table below. How can this sequence be modeled recursively?

1) $a_1 = 12$

$a_n = 1.2a_{n-1}$

2) $a_n = 12(1.2)^{n-1}$

3) $a_1 = 12$

$a_n = a_{n-1} + 2.4$

4) $a_n = 12(2.4)^n$

Week	Distance (In Miles)
1	12
2	14.4
3	17.28
4	20.736

$\frac{14.4}{12} = 1.2$ $\quad \frac{17.28}{14.4} = 1.2$

9. Kina earns a \$27,000 salary for the first year of work at her job. She earns annual increases of 2.5%. What is the total amount, to the *nearest cent*, that Kina will earn for the first eight years at this job?

a_1 $\quad r = 1.025$ $\quad S_n$

$a_1 = 27000$

$r = 1.025$

$n = 8$

$$S_n = \frac{a_1 - a_1(r)^n}{1-r}$$

$$S_8 = \frac{27000 - 27000(1.025)^8}{1-1.025}$$

$S_8 = 235875.13$

10. A fisherman harvests 350 kilograms of crab on Monday. From Monday to Friday, the fisherman harvests 8% less kilograms of crab per day. To the *nearest tenth of a kilogram*, what is the total amount of crab harvested between Monday and Friday? $n=5$

$\to .92$ $\quad S_n$

$a_1 = 350$

$r = .92$

$n = 5$

$$S_n = \frac{a_1 - a_1(r)^n}{1-r}$$

$$S_5 = \frac{350 - 350(.92)^5}{1-.92}$$

$S_5 = 1491.5$

11. Kristin wants to increase her running endurance. According to experts, a gradual mileage increase of 10% per week can reduce the risk of injury. If Kristin runs 8 miles in week one, 8.8 miles in week two, and 9.68 miles in week three, which expression can help her find the total number of miles she will have run over the course of her 6-week training program?

1) $\sum_{n=1}^{6} 8(1.10)^{n-1}$

2) $\sum_{n=1}^{6} 8(1.10)^{n}$

3) $\sum_{n=1}^{8} 6(1.1)^{n-1}$

4) $\sum_{n=1}^{6} 8(1.1)^{n}$

$\sum_{n=1}^{K} a_1(r)^{K-1}$

$8, 8.8, 9.68$

$a_1 = 8$

$r = 1.1$

$n = 6$

$\frac{8.8}{8} = 1.1$

$\frac{9.68}{8.8} = 1.1$

$\sum_{n=1}^{6} 8(1.1)^{n-1}$

12. In his first year running track, Brendon earned 8 medals. In the second year, he earned 12 medals and in the third year, he earned 18 medals. Which of the following expressions can be used to determine how many total medals Brendon will have after four years of high school?

1) $\sum_{n=1}^{8} 4(1.5)^{n}$

2) $\sum_{n=1}^{8} 4(1.5)^{n-1}$

3) $\sum_{n=1}^{4} 1.5(4)^{n-1}$

4) $\sum_{n=1}^{4} 8(1.5)^{n-1}$

$a_1 = 8$

$r = 1.5$

$n = 4$

$8, 12, 18$

$\sum_{n=1}^{K} a_1(r)^{K-1}$

$\sum_{n=1}^{4} 8(1.5)^{n-1}$

$\frac{12}{8} = 1.5$

$\frac{18}{12} = 1.5$

10 STO→x

15 STO→y

13. Which expression is equivalent to $2xy^2\sqrt[3]{x^2y}$? 51512...

1) $2x^{\frac{5}{3}}y^{\frac{7}{3}}$ 51512...

2) $2xy$

3) $2x^{\frac{2}{3}}y^{\frac{2}{3}}$

4) $2x^7y^4$

14. Which equation is equivalent to $P = 210x^{\frac{4}{3}}y^{\frac{7}{3}}$ 2510530.87

1) $P = \sqrt[3]{210x^4y^7}$

2) $P = 70xy^2\sqrt[3]{xy}$

3) $P = 210xy^2\sqrt[3]{xy}$ 2510530.87

4) $P = 210xy^2\sqrt[3]{x^3y^5}$

Express in simplest form with a rational exponent:

Radicals are fractional exponents

15. $\sqrt[5]{x^2} \bullet \sqrt{x^3}$

$\frac{2}{5} + \frac{3}{2} = \frac{19}{10}$

$x^{\frac{2}{5}} \cdot x^{\frac{3}{2}}$

$x^{\frac{19}{10}}$

16. $\sqrt[4]{a^7} \bullet \sqrt{a^5}$

$a^{\frac{7}{4}} \cdot a^{\frac{5}{2}}$

$a^{\frac{17}{4}}$

clean it up: multiply (add), divide (subtract), evaluate

$\frac{7}{4} + \frac{5}{2} = \frac{17}{4}$

17. Mr. and Mrs. Jenkins just closed on a new home whose purchase price was \$380,000. At the closing, they supplied a down payment of \$76,000. If on the day of the closing the monthly interest rate was .3125%, determine the Jenkins' monthly mortgage payment, to the *nearest cent*, if they were approved for a 30-year loan.

Use the formula $M = P \bullet \frac{r(1+r)^n}{(1+r)^n - 1}$ where M is the mortgage payment, P is the principal amount of the loan, r is the monthly interest rate, and n is the number of monthly payments.

$M = m$

$P = 380000 - 76000 = 304000$

$r = .003125$

$n = 30(12) = 360$

$$m = 304000 \frac{(.003125(1+.003125)^{360}}{(1+.003125)^{360}}$$

$$m = 1407.87$$

18. Monthly mortgage payments can be found using the formula below:

$$M = \frac{P\left(\frac{r}{12}\right)\left(1+\frac{r}{12}\right)^n}{\left(1+\frac{r}{12}\right)^n - 1}$$

M = monthly payment $= m$

P = amount borrowed $= 120000$

r = annual interest rate $.048$

n = number of monthly payments $15(12) = 180$

The Banks family would like to borrow \$120,000 to purchase a home. They qualified for an annual interest rate of 4.8%. If they plan to spend 15 years to repay the loan, what will be the monthly payment rounded to the *nearest* cent?

$$m = \frac{120000\left(\frac{.048}{12}\right)\left(1+\frac{.048}{12}\right)^{180}}{\left(1+\frac{.048}{12}\right)^{180} - 1}$$

$$m = 936.50$$

Write a recursive formula for each of the following

19. $a_n = 16\left(\frac{1}{2}\right)^{n-1}$ $y=16(\frac{1}{2})^{x-1}$

$\frac{8}{16}=\frac{1}{2}$ 16, 8, 4, ...

$\frac{4}{8}=\frac{1}{2}$

$a_1 = 16$

$a_n = \frac{1}{2}a_{n-1}$

20. $a_n = 3n+6$ $y=3x+6$

9, 12, 15, ...

$12-9=3$

$15-12=3$

$a_1 = 9$

$a_n = a_{n-1} + 3$

21. Write an expression in summation form to find the sum of the first n terms of the sequence
$11 \overset{+7}{+} 18 \overset{+7}{+} 25 \overset{+7}{+} 32 + \ldots$
Use your formula to find the sum of the first three terms.

$n=3$
$a_1=11$
$a_n=25$

$S_n = \frac{n(a_1+a_n)}{2}$

$S_3 = \frac{3(11+25)}{2}$

$S_3 = 54$

$a_n = a_1 + d(n-1)$
$a_3 = 11+7(3-1)$
$a_3 = 25$

22. Write an expression in summation form to find the sum of the first n terms of the sequence
$8 \overset{-2}{+} 6 \overset{-2}{+} 4 \overset{-2}{+} 2 + \ldots$
Use your formula to find the sum of the first three terms.

$n=3$
$a_1=8$
$a_n=4$

$S_n = \frac{n(a_1+a_n)}{2}$

$S_3 = \frac{3(8+4)}{2}$

$S_3 = 18$

$a_n = a_1 + d(n-1)$
$a_3 = 8-2(3-1)$
$a_3 = 4$

Recursive: $a_1 =$
$a_n = a_{n-1}$

Algebra II Reference Sheet (NGLS)

Quadratic Formula	$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Trigonometric Identities	$\sin^2(\theta) + \cos^2(\theta) = 1$ $\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$ $\cot(\theta) = \dfrac{\cos(\theta)}{\sin(\theta)}$ $\csc(\theta) = \dfrac{1}{\sin(\theta)}$ $\sec(\theta) = \dfrac{1}{\cos(\theta)}$
Cubic Factorizations	$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
Probability	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $P(A\|B) = \dfrac{P(A \cap B)}{P(B)}$
Independence	$P(A \cap B) = P(A) \cdot P(B)$ $P(A\|B) = P(A)$

Arithmetic Sequence	$a_n = a_1 + d(n-1)$
Arithmetic Series	$S_n = \dfrac{n(a_1 + a_n)}{2}$ n $S_n = \sum_{k=1}^{n} a_1 + d(k-1)$
Geometric Sequence	$a_n = a_1 r^{n-1}$
Geometric Series	$S_n = \dfrac{a_1(1-r^n)}{1-r}, r \neq 1$ $S_n = \sum_{k=1}^{n} a_1 r^{k-1}, r \neq 1$
Exponential Growth and Decay	$A = P\left(1 + \dfrac{r}{n}\right)^{nt}$ $A = Pe^{rt}$ $A = A_0\left(\dfrac{1}{2}\right)^{\frac{t}{h}}$

Normal Curve