- Isolate 1 variable in at least 1 equation
- Substitute one equation into the other
- Solve equation
- Substitute into 1 of original equations to find other variable

Name Schlansky
Mr. Schlansky

Date _____
Algebra II

Solving Non-Linear Systems Algebraically

Solve each of the following systems of equations for all values of x and y

1. $y = -2x + 1$
   $y = -2x^2 + 3x + 1$

$-2x+1 = -2x^2+3x+1$
$+2x^2-3x-1 \quad +2x^2-3x-1$
$2x^2-5x=0$
$x(2x-5)=0$
$x=0 \mid 2x-5=0$
$+5 \quad +5$
$\frac{2x}{2}=\frac{5}{2}$
$x=\frac{5}{2}$

2. $y = x^2 - 4x + 3$
   $y + 1 = x$
   $-1 \quad -1$
   $y = x - 1$

$x=4$ $\quad x=1$
$y=x-1$ $\quad y=x-1$
$y=4-1$ $\quad y=1-1$
$y=3$ $\quad y=0$
$(4,3)$ $\quad (1,0)$

$x+1 = x^2-4x+3$
$-x+1 \quad -x+1$
$0 = x^2-5x+4$
$0=(x-4)(x-1)$
$x=4 \quad x=1$

3. $x^2 - y = 5$
   $y = 3x - 1$

$x^2-y=5$
$+y \quad +y$
$x^2=y+5$
$-5 \quad -5$
$x^2-5=y$

$x^2-5=3x-1$
$-3x+1 \quad -3x+1$
$x^2-3x-4=0$
$(x-4)(x+1)=0$
$x=4 \mid x=-1$

$x=4$: $y=3x-1$, $y=3(4)-1$, $y=11$, $(4,11)$
$x=-1$: $y=3(-1)-1$, $y=-4$, $(-1,-4)$

4. $5 = y - x$
   $4x^2 = -17x + y + 4$

$5=y-x$ $\quad 4x^2=-17x+y+4$
$+x \quad +x$ $\quad +17x-4 \quad +17x \quad -4$
$x+5=y$ $\quad 4x^2+17x-4=y$

$x+5 = 4x^2+17x-4$
$-x-5 \quad -x-5$
$0=4x^2+16x-9$ PT
$x^2+16x-36$
$(x+18)(x-2)$
$(x+\frac{9}{2})(x-\frac{1}{2})$

$(2x+9)(2x-1)=0$
$2x+9=0 \mid 2x-1=0$
$-9 \quad -9 \quad +1 \quad +1$
$\frac{2x}{2}=\frac{-9}{2} \quad \frac{2x}{2}=\frac{1}{2}$
$x=-4.5 \quad x=.5$
$y=x+5 \quad y=x+5$
$y=-4.5+5 \quad y=.5+5$
$y=.5 \quad y=5.5$
$(-4.5,.5) \quad (.5,5.5)$

5. $y^2 - x^2 + 32 = 0$
   $3y - x = 0$
   $+x \quad +x$
   $3y = x$

$y^2-(3y)^2+32=0$
$y^2-9y^2+32=0$
$\frac{-8y^2}{-8}+\frac{32}{-8}=\frac{0}{-8}$
$y^2-4=0$
$(y+2)(y-2)=0$
$y=-2 \mid y=2$

$y=-2$: $x=3y$, $x=3(-2)$, $x=-6$, $(-6,-2)$
$y=2$: $x=3y$, $x=3(2)$, $x=6$, $(6,2)$

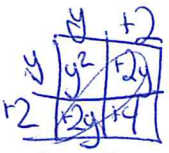
6. $x^2 + y^2 = 10$
   $x = y - 4$

$(y-4)^2+y^2=10$
$y^2-8y+16+y^2=10$
$2y^2-8y+16=10$
$-10 \quad -10$
$\frac{2y^2}{2}-\frac{8y}{2}+\frac{6}{2}=\frac{0}{2}$
$y^2-4y+3=0$
$(y-3)(y-1)=0$
$y=3 \mid y=1$

$y=3$: $x=y-4$, $x=3-4$, $x=-1$, $(-1,3)$
$y=1$: $x=y-4$, $x=1-4$, $x=-3$, $(1,-3)$

	y	-4
y	y^2	$-4y$
-4	$-4y$	$+16$

$y^2-8y+16$



y^2+4y+4

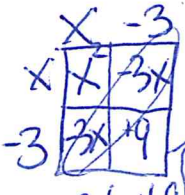
7. $x^2 + y^2 = 2$
$y + 2 = x$

$(y+2)^2+y^2=2$
$y^2+4y+4+y^2=2$
$2y^2+4y+4=2$
$-2\quad -2$
$2y^2+4y+2=0$
$\frac{2}{2}\quad\frac{2}{2}\quad\frac{2}{2}\quad\frac{0}{2}$

$y^2+2y+1=0$
$(y+1)(y+1)=0$
$y=-1 \mid y=-1$

$x=y+2$
$x=-1+2$
$x=1$
$(1,-1)$

8. $3x^2 + y^2 = 13$
$y = x - 3$



$x-3=0 \quad 2x+1=0$
$+3 \quad\quad -1 \quad -1$
$x=3 \quad \frac{2x}{2}=\frac{-1}{2}$
$x=-\frac{1}{2}$

$3x^2+(x-3)^2=13$
$3x^2+x^2-6x+9=13$
$4x^2-6x+9=13$
$-13\quad -13$
$\frac{4x^2-6x-4}{2}=\frac{0}{2}$

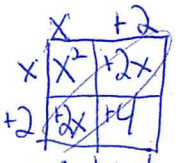
x^2-6x+9

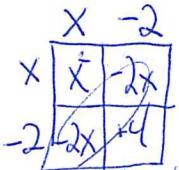
PT
$2x^2-3x-2=0$
$x^2-3x-4=0$
$(x-4)(x+1)=0$
$(x-2)(2x+1)$

$y=x-3$
$y=4-3$
$y=1$
$(4,1)$

$y=x-3$
$y=-\frac{1}{2}-3$
$y=-\frac{7}{2}$
$(-\frac{1}{2},-\frac{7}{2})$

9. $(x+2)^2 + (y-4)^2 = 40$
$y = x + 2$





$(x+2)^2+(x+2-4)^2=40$
$(x+2)^2+(x-2)^2=40$
$x^2+4x+4+x^2-4x+4=40$
$2x^2+8=40$
$-40\quad -40$
$\frac{2x^2-32}{2}=\frac{0}{2}$
$x^2-16=0$

x^2+4x+4
x^2-4x+4

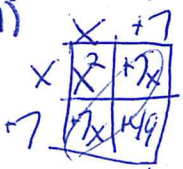
$(x+4)(x-4)=0$
$x=-4 \mid x=4$

$y=x+2$
$y=-4+2$
$y=-2$
$(-4,-2)$

$y=x+2$
$y=4+2$
$y=6$
$(4,6)$

10. $x^2 + (y+4)^2 = 109$
$y = x + 3$

$x^2+(x+3+4)^2=109$
$x^2+(x+7)^2=109$
$x^2+x^2+14x+49=109$
$2x^2+14x+49=109$
$-109\quad -109$
$\frac{2x^2+14x-60}{2}=\frac{0}{2}$



$x^2+14x+49$

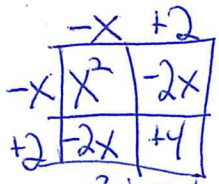
$x^2+7x-30=0$
$(x+10)(x-3)=0$
$x=-10 \mid x=3$

$y=x+3$
$y=-10+3$
$y=-7$
$(-10,-7)$

$y=x+3$
$y=3+3$
$y=6$
$(3,6)$

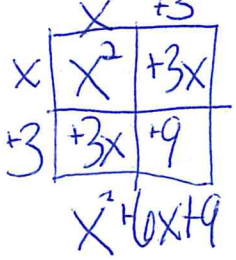
11. $x + y = 5$
$-x \quad -x$
$(x+3)^2 + (y-3)^2 = 53$

$y=5-x$



$(x+3)^2+(5-x-3)^2=53$
$(x+3)^2+(-x+2)^2=53$
$x^2+6x+9+x^2-4x+4=53$
$2x^2+2x+13=53$
$-53\quad -53$
$\frac{2x^2+2x-40}{2}=\frac{0}{2}$
$x^2+x-20=0$
$(x+5)(x-4)=0$
$x=-5 \mid x=4$

x^2-4x+4

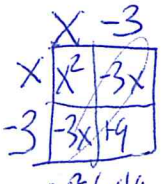


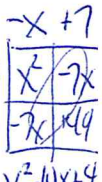
x^2+6x+9

$x=-5$
$y=5-x$
$y=5--5$
$y=10$
$(-5,10)$

$x=4$
$y=5-x$
$y=5-4$
$y=1$
$(4,1)$

12. $(x-3)^2 + (y+2)^2 = 16$
$2x + 2y = 10$
$-2x \quad -2x$
$\frac{2y}{2}=\frac{-2x+10}{2}$
$y=-x+5$





$(x-3)^2+(-x+5+2)^2=16$
$(x-3)^2+(-x+7)^2=16$
$x^2-6x+9+x^2-14x+49=16$
$2x^2-20x+58=16$
$-16\quad -16$
$\frac{2x^2-20x+42}{2}=\frac{0}{2}$
$x^2-10x+21=0$
$(x-7)(x-3)=0$
$x=7 \mid x=3$

x^2-6x+9
$x^2-14x+49$

$x=7$
$y=-x+5$
$y=-7+5$
$y=-2$
$(7,-2)$

$x=3$
$y=-x+5$
$y=-3+5$
$y=2$
$(3,2)$